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# The Transformation of a Handbook into Tables: The *Brahmatulyasāraṇī* and the *Karaṇakutūhala* of Bhāskara

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## I Introduction

*Brahmatulyasāraṇī* is the name most often given to a set of tables (Sanskrit *sāraṇī/sāriṇī*, *koṣṭhaka*) based on Bhāskara II's astronomical handbook *Karaṇakutūhala* or *Brahmatulya* (epoch 1183 CE), which in turn is a condensed and simplified adaptation of the same author's treatise *Siddhāntaśiromaṇi*. The name *Brahmatulya* means "equal or corresponding to the Brāhma," i.e., the Brāhmapakṣa school of astronomy adhered to by Bhāskara II, which follows the parameters of the *Brāhmasphuṭasiddhānta* of Brahmagupta (628 CE). The *Brahmatulyasāraṇī* tables record Brāhmapakṣa-derived values of planetary mean motions with orbital and geographical corrections for computing their true motions for a given terrestrial location, topics which are addressed in chapters 1–2 of the *Karaṇakutūhala*.

There are at least five extant manuscripts of the tables of the *Brahmatulyasāraṇī*, some with occasional expository details in table headers and marginal notes. A brief description of their contents has been published by Pingree [1968, 36–37] based on the manuscripts described in tables 1–4 below; we have used also the so-called *Karaṇakutūhala-sāriṇī* in BORI 501/1895–1902. A critical edition of the tables based on these five manuscripts is currently in preparation.

One of the *Brahmatulyasāraṇī* manuscripts also contains (S29, ff. 6r–6v) the only currently known copy of ten verses explaining the use of the tables, plus a colophon and two post-colophon verses on astrological matters; it was copied by an otherwise unknown scribe named Malūkacandra. It consists of an invocation and prescribed algorithms for accomplishing fundamental tasks of astronomy: computing planetary mean longitudes measured along the ecliptic for a given date, correcting them for orbital anomalies reckoned from the so-called *manda* and *śīghra* apogees (see the technical analysis for verses 4–7) while interpolating linearly between tabulated values, and reducing arcs to the appropriate trigonometric quadrant. A few different verse meters are used, primarily *śārdūlavikrīḍita* and *upajāti*: several of the verses draw on corresponding text in *Karaṇakutūhala* chapter 2 for their style and/or content.

In the present paper, we provide a critical edition with transcription, translation, and technical commentary for this very terse explanatory text.

## I.1 The Relationship of the *Brahmatulyasāraṇī* and the *Karaṇakutūhala*

There are a few known instances of second-millennium *karaṇa* works giving rise to eponymous table texts using the same epoch date, composed either by the *karaṇa*'s author or by a later compiler.[1] The *Brahmatulyasāraṇī*, although it does not explicitly mention the *Karaṇakutūhala* or its author Bhāskara, conforms closely to this pattern. The name *Brahmatulya* is well attested as one of the alternate titles of the *Karaṇakutūhala* [Pingree 1970–94, A4, 322], and as noted above, one of the known *Brahmatulyasāraṇī* manuscripts is actually titled *Karaṇakutūhala-sāriṇī*. The evident (though not entire) reliance of the *Brahmatulyasāraṇī* on the *Karaṇakutūhala*'s epoch date (discussed in section I.2), and its allusion in verse 2 to an unspecified algorithm "stated in the handbook" (*karaṇokta*), corroborate the inference that this *koṣṭhaka* work is largely derived from an earlier *karaṇa* and that the *karaṇa* in question is the *Karaṇakutūhala*. Close resemblances between the two texts in the content and phrasing of some verses and table headers (described in detail in section II) further confirm this conclusion.

Other than these examinations of some technical details concerning computational methods and textual borrowings (see also Montelle [2013]), we know almost nothing about the historical context of the conversion process from *karaṇa* to *sāraṇī*, i.e., when, where and by whom the *Brahmatulyasāraṇī* was compiled as a separate work. The possible identity of its author with one Nāgadatta to whom is attributed a *Karaṇakutūhala-gata-sāraṇī* has been suggested, but not yet investigated [Pingree 1970–94, A5, 166].

## I.2 A Description of the Sources

The contents and organization of each of the five known manuscripts of the *Brahmatulyasāraṇī* are outlined in tables 1–5; the quantities they refer to are discussed in section II. (Note that the abbreviation 20YP stands for "20-year-periods" and a celestial longitude in the form $a^s\, b^\circ$ represents $a$ zodiacal signs of 30° each plus $b$ degrees within a sign.)

The manuscripts' table data contain a few scraps of indirect evidence bearing on the date of their compilation. In most of the mean motion tables, for instance, initial mean positions correspond to the *Karaṇakutūhala*'s 1183 epoch. However,

[1] E.g., the *karaṇa* and *koṣṭhaka* both entitled *Rāmavinoda*, composed by Rāma in the late sixteenth century, and the numerous versions of *Grahalāghava-sāraṇī* based on the 1520 *Grahalāghava* of Gaṇeśa [Pingree 1981, 37–43].

| Folio No. | Contents |
|---|---|
| 1r-2v | The longitude of the moon and its anomaly (argument 0–360°) |
| 2v-3r | Mean motion of the moon per *ghaṭikā* (argument 1–60 *ghaṭikās*; no title) |
| 3r-6r | Lunar *manda*-equation, differences, and *gatiphala* (argument $0^s0°$–$11^s29°$) |
| 6r-6v | Instructional verses 1–10 |

Table 1: MS S29, Poleman 4952 (Smith Indic 29)

| Folio No. | Contents |
|---|---|
| 1r | Solar declination and lunar latitude (argument 1–90°) |
| 1r-1v | Correction for Mars' apogee (argument 1–45°) |
| 1v-2r | Solar *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 2r- 2v | Lunar *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 2v-3r | Mars *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 3r-3v | Mercury *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 4r-4v | Jupiter *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 4v-5r | Venus *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 5r-5v | Saturn *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 6r-7v | Mars *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |
| 8r- 9r | Mercury *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |
| 9v-10v | Jupiter *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |
| 11r-12r | Venus *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |
| 12v-13v | Saturn *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |

Table 2: MS S43, Poleman 4876 (Smith Indic 43)

some manuscript details relating to annual mean longitude corrections (*abdabīja* and *rāmabīja*; see the technical analysis for verse 3) suggest that their tables were designed for users several centuries after 1183. In the first place, the oldest known allusion to *rāmabīja* corrections occurs no earlier than 1519 [Pingree 1996, 169]. Furthermore, some *abdabīja* values for Venus recorded in a table header in MS S45 f. 5v are assigned to increments of 432, 441, and 450 years. Assuming these refer to years elapsed since the 1183 epoch, we should infer a compilation date in the range 1615–1633.

In addition, MS S45's version of the 20-year mean motion tables applies an initial-position adjustment corresponding to a date as much as 600 years after 1183 (see the technical analysis for verse 2). These variations from one manuscript to another

| Folio No. | Contents |
|---|---|
| 1r | Mean motions of the sun in days, months, years, and 20YP |
| 1v | Mean motions of the moon in days, months, years, and 20YP |
| 2r | Mean motions of the lunar apogee in days, months, years, and 20YP |
| 2v | Mean motions of the node in days, months, years, and 20YP |
| 3r | Mean motions of Mars in days, months, years, and 20YP |
| 3v | Mean motions of Mercury's *śīghra*-apogee in days, months, years, and 20YP |
| 4r | Mean motions of Jupiter in days, months, years, and 20YP |
| 4v | Mean motions of Venus' *śīghra*-apogee in days, months, years, and 20YP |
| 5r | Mean motions of Saturn in days, months, years, and 20YP |
| 5v | Solar declination |
| 5v | Lunar latitude |

Table 3: MS SMB, Poleman 4946 (Smith Indic MB LVIII)

suggest that at least some of their author-scribes saw fit to adjust crucial data for their particular circumstances, although the basic structure and use of the tables remained unchanged.

Two of the manuscripts mention their date of copying in colophons, although neither is much help in pinpointing the work's composition date. In MS S45 (f. 17v margin):

*saṃvat 1855 varṣe śāke 1720 pravarttamāni kārttikavid 11 some*

Śaka 1720, or Saṃvat 1855, Kārttika śuklapakṣa 11 corresponds to the date 19 November 1798, which was indeed a Monday or *somavāra*, as the scribe asserts. MS B has (f. 28v):

*saṃvat 1734 || varṣe kātī sudī 2 budhavāre pothīlaṣītaṃ caraṃbagasu ||*

Saṃvat 1734 Kārttika śuklapakṣa 2 corresponds to 28 October 1677 CE which was in fact a Thursday and not, *pace* the scribe, a *budhavāra* or Wednesday. The closing phrase may indicate that the manuscript (*pothī* in various northern Indian vernaculars) was copied in a location called something like "Rambag" (possibly Ram Bagh near Agra?), but the interpretation is very tentative.

| Folio No. | Contents |
|---|---|
| 2r | Mean motions of the sun in days, months, years, and 20YP |
| 2v | Mean motions of the moon in days, months, years, and 20YP |
| 3r | Mean motions of the lunar apogee in days, months, years, and 20YP |
| 3v | Mean motions of the node in days, months, years, and 20YP |
| 4r | Mean motions of Mars in days, months, years, and 20YP |
| 4v | Mean motions of Mercury's *śīghra*-apogee in days, months, years, and 20YP |
| 5r | Mean motions of Jupiter in days, months, years, and 20YP |
| 5v | Mean motions of Venus' *śīghra*-apogee in days, months, years, and 20YP |
| 6r | Mean motions of Saturn in days, months, years, and 20YP |
| 6v | Solar *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 7r | Lunar *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 7v | Solar declination and lunar latitude (argument 2–90°, every second degree) |
| 7v | Correction for Mars' apogee (argument 1–45°) |
| 8r | Mars *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 8v-9v | Mars *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |
| 10r | Mercury *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 10v-11v | Mercury *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |
| 12r | Jupiter *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 12v-13v | Jupiter *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |
| 14r | Venus *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 14v-15v | Venus *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |
| 16r | Saturn *manda*-equation, differences, and *gatiphala* (argument 1–90°) |
| 16v-17v | Saturn *śīghra*-equation, differences, and hypotenuse (argument 1–180°) |

Table 4: MS S45, Poleman 4735 (Smith Indic 45)

## I.3 Typographic Conventions

In the edited text as well as in the transliteration, translation and commentary we employ the following editorial conventions (see also Montelle and Plofker [2013]):

- Square brackets [ ] indicate an editorial addition or proposed reconstruction of missing text.
- Scribal variants of *nāgarī* orthography which are emended silently and not noted in the critical apparatus (except where the meaning of the original read-

| Folio No. | Contents |
|---|---|
| 2r-2v | Mean motions of the sun in days, months, years, and 20YP |
| 2v-3v | Mean motions of the moon in days, months, years, and 20YP |
| 3v-4v | Mean motions of the lunar apogee in days, months, years, and 20YP |
| 4v-5v | Mean motions of the node in days, months, years, and 20YP |
| 5v-6v | Mean motions of Mars in days, months, years, and 20YP |
| 6v-8v | Mean motions of Mercury's *śīghra*-apogee in days, months, years, and 20YP |
| 8v-9r | Mean motions of Jupiter in days, months, years, and 20YP |
| 9r-10r | Mean motions of Venus' *śīghra*-apogee in days, months, years, and 20YP |
| 10r-11r | Mean motions of Saturn in days, months, years, and 20YP |
| 11r-12r | Solar *manda*-equation and *gatiphala* (argument 1–90°) |
| 12r-13r | Lunar *manda*-equation and *gatiphala* (argument 1–90°) |
| 13r | Correction for Mars' apogee (argument 1–45°) |
| 13v -14v | Mars *manda*-equation and *gatiphala* (argument 1–90°) |
| 14v-16v | Mars *śīghra*-equation and hypotenuse (argument 1–180°) |
| 16v -17v | Mercury *manda*-equation and *gatiphala* (argument 1–90°) |
| 17v-19v | Mercury *śīghra*-equation and hypotenuse (argument 1–180°) |
| 19v-20v | Jupiter *manda*-equation and *gatiphala* (argument 1–90°) |
| 20v-22v | Jupiter *śīghra*-equation and hypotenuse (argument 1–180°) |
| 22v-23v | Venus *manda*-equation and *gatiphala* (argument 1–90°) |
| 23v-25v | Venus *śīghra*-equation and hypotenuse (argument 1–180°) |
| 26r-26v | Saturn *manda*-equation and *gatiphala* (argument 1–90°) |
| 27r-28v | Saturn *śīghra*-equation and hypotenuse (argument 1–180°) |

Table 5: MS B, BORI 501/1895–1902

ing may be ambiguous) include the following: *anusvāra* used for a nasal consonant or an incorrect nasal substituted, omitted *visarga*, *virāma* or *avagraha*, misplaced *daṇḍas*, reversed conjunct consonants (e.g., *adba* for *abda*), conjunct consonants that we cannot reproduce in our *nāgarī* typesetting, doubled consonants after *r* or across a *pāda* break, and routinely confused consonant pairs (e.g., *ba* for *va*, *ṣa* for *kha*).

- Fragments of Sanskrit words or compounds in *nāgarī* are indicated with a small circle ∘ at the breakpoint.
- Folio breaks are indicated by a single vertical stroke |.
- In the critical apparatus, text followed by a single square close-bracket ] indicates the edited version of the manuscript reading that follows it.
- The symbol × within *nāgarī* text indicates an *akṣara* (syllable) that is too illegible or indefinite in the manuscript to reconstruct confidently; in square

brackets, it signifies a missing *akṣara* (in a metrically deficient verse).

- Numerals in sexagesimal or base-60 notation are shown with a semicolon separating their integer and fractional parts, and commas separating their successive sexagesimal digits. The superscripts $^s$ and $^\circ$ and $'$ indicate zodiacal signs (i.e., 30-degree arcs of longitude), degrees and minutes of arc, respectively.

# II Text and Translation

oṃ || śrīgaṇeśāya namaḥ ||

OṂ. Homage to Lord Gaṇeśa.

### Verse One: Invocation

natvā vallabhanandanaṃ tadanugopālāṃhripadmadvayaṃ
jñātvā śrīguruvākyato hy aharniśaṃ [dṛṣṭvā] dyum evādhunā ||
siddhānteṣu yathoktakhecaravidhi[bhya]ḥ spaṣṭakoṣṭaṃ muhur
madhyaspaṣṭavibhāgato grahagaṇāt kurve dinaughād aham || 1 ||

Saluting Vallabhanandana and after him the two lotus feet of Gopāla, having learned from the word of the revered teacher and having observed the heavens themselves by day and night, now I shall compute an accurate set of tables from the rules of the planets as spoken in the Siddhāntas, separately for mean and true [quantities], for the various planets, from the accumulated days.

*Verse Analysis*

Meter: *śārdūlavikrīḍita*.

Gopāla is a well-known epithet for Kṛṣṇa, but we cannot identify more precisely the deity referred to as Vallabhanandana. The word *dṛṣṭvā* "having seen" or "having observed" is speculatively suggested for the defective second *pāda* (quarter-verse) to preserve the meter and the sense.

### Verse Two: Computing the Mean Longitudes for a Given Date

kṛtvādau karaṇoktavāsaragaṇaṃ śiṣṭaiḥ suhṛṣṭātmabhir
bhājyaṃ khāgni 30 mitair avāptakam idaṃ sūryair 12 vibhājyaṃ punaḥ ||
labdhaṃ viṃśati 20 bhir bhajed atha catuḥśeṣāṅkasaṃjñā dhruvaṃ
aṅkās te militāḥ svakoṣṭakagatā laṅkānagaryāṃ khagāḥ || 2 ||

Firstly, computing the number of accumulated days as stated in the handbook, the learned who are cheerful in nature are to divide [it] by the amount 30; again this quotient should be divided by 12; one should divide the result by 20. Now, precisely these numbers called the four remainder-numbers, [entered into] their respective tables [with the corresponding entries] combined, are the [mean longitudes of the] planets at the city of Laṅkā [i.e., for zero degrees of terrestrial longitude].

*Verse Analysis*

Meter: *śārdūlavikrīḍita*.

*Technical Analysis*

This verse explains how to manipulate the entries in a set of four mean motion tables provided by the *Brahmatulyasāraṇī* for each of the specified celestial bodies to reckon up the body's mean longitude at a desired time. (MS SMB's version of these four tables for the sun is shown in Figure 1 to illustrate the arrangement.) First, the user must know the *ahargaṇa* or number of civil days elapsed since the tables' epoch date. Dividing this *ahargaṇa* by 30 produces an integer number of completed (ideal) "months" of 30 days each and a remainder $D$ in days. That number of "months" divided by 12 in turn yields an integer number of ideal "years" of 360 days each and a remainder $M$ in "months." The number of "years" divided by 20 gives the number $T$ of elapsed 20-"year" periods and a remainder $Y$ in "years."

The first of the four mean motion tables contains successive multiples from 1 through 30 of the body's mean daily motion, i.e., the amount of change in its mean longitude over the corresponding number of days. The next three tabulate similar longitude increments for 1 through 12 ideal 30-day "months," 1 through 20 ideal 360-day "years," and 1 through 30 successive 20-"year" periods. The values in this last table include an epoch correction computed for 20 (ideal) "years" after the epoch date of the *Karaṇakutūhala*: i.e., the body's epoch longitude as given in *Karaṇakutūhala* 1.4–6 and 13, plus an amount equal to its mean daily motion multiplied by $360 \times 20$ [Mishra 1991, 5, 11; Rao and Uma 2008, S4, S12]. Thus, after entering into each of the appropriate mean motion tables with $D$, $M$, $Y$ and $T$, the user simply adds up the four corresponding table entries (modulo 360°) to get the mean longitude for the body in question since epoch.

In one manuscript (MS S45) the argument values in each planet's 20-"year"-periods table are numbered 31–59 rather than 1–30, and its entries incorporate an epoch correction equal to the *Karaṇakutūhala* epoch longitude plus the mean daily motion multiplied by $360 \times 20 \times 30$. The apparent implication is that the tables were expected (at least by the scribe of MS S45) to be used beginning at some time nearly 600 years *after* the *Karaṇakutūhala* epoch date.

It is rather striking that the *Brahmatulyasāraṇī* procedure demands an *ahargaṇa* already converted from a date in actual Indian calendar units, such as synodic months, thirtieths of a synodic month (*tithis*), and luni-solar years, to total civil days. Evidently the conversion procedure "stated in the handbook" (presumably

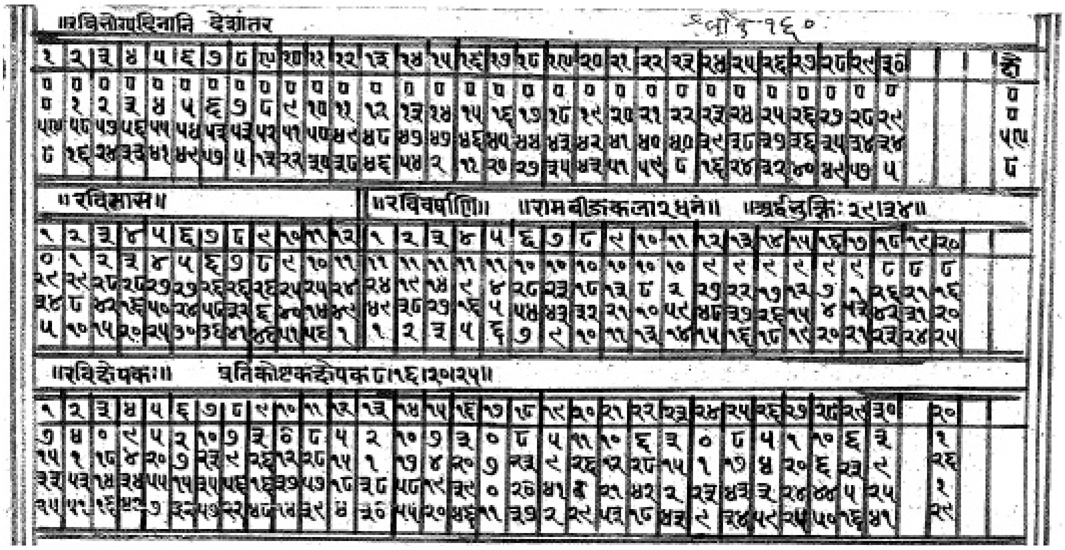

**Figure 1**: Tables of increments to the mean position of the sun for a given number of days, "months," "years," and 20-"year" periods (entitled *ravikṣepakāḥ* "Additive (positions) of the sun") elapsed since the *Karaṇakutūhala*'s epoch (MS SMB f. 1r).

*Karaṇakutūhala* 1.2–3 [Mishra 1991, 2; Rao and Uma 2008, S1]) can simply be prescribed rather than explained. The resulting total must then be apportioned into the appropriate multiples of 7200, 360, 30 and single civil days for convenience in computing the corresponding mean motion increments. These mean motion tables with their idealized "months" and "years" in round numbers of civil days are in fact more reminiscent of some Islamic *zīj* calendar conversion tables than of the standard *ahargaṇa* algorithms in Sanskrit texts.[2]

## Verse Three: Mean Planetary Positions and their Corrections

madhyāḥ svadeśīyakhagā bhaveyur
deśāntareṇābda x [x] x rāma- ||
bījena yuktā gaṇakais tataś ca
spaṣṭāḥ kriyante phalayugmakena || 3 ||

[2] The *Brahmatulyasāraṇī*'s allotment of the standard integer numbers of civil days in the "month" and the "year" resembles in particular the tradition of *zījes* using Persian and/or Ptolemaic years in blocks of 20, such as those of al-Manṣūr and al-Battānī. Other *zījes* including those of al-Hāsib and al-Khāzinī commonly use *hijra* calendar months and blocks of 30 (*hijra*) years; see, e.g., [Kennedy 1956, 145–167].

The mean [longitudes] should become [longitudes of] the planets for one's own locality [when] adjusted by the longitude-correction [and] by the *rāmabīja* [correction] with the annual [correction?]. And from [those], the true [longitudes] are made by the calculators by means of the two equations [i.e., *manda* and *śīghra* equations].

*Verse Analysis*

Meter: *upajāti.*

The manuscript appears to have for the second *pāda* of the verse the metrically and logically deficient *deśāntareṇāprabhatharāma* (see Figure 2), which we hypothesize originally enumerated the *deśāntara*, *abdabīja* and *rāmabīja* corrections. But we cannot fully reconstruct the allusion to the *abdabīja* (one possibility would be *deśāntareṇābdaviliptarāma*).

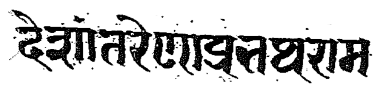

**Figure 2**: Manuscript rendering of verse 3, *pāda* 2 (MS S29 f. 6r).

*Technical Analysis*

This verse describes various modifications to the computed mean positions. The first adjusts them from the default locality at the notional Indian zero-point of latitude and longitude, i.e., the ideal position of Laṅkā at the intersection of the equator and the prime meridian, to the terrestrial longitude of one's own locality by means of the so-called *deśāntara* or longitudinal difference correction. In *Karaṇakutūhala* 1.14–15, Bhāskara declares the *deśāntara* to be this longitudinal difference measured in *yojanas*, multiplied by the daily motion of the planet in question in arcminutes (*kalās*) per day and divided by 80 [Mishra 1991, 12; Rao and Uma 2008, S13]. (The factor of 80 can be explained as follows: The circumference of the earth is taken to be 4800 *yojanas*, which are passed over in each revolution of the celestial equator during one day or 60 *ghaṭikās* [Plofker forthcoming, ch. 1]. Therefore, the revolution takes $\frac{60}{4800} = \frac{1}{80}$ *ghaṭikās* per *yojana* of terrestrial longitude at the equator.) The result is measured in *vikalās* or arcseconds and is to be applied positively or negatively at longitudes west or east of the prime meridian, respectively.

As we reconstruct it, the *Brahmatulyasāraṇī* here mentions a second correction also discussed by Bhāskara (*Karaṇakutūhala* 1.16 [Mishra 1991, 13; Rao and Uma 2008, S13]), the *abdabīja* (literally "yearly correction"). This serves merely to correct computational inaccuracies in the standard mean motion values based on the more precise long-period parameters of Brāhmapakṣa astronomy [Plofker forthcoming, ch. 1].

No *abdabīja* is applied to the mean sun, Jupiter or Saturn. For each of the other bodies, the number of years elapsed since the epoch (*gatābda*) is divided by an appropriate integer to produce a correction in seconds of arc.

The derivation and purpose of *rāmabīja* corrections to the epoch mean longitudes are still not fully explicated. They appear to be a post-Bhāskara innovation attributed to one Rāma, and are attested in slightly different form in another authority named Rāmacandra [Pingree 1996, 168–171].

## Verse Four: Interpolation Procedure

kendrasya doraṃśamitiś ca koṣṭe
bhuktaṃ tadagraṃ parabhogyakaṃ ca ||
kalādikaṃ tadvivarāhataṃ tu
ṣaṣṭyuddhṛtaṃ bhuktakamānakena || 4 ||

The amount in degrees of the arc of the [desired] anomaly (*kendra*) is [entered] in the table. [The table entry for the degree] before that is the "elapsed" (*bhukta*) and [then] the following "future" (*bhogya*). The minutes etc. [of the argument] are multiplied by the difference of those [i.e., the two table entries] and divided by sixty, [and the result increased] by the amount of the "elapsed."

### *Verse Analysis*

Meter: *upajāti*.

### *Technical Analysis*

This verse appears to be nothing more than an explanation of linear interpolation between two values of a function tabulated for each successive integer degree of its argument. The argument in the remaining tables of the *Brahmatulyasāraṇī* is typically the planet's orbital displacement or "anomaly" (*kendra*; see the technical analysis for the following verse).

Two standard technical terms relating to interpolation are introduced: the *bhukta* or "past" value refers to the table entry for the integer degree immediately preceding the desired argument, and the *bhogya* or "future" value to the entry for the degree immediately after it.[3] The concise instructions prescribe scaling the fractional difference between the desired argument value and the next lower integer degree by the difference between the two neighbouring table entries to give the required increment

[3] A similar verse using the same two terms for the differences between a given argument value and the two table entries surrounding it is used to explain linear interpolation within a crude sine table in *Karaṇakutūhala* 2.6 [Mishra 1991, 21; Rao and Uma 2008, S17–S18].

for the interpolated function value:

$$\text{desired value } = bhukta + \frac{bhogya - bhukta}{60} \cdot (\text{fractional part of desired argument})$$

## Verse Five: Application of the *manda*-Correction

yuktaṃ bhaven mandaphalaṃ grahāṇāṃ
svarṇaṃ kramān meṣatulādikendre ||
grahasya bhuktir vivarāhataṃ ca
ṣaṣṭyuddhṛtaṃ kendravaśād dhanarṇam || 5 ||

The *manda*-equation (*mandaphala*) of the planets should be applied positively or negatively [to the mean longitude of the planet] when the anomaly (*kendra*) is in [the semicircle] beginning with Aries or Libra respectively [i.e., when the anomaly is between 0 and 180 or between 180 and 360 degrees]. The velocity of a planet [is modified by the *manda*-correction as follows: the fractional part of the desired value of anomaly], multiplied by the difference [between successive entries in the table of velocity-correction (*gatiphala*)] and divided by sixty, [is the increment to the appropriate tabulated *gatiphala* entry. The resulting *gatiphala* is applied to the mean daily velocity] positively or negatively according to [whether] the anomaly [is in quadrants II and III or quadrants IV and I respectively].

*Verse Analysis*

Meter: *upajāti*.

*Technical Analysis*

In order to determine the true longitudes of the planets, their mean longitudes need to be adjusted for the inequalities of their orbits. The *manda* "slow" and *śīghra* "fast" equations mentioned in verse 3 are used to correct the mean position of a planet to its true one based on its anomaly or angular displacement in longitude from the direction of the corresponding apogee. The sun and moon have only one anomaly each and thus are not *śīghra*-corrected.

The present verse describes corrections due to the *manda*-anomaly, i.e., the difference between the mean position of the planet $\bar{\lambda}$ and that of its *manda*-apogee $\lambda_{A_M}$. This corresponds to the assumption, in an eccentric geocentric orbital model such as the one illustrated in Figure 3, that the orbiting body is moving with uniform velocity upon a circle whose center is displaced from the earth by an amount of eccentricity $r_M$. This displacement produces changes in speed and position over the course of the body's revolution that are qualitatively similar to the effect of an elliptical orbit with the earth at one focus. The *manda*-equation $\mu$ (*mandaphala*) is the displacement in ecliptic longitude from the planet's mean position resulting from the *manda*-anomaly $\kappa_M = \lambda_{A_M} - \bar{\lambda}$.

As shown by the diagram on the left in Figure 3, the correction $\mu$ (represented by the angle $\angle\bar{P}OP$ between the mean position $\bar{P}$ and the corrected position $P$ as viewed from the earth) is zero when the planet is on the apsidal line defined by its apogee-point and the earth: that is, when its anomaly is 0 or 180 degrees. The *mandaphala* values are symmetric about the absolute maximum that occurs when the anomaly is either 90 or 270 degrees; they are positive (meaning that the planet's corrected longitude will be larger than its mean longitude) when the anomaly falls in the first two quadrants, and negative thereafter.

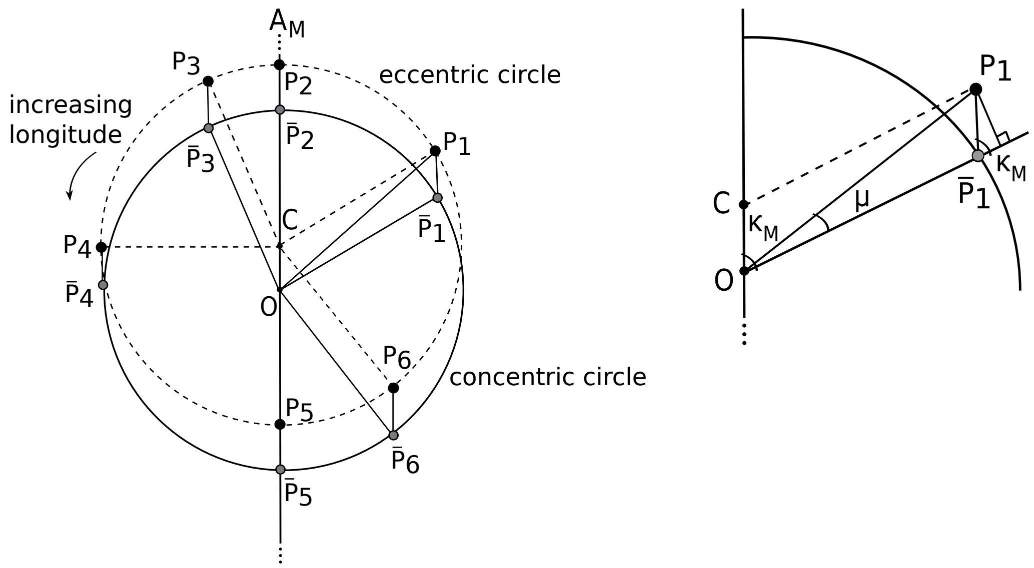


**Figure 3**: The *manda* correction interpreted geometrically via an eccentric orbit. **Left:** The point $O$ is the observer's position at the center of the concentric circle on which the mean planet $\bar{P}$ moves, $C$ the center of the eccentric representing the actual path of the planet $P$, the distance $OC$ the amount of eccentricity $r_M$, and $A_M$ the position of the *manda*-apogee from which the anomaly $\angle A_M O\bar{P}$ or $\kappa_M$ is computed. **Right:** The *manda*-equation $\mu$ is computed trigonometrically from the right triangle with *manda*-hypotenuse $H_M = OP$.

The diagram on the right in Figure 3 shows the trigonometric definition of the *mandaphala* $\mu$ based on the right triangle containing acute angle $\mu$, its opposite side $\mathrm{Sin}\,\kappa_M \cdot r_M/R$, and its adjacent side $R \pm \mathrm{Cos}\,\kappa_M \cdot r_M/R$ (where $R$ is the radius of the Sine-table and the capitalized Sine function is just $R$ times the modern sine function with unit radius). The ratio of the opposite side to the hypotenuse then gives (bearing in mind that the modern cosine function can be either positive or

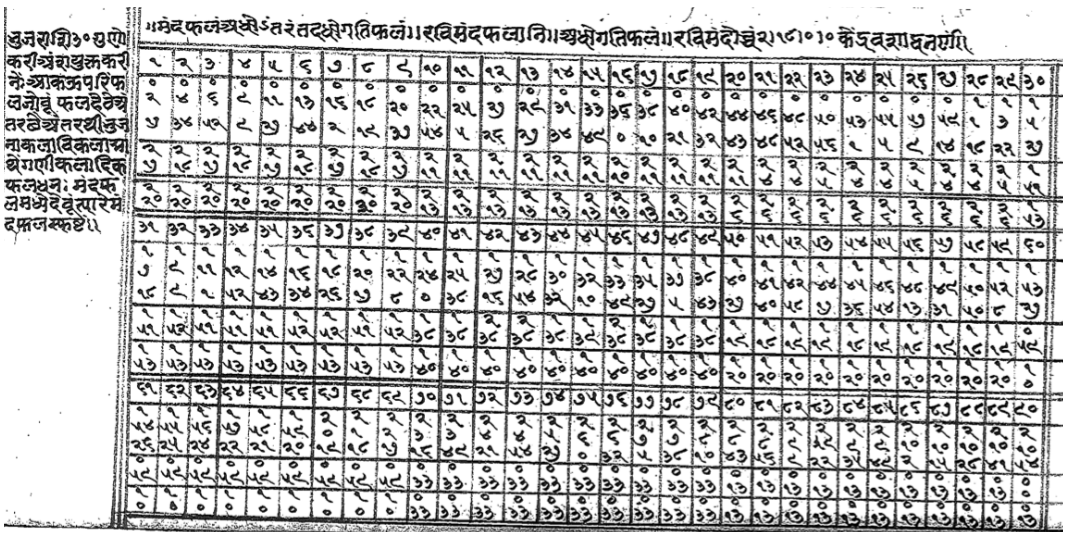

**Figure 4**: First half of the table in MS S45 f. 6v for determining the sun's *manda*-equation.

negative, so the ± symbol becomes unnecessary):

$$\sin\mu = \frac{\operatorname{Sin}\kappa_M \cdot \dfrac{r_M}{R}}{\sqrt{\left(\operatorname{Sin}\kappa_M \cdot \dfrac{r_M}{R}\right)^2 + \left(R \pm \operatorname{Cos}\kappa_M \cdot \dfrac{r_M}{R}\right)^2}}$$
$$= \frac{r_M \sin\kappa_M}{\sqrt{(r_M\sin\kappa_M)^2 + (R + r_M\cos\kappa_M)^2}}\,. \tag{1}$$

Figure 4 shows the sun's *manda*-equation table from MS S45, with the following header text:

> *mandaphalaṃ adho 'ntaraṃ tadadho gatiphalaṃ || ravimandaphalāni || adho gatiphalaṃ || ravimandoccaṃ 2 | 18 | 0 | 0 kendravaśād dhanarṇaṃ ||*
>
> The *manda*-equation; below, the difference; below that, the *gatiphala*. The *manda*-equations of the sun. Below, the *gatiphala*. The *manda*-apogee of the sun (is) 2 (zodiacal signs) 18 (degrees) 0 (minutes) 0 (seconds); (the *manda*-equation is) positive or negative according to (the amount of) the anomaly.

The longitude $\lambda_{A_M}$ of the sun's *manda*-apogee, $2^s18^\circ$ or $78^\circ$ total, is the value stated in *Karaṇakutūhala* 2.1 [Mishra 1991, 18; Rao and Uma 2008, S15]. (Because most of the planets' *manda*-apogees move so slowly, their change of position $v_{A_M}$ over a few hundred years or so may be neglected.) The first row in each horizontal segment of the table contains the degree of *manda*-anomaly as the table argument, running from 0 to 90. The second row is the *manda*-equation, whose maximum value at $90^\circ$ of anomaly is $2^\circ\ 10'\ 54''$.

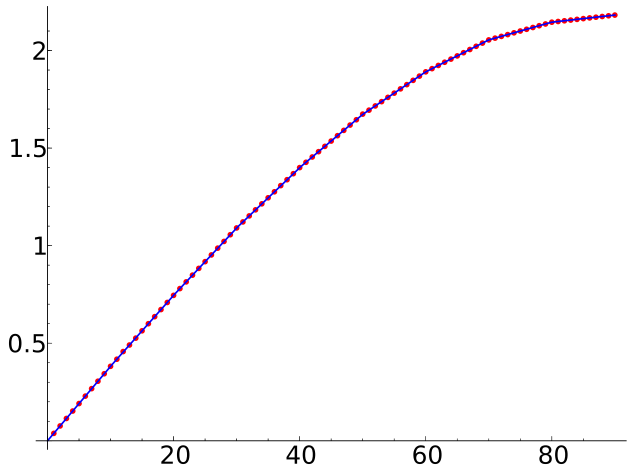


**Figure 5**: The sun's *manda*-equation in degrees versus degrees of *manda*-anomaly, as computed via *Karaṇakutūhala* 2.9–10 (blue line), and its tabulated values in *Brahmatulyasāraṇī* MS S45 f. 6v (red dots).

The graph in Figure 5 indicates that its tabulated values, between which the user is instructed to interpolate linearly, correspond very closely to the algorithm prescribed in *Karaṇakutūhala* 2.9–10 [Mishra 1991, 23; Rao and Uma 2008, S19]. It approximates the *manda*-equation by the Sine of the *manda*-anomaly scaled to the maximum value of $\mu$ [Plofker forthcoming, ch. 2]. Since the *Karaṇakutūhala* gives Sine values only for integer multiples of 10° and interpolates linearly between them, the resulting function comes out piecewise-linear over 10° intervals. Hence the third row of the table shows the differences between successive entries in the second row changing only at every tenth entry.

The table's fourth row is the so-called *gatiphala* or velocity-correction of the sun, beginning with the maximum value of 2,20 arcminutes and ending at the minimum of 0,13. The algorithm by which these *gatiphala* values were apparently determined approximates a more accurate function described in *Siddhāntaśiromaṇi* 2.36–38 [Śāstrī 1866, 52–53], which uses the Cosine of a planet's *manda*-anomaly $\kappa_M$ to produce the sinusoidal variation of the velocity-correction:

$$\bar{v}_M = \bar{v} \pm \frac{(\bar{v} - v_{A_M}) \cdot \mathrm{Cos}\,\kappa_M \cdot (r_M/R)}{R}\,. \tag{2}$$

The planet's *manda*-corrected angular velocity $\bar{v}_M$ will increase from its minimum value when the planet is most distant, at the apogee, to equal its mean velocity $\bar{v}$ when the anomaly is 90°. It reaches its maximum at closest distance or perigee when the anomaly equals 180°, subsequently slowing down to its minimum again when it returns to the apogee—hence the decrease of the tabulated *gatiphala* (absolute) values to nearly zero at the end of the first (or third) quadrant of anomaly, and the

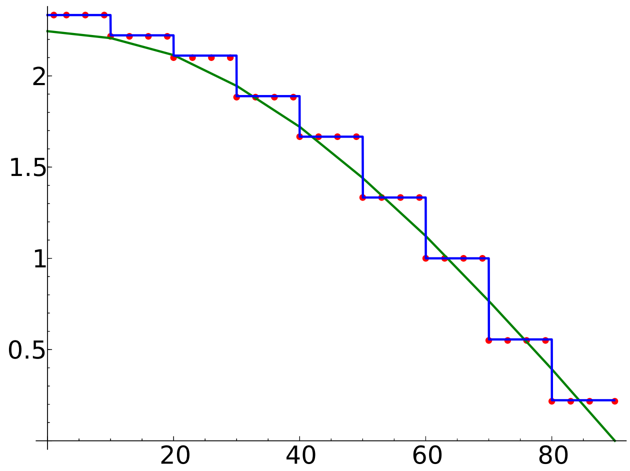


**Figure 6**: The sun's *manda*-derived velocity correction in arcminutes versus degrees of *manda*-anomaly, as computed via *Karaṇakutūhala* 2.11–12 (blue line), and a sample of its tabulated values in *Brahmatulyasāraṇī* MS S45 f. 6v (red dots). The more accurate smooth version of the correction-term function from *Siddhāntaśiromaṇi* 2.36–38 (green line, Equation 2) is shown for comparison.

requirement that they be applied positively to accelerate the mean velocity in the perigee-half of the anomalistic circle but negatively to decelerate it in the apogee-half.

The approximate formula stated in *Karaṇakutūhala* 2.11–12 replaces the above velocity-correction term with a scale factor multiplied by the difference between the tabulated Sine values of the 10° interval in which the anomaly $\kappa_M$ falls (this Sine-difference very roughly approximates the Cosine in the exact formula) [Mishra 1991, 24; Rao and Uma 2008, S23–S24]. Since the rule does not call for interpolating within that 10° interval, it amounts to a step function rather than a continuous one, as illustrated in Figure 6.

## Verses Six and Seven: Computing the *śīghra*-Correction; Iterated Corrections

grahoṇam uccaṃ ca phalaṃ rasādhikaṃ
cet sūryataḥ śodhya lavādikaṃ kṛtam ||
bhāgāṅkasaṃkhyāgatakoṣṭakaṃ tayoḥ
kalādikaṃ śeṣaṃ vivarāhataṃ tat || 6 ||

ṣaṣṭyā vibhaktaṃ svam ṛṇaṃ ca bhogyāt
kāryaṃ vihīnādhi[ka]tatkrameṇa ||
ādau hi mandārdha [x] kena tasmāt

samagraṃ [x x x] punaḥ punaś ca || 7 ||

[The longitude of] the apogee is diminished by [the longitude of] the planet. Having subtracted the result from 12 [signs] if it is greater than 6, it is made into degrees etc. [Subtract from this reduced *śīghra*-anomaly] the previous table entry for the number [equal to its] number of degrees; the remainder [from the subtraction] of those two is in arcminutes and so on. That is multiplied by the difference [between the previous and the next table entries, and] divided by sixty. [The result is] applied [to the previous entry] positively or negatively, according as that is respectively less or greater than the next entry. At first, [the mean longitude is corrected] with half the *manda*-equation and afterwards with the whole, repeatedly.

### *Verse Analysis*

The meter is *vaṃśamālā* in verse 6. The scansion of the final *pāda* of verse 6 is wrong: where the pattern of *gaṇas* should be either *ta-ta-ja-ra* or *ja-ta-ja-ra*, it is *ja-ma-sa-ya*.

The intended meter of verse 7 is apparently *upajāti*, but the text is evidently corrupt. The second *pāda* requires an interpolated syllable that is quite plausibly restored as *ka* to reach the required total of eleven, but there is no equally obvious way to emend the last two deficient *pādas* of ten and eight syllables respectively.

### *Technical Analysis*

The second planetary longitude correction is the *śīghra*-equation $\sigma$, roughly corresponding to the correction for synodic anomaly in western geocentric astronomy. It accounts for the phenomena of planetary stations and retrogradation, heliocentrically explained by the fact that the other planets as well as the earth are revolving about the sun. Thus a planet seen from the earth as they pass in their orbits can appear to pause and go backwards temporarily. Since the sun and moon do not retrograde and thus do not have a *śīghra*-anomaly, as noted previously, this correction applies only to the five star-planets.

The concept used in Indian astronomy to model this effect is a second anomaly or *śīghra-kendra* $\kappa_S$ measured from a notional point called the *śīghra*-apogee. Its position coincides with that of the mean sun in the case of superior planets, and with the planet itself in the case of inferior planets (for which the sun's mean position does duty as their mean longitude for the purpose of computing the anomaly).

The *śīghra*-anomaly $\kappa_S$ is determined by subtracting the longitude of the planet corrected by the *manda*-equation, or $\bar{\lambda}_M$, from that of its *śīghra*-apogee, $\lambda_{A_S}$. Since the *śīghra*-apogee revolves about the earth faster than the mean planet does, the planet periodically appears to go backwards while it is close to its opposition (or in the case of an inferior planet, its inferior conjunction) with respect to the sun. Figure 7 qualitatively illustrates the *śīghra* for a superior planet, neglecting the effect of the *manda*: as the *śīghra*-anomaly angle goes from 0° to 180°, the planet's apparent motion follows the dotted path, whose loops represent the apparent retrogradations.

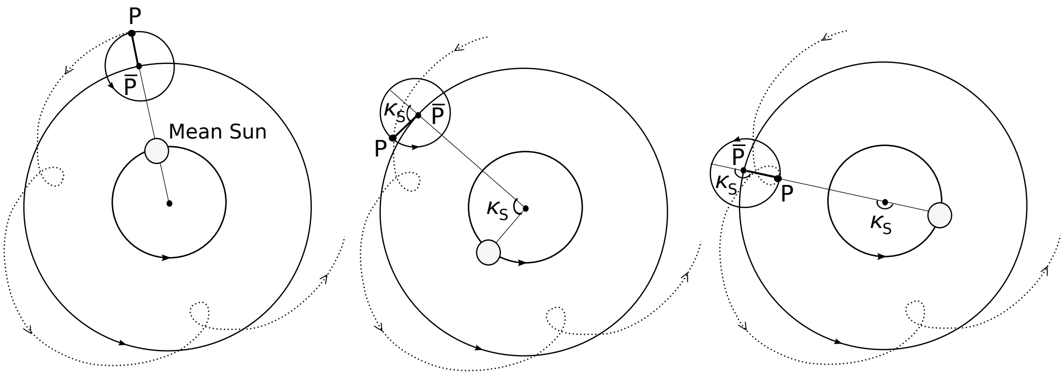


**Figure 7**: The *śīghra*-correction for a superior planet. **Left:** The *śīghra*-anomaly $\kappa_S$ and the corresponding correction are zero when the planet is in conjunction with the sun. **Center:** The direction of the planet stays parallel to that of the sun, which is revolving faster than the mean planet on its orbit, so the planet appears to slow down in its forward motion. **Right:** The continued motion of the mean sun appears to drag the planet backwards, so that it reaches the center of its retrograde motion in opposition to the sun, with anomaly 180°.

This cyclic "looping" means that the *śīghra*-equation values are symmetric about the end of the second quadrant of anomaly. More precisely, they are given by the formula stated in *Karaṇakutūhala* 2.13 [Mishra 1991, 25; Rao and Uma 2008, 26], which is analogous to that for the *manda*-correction defined in Equation 1 and equivalent to the following expression:

$$\operatorname{Sin}\sigma = \frac{R \cdot r_S \sin\kappa_S}{\sqrt{(r_S \sin\kappa_S)^2 + (R + r_S\cos\kappa_S)^2}} = \frac{R \cdot r_S \sin\kappa_S}{H_S}\,. \tag{3}$$

Here, $r_S$ is the radius of the planet's *śīghra*-epicycle while the so-called *śīghra*-hypotenuse

$$H_S = \sqrt{(r_S \sin\kappa_S)^2 + (R + r_S\cos\kappa_S)^2}$$

extends from the planet's true position to the earth. Since the Cosine of the *śīghra*-equation is similarly given by

$$\operatorname{Cos}\sigma = \frac{R \cdot (R + r_S\cos\kappa_S)}{\sqrt{(r_S \sin\kappa_S)^2 + (R + r_S\cos\kappa_S)^2}} = \frac{R \cdot (R + r_S\cos\kappa_S)}{H_S}, \tag{4}$$

we can write

$$
\begin{aligned}
\sin\sigma &= \frac{r_S \sin\kappa_S}{\sqrt{(r_S \sin\kappa_S)^2 + (R + r_S\cos\kappa_S)^2}} \\
&= \frac{r_S\sin\kappa_S}{\sqrt{{r_S}^2\sin^2\kappa_S + R^2 + 2Rr_S\cos\kappa_S + {r_S}^2\cos^2\kappa_S}} \\
&= \frac{r_S\sin\kappa_S}{\sqrt{{r_S}^2 + 2Rr_S\cos\kappa_S + R^2}} \\
&= \frac{r_S\sin\kappa_S}{H_S},
\end{aligned}
\tag{5}
$$

and likewise

$$
\cos\sigma = \frac{R + r_S\cos\kappa_S}{\sqrt{{r_S}^2 + 2Rr_S\cos\kappa_S + R^2}} = \frac{R + r_S\cos\kappa_S}{H_S}. \tag{6}
$$

It is clear, as the tables of the planets' *śīghra*-equations illustrate, that $\sigma$ is zero when the anomaly is zero (at conjunction or superior conjunction for a superior or inferior planet respectively) or 180° (opposition/inferior conjunction). To find where the maximum $\sigma$-values occur, we set the derivative of $\sigma = \arcsin(\sin\sigma)$ to zero and solve for $\kappa_S$:

$$
\begin{aligned}
\frac{d}{d\,\kappa_S}(\arcsin(\sin\sigma)) \quad &= \quad \frac{d}{d\,\kappa_S}\left(\arcsin\left(\frac{r_S\sin\kappa_S}{\sqrt{{r_S}^2 + 2Rr_S\cos\kappa_S + R^2}}\right)\right) \\
= \frac{d}{d\,\kappa_S}&\left(\arcsin\left(\frac{r_S\sin\kappa_S}{H_S}\right)\right) \\
= \frac{1}{\sqrt{1 - \frac{{r_S}^2\sin^2\kappa_S}{{H_S}^2}}} &\cdot \frac{(r_S\cos\kappa_S)(H_S) - \frac{r_S\sin\kappa_S(-2Rr_S\sin\kappa_S)}{2H_S}}{{H_S}^2} \\
= \left(\frac{1}{\sqrt{1 - \frac{{r_S}^2\sin^2\kappa_S}{{H_S}^2}}}\right) &\cdot \\
&\left(\frac{(r_S\cos\kappa_S)\sqrt{{r_S}^2 + 2Rr_S\cos\kappa_S + R^2} - \frac{r_S\sin\kappa_S(-2Rr_S\sin\kappa_S)}{2\sqrt{{r_S}^2 + 2Rr_S\cos\kappa_S + R^2}}}{{H_S}^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{\dfrac{{H_S}^2 - {r_S}^2 \sin^2 \kappa_S}{{H_S}^2}}} \cdot \frac{r_S \cos \kappa_S ({H_S}^2) + R{r_S}^2 \sin^2 \kappa_S}{({H_S}^2) H_S} \\
&= \left( \frac{H_S}{\sqrt{{r_S}^2 \cos^2 \kappa_S + 2 r_S \cos \kappa_S + R^2}} \right) \cdot \\
&\qquad \left( \frac{{r_S}^3 \cos \kappa_S + 2R{r_S}^2 \cos^2 \kappa_S + R^2 r_S \cos \kappa_S + R{r_S}^2 \sin^2 \kappa_S}{({H_S}^2) H_S} \right) \\
&= \frac{{r_S}^3 \cos \kappa_S + R{r_S}^2 \cos^2 \kappa_S + R^2 r_S \cos \kappa_S + R{r_S}^2}{\sqrt{(r_S \cos \kappa_S + R)^2} \cdot ({H_S}^2)} \\
&= \frac{(r_S \cos \kappa_S + R)({r_S}^2 + R r_S \cos \kappa_S)}{\sqrt{(r_S \cos \kappa_S + R)^2} \cdot ({r_S}^2 + 2R r_S \cos \kappa_S + R^2)} \\
&= \frac{{r_S}^2 + R r_S \cos \kappa_S}{{r_S}^2 + 2R r_S \cos \kappa_S + R^2} \, .
\end{aligned}
\tag{7}
$$

When $\sigma$ is at its maximum, this reduces to

$$
\begin{aligned}
0 &= \frac{{r_S}^2 + R r_S \cos \kappa_S}{{r_S}^2 + 2R r_S \cos \kappa_S + R^2} \\
&= {r_S}^2 + R r_S \cos \kappa_S \\
&= r_S + R \cos \kappa_S \\
\cos \kappa_S &= -\frac{r_S}{R} \, .
\end{aligned}
$$

Since the ratio of the two radii is between about 0.1 and 0.7, depending on the planet, this tells us that the maximum *śīghra*-equation occurs when the anomaly attains a certain value in the second quadrant (and again when the anomaly is zero minus that value, in the third quadrant). Thus the linear interpolation procedure for the *śīghra*-tables must specify whether the interpolated increment of equation is to be added to or subtracted from the previous tabulated value, according as the equation is increasing or decreasing respectively.

Figure 8 shows a selection of the *Brahmatulyasāraṇī*'s tabulated *śīghra*-function values for Jupiter, and Figure 9 compares them to the results of the *Karaṇakutūhala*'s formulas. It is not quite clear why the compiler of the tables bothered to tabulate the values of the *śīghra*-hypotenuse $H_S$, as none of the rules specified in the *Brahmatulyasāraṇī* requires the user to employ it; however, the values shown confirm that the *Brahmatulyasāraṇī* follows the *Karaṇakutūhala* in using $R = 120$ for the trigonometric radius.

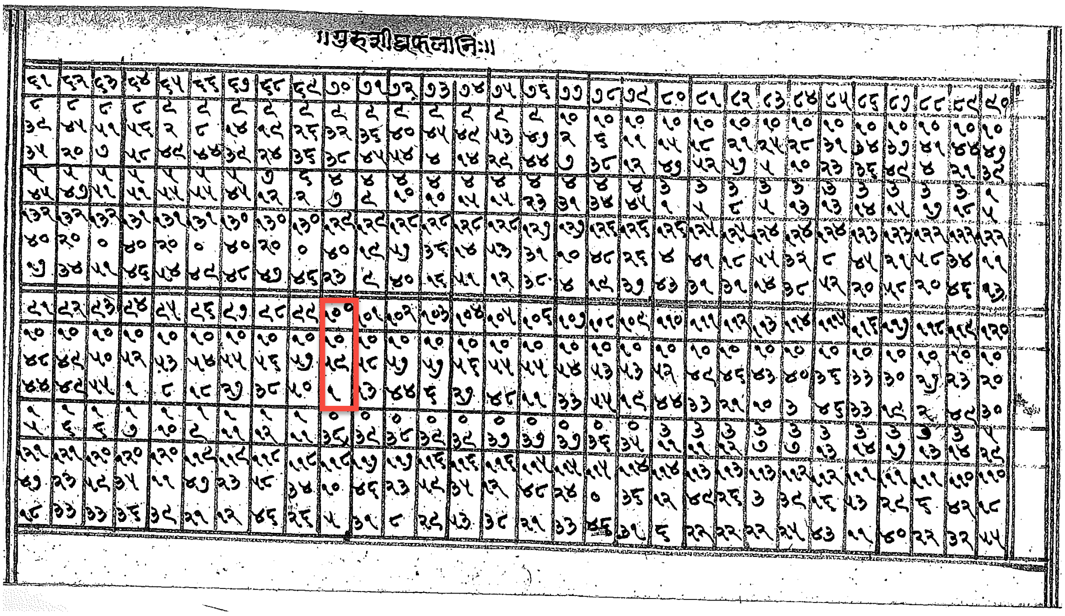

**Figure 8**: Part of the *śīghra*-equation table for Jupiter in MS S45 f. 13r, showing the maximum value of the equation $\sigma^{(\circ)}$ at 100° of anomaly, outlined in red. The second row of the table contains the differences $\Delta\sigma^{(\prime)}$ between the successive entries, and the third row the corresponding values of the hypotenuse $H_S$.

The procedure in *Karaṇakutūhala* 2.14 specifies that the initial equations for both *manda*- and *śīghra*-corrections should be halved before applying them to the planet's longitude—but only in the case of Mars [Mishra 1991, 26; Rao and Uma 2008, S28]. This *Brahmatulyasāraṇī* algorithm, on the other hand, seems to generalize the initial halving of the *manda*-equation to all the planets. It may be that some of the missing syllables that make the verse metrically deficient originally specified Mars, but we have not taken it upon ourselves to restore them.

## Verses Eight and Nine: Application of the *śīghra*-Velocity Correction; Retrograde Motion and Combination of Corrections

drākkendrabhuktir vivareṇa nighnā
ṣaṣṭyuddhṛtaṃ svaṃ ca phalasya vṛddhau ||
hrāsa ṛṇaṃ mandagater grahāṇāṃ
kṛtām iti syāt sphuṭakheṭabhuktiḥ || 8 ||

yadā na śuddhā tu vilomaśodhyā
śeṣeṣu vakrā bhavatīha bhuktiḥ ||
bhaumādikāḥ karmacatuṣṭayena

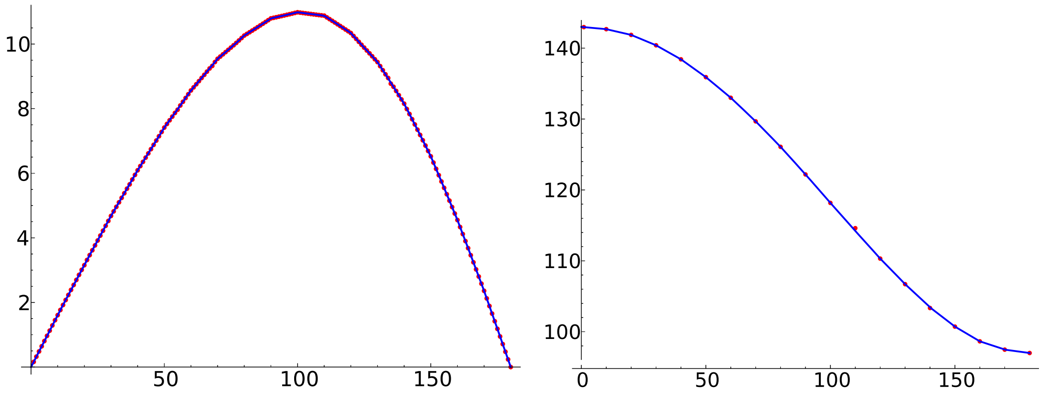


**Figure 9**: Functions for 0–180° of *śīghra*-anomaly for Jupiter, as represented by the *Karaṇakutūhala*'s algorithms (blue line) and sample values tabulated in *Brahmatulyasāraṇī* MS S45 ff. 12v–13r (red dots; see Figure 8). **Left:** The *śīghra*-equation $\sigma$ in degrees. **Right:** The *śīghra*-hypotenuse $H_S$ in radial units ($R = 120$).

kujas tu yāvat sthiratām upeti || 9 ||

The velocity of the *śīghra*-anomaly is multiplied by the difference [between successive *śīghra*-equation values corresponding to that *śīghra*-anomaly] and the quotient with sixty [is applied] positively with respect to the *manda*[-corrected] velocity of the computed planets when there is increase of the equation [in successive tabulated values], negatively when there is decrease. Thus the velocity of the true planet should be [computed].

When [the modified *śīghra*-anomaly velocity] is not [capable of being] subtracted [from the *manda*-corrected mean velocity, it] is to be reverse-subtracted. The velocity here becomes retrograde in [the amount of] the remainders. The [star-planets] beginning with Mars [are corrected] by four procedures, but Mars [itself] until [it] attains fixedness.

### *Verse Analysis*

Meter: *upajāti*.

The text of the third *pāda* of verse 8 is very unclear and we have taken several liberties with its interpretation. The manuscript reads *hrāso ṛṇaṃ maṃdārdhagatagrahāṇāṃ*, which is both hypermetric and ungrammatical, as well as difficult to make sense of. We speculate that *hrāso* resulted from a faulty sandhi correction of *hrāsa* for locative absolute *hrāse* before *ṛṇaṃ*, and that the scribe may have written *maṃdārdha* in unconscious imitation of the phrase *maṃdārdhakena* from verse 7 that appears just above it in the preceding line.

*Technical Analysis*

The algorithm in verse 8, unlike those in the previous verses, diverges markedly from its counterpart in the *Karaṇakutūhala* (2.14–2.16ab).[4] The *Brahmatulyasāraṇī* determines and applies the velocity correction due to the *śīghra*-anomaly to produce the planet's true velocity $v$ by a linear interpolation, as follows:

$$v = \bar{v}_M + v_{\kappa_S} \cdot \frac{\Delta\,\sigma^{(\prime)}}{60}\,. \tag{8}$$

Here, $\bar{v}_M$ is the planet's mean velocity $\bar{v}$ corrected by the *manda-gatiphala* from Equation 2 in the discussion of verse 5; $v_{\kappa_S}$ is the so-called velocity of the *śīghra*-anomaly, i.e., the difference between two successive values of the *śīghra*-anomaly $\kappa_S = \lambda_{A_S} - \bar{\lambda}_M$; and $\Delta\sigma$ is the difference between two successive tabulated values of the *śīghra*-equation $\sigma$. We take $\Delta\sigma$ to be positive when $\sigma$ is increasing and negative when $\sigma$ is decreasing, so we write "+" instead of "±" in the velocity formula.

The "anomaly velocity" $v_{\kappa_S}$ can be shown to be equivalent to $(v_{A_S} - \bar{v}_M)$, the difference between the velocity $v_{A_S}$ of the *śīghra*-apogee (which, unlike that of the much slower *manda*-apogee, cannot be assumed to be zero) and that of the *manda*-corrected planet. To wit: Understanding each of these velocities as simply a change

[4]The *Karaṇakutūhala*'s procedure is stated in the following verses which are differently arranged and numbered in different editions [Mishra 1991, 26–27; Rao and Uma 2008, S197]:

tadutthamāndena calena madhyaś
cet saṃskṛtaḥ spaṣṭataras tadā syāt ||
dalīkṛtābhyāṃ prathamaṃ phalābhyāṃ
tato 'khilābhyāṃ tu punaḥ kujas tu ||
gateḥ phalenaiva tu saṃskṛtā yā
madhyā gatir mandagatir bhavet sā ||
grahasya mandasphuṭabhuktivarjitā
svāśīghrakendrasya gatir bhavet sā ||

drākkendrabhuktir guṇitāśucāpa-
bhogyajyayā khābdhiguṇā ca bhaktā ||
saptaghnakarṇena caloccabhukteḥ
śodhyāviśiṣṭaṃ sphuṭakheṭabhuktiḥ ||
yadā na śuddhā viparītaśodhyā
śeṣaṃ bhaved vakragatis tadānīm ||

between two successive positions in longitude with superscripts $i$ and $ii$, we write

$$\begin{aligned}\left({\lambda_{A_S}}^{ii} - {\bar{\lambda}_M}^{ii}\right) - \left({\lambda_{A_S}}^{i} - {\bar{\lambda}_M}^{i}\right) &= \left({\lambda_{A_S}}^{ii} - {\lambda_{A_S}}^{i}\right) - \left({\bar{\lambda}_M}^{ii} - {\bar{\lambda}_M}^{i}\right)\\ &= v_{A_S} - \bar{v}_M \quad = \quad v_{\kappa_S}\,.\end{aligned}$$

Moreover, since the true planetary longitude $\lambda$ is given by

$$\lambda = \bar{\lambda}_M + \sigma\,,$$

and we can regard velocity as just the derivative of longitude with respect to time, the true velocity $v$ can be defined thus:

$$\begin{aligned}v &= \frac{d}{dt}(\lambda) = \frac{d}{dt}(\bar{\lambda}_M + \sigma) = \frac{d}{dt}(\bar{\lambda}_M) + \frac{d}{dt}(\sigma)\\ &= \frac{d}{dt}(\bar{\lambda}_M) + \frac{d}{dt}\left(\arcsin\left(\frac{r_S \sin\kappa_S}{H_S}\right)\right)\\ &= \frac{d}{dt}(\bar{\lambda}_M) + \frac{d}{d\kappa_S}\left(\arcsin\left(\frac{r_S \sin\kappa_S}{H_S}\right)\right)\cdot\frac{d}{dt}(\kappa_S)\\ &= \bar{v}_M + \frac{d}{d\kappa_S}\,(\sigma)\cdot v_{\kappa_S}\,,\end{aligned} \tag{9}$$

where $H_S$ as before denotes the *śīghra*-hypotenuse. Recalling that $\Delta\,\sigma^{(')}/60 = \Delta\,\sigma^{(\circ)}$, we can see that the final expression in Equation 9 is identical to the formula for $v$ stated in Equation 8, up to the equivalence of the derivative $\dfrac{d}{d\kappa_S}\,(\sigma)$ with its finite-difference approximation $\Delta\sigma$.

The corresponding rule for true velocity $v$ in *Siddhāntaśiromaṇi* 2.39 prescribes [Śāstrī 1866, 54]:

$$v = v_{A_S} - \frac{v_{\kappa_S}\cdot \operatorname{Cos}\sigma}{H_S}\,. \tag{10}$$

If we substitute $v_{A_S} = \bar{v}_M + v_{\kappa_S}$ into this formula, we obtain a rule identical to that of the *Brahmatulyasāraṇī* in Equation 8 except that it appears to use a different scale factor for the anomaly velocity $v_{\kappa_S}$:

$$v = \bar{v}_M + v_{\kappa_S} - \frac{v_{\kappa_S}\cdot\operatorname{Cos}\sigma}{H_S} = \bar{v}_M + v_{\kappa_S}\left(1 - \frac{\operatorname{Cos}\sigma}{H_S}\right)\,. \tag{11}$$

In fact, the two multipliers in Equations 8 and 11 are mathematically equivalent, as we show by further manipulating the expression for the *śīghra*-equation difference derived in Equation 7:

$$\begin{aligned}
\Delta\sigma \quad &\approx \quad \frac{d}{d\,\kappa_S}(\arcsin(\sin\sigma)) \quad = \quad \frac{{r_S}^2 + Rr_S\cos\kappa_S}{{r_S}^2 + 2Rr_S\cos\kappa_S + R^2} \\
&= \frac{{r_S}^2 + 2Rr_S\cos\kappa_S + R^2 - (Rr_S\cos\kappa_S + R^2)}{{r_S}^2 + 2Rr_S\cos\kappa_S + R^2} \\
&= \frac{{H_S}^2 - (Rr_S\cos\kappa_S + R^2)}{{H_S}^2} \\
&= 1 - \frac{R(R + r_S\cos\kappa_S)}{{H_S}^2} \quad = 1 - \frac{R}{H_S}\cdot\frac{R + r_S\cos\kappa_S}{H_S} \quad = 1 - \frac{R}{H_S}\cdot\cos\sigma \\
&= 1 - \frac{\mathrm{Cos}\,\sigma}{H_S}\,.
\end{aligned}$$

It is not clear exactly how the relationship between these two attested forms of the scale factor, $\Delta\,\sigma^{(\circ)}$ and $\left(1 - \dfrac{\mathrm{Cos}\,\sigma}{H_S}\right)$, was understood by Bhāskara and his successors. It could easily have been noticed that their behavior is qualitatively very similar, as both are zero when the *śīghra*-equation is at its maximum and have their largest absolute value when the planet is at perigee: i.e., $\sigma = 0$ so $\mathrm{Cos}\,\sigma = R$ and $H_S$ takes its minimum value $R - r_S$.

The *Karaṇakutūhala* modifies the *Siddhāntaśiromaṇi* formula to the following expression:

$$v = v_{A_S} - \frac{v_{\kappa_S}\cdot\text{Sine-difference}(\sigma)}{H_S}\cdot\frac{40}{7}\,. \tag{12}$$

This algorithm, like its counterpart for the *manda*-derived velocity correction discussed in verse 5 (Equation 2), merely replaces the Cosine of the equation $\sigma$ by the appropriately scaled Sine-difference for the 10° interval in which $\sigma$ falls, thus:

$$\mathrm{Cos}\,\sigma \approx \frac{\text{Sine-difference}(\sigma)\cdot R}{\text{max. Sine-difference}} = \frac{\text{Sine-difference}(\sigma)\cdot 120}{21} = \frac{\text{Sine-difference}(\sigma)\cdot 40}{7}\,.$$

The graph on the left in Figure 10 compares the exact and approximate versions of the velocity-correction term from Equations 11 and 12 respectively. (Note that the sharp discontinuities in the approximation occur where the *śīghra*-equation $\sigma$ changes its 10° interval). The graph on the right compares the *Siddhāntaśiromaṇi* version to the values in the *Brahmatulyasāraṇī*.

We presume that the author of the *Brahmatulyasāraṇī* algorithm, or perhaps an earlier innovator from whom he copied the technique, was familiar with the *Karaṇakutūhala* formula for correcting the planetary velocity. This is corroborated by the statement of the condition for retrograde velocity in verse 9, which is clearly heavily indebted to the corresponding *Karaṇakutūhala* verse.

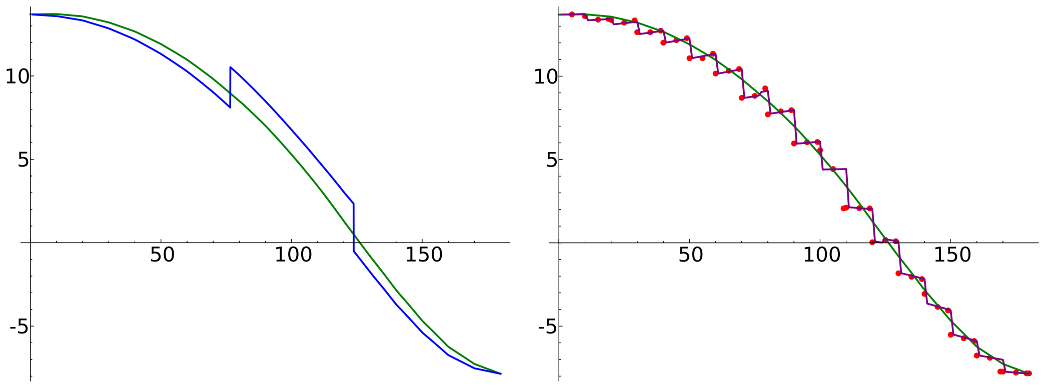


**Figure 10**: The *śīghra* velocity correction in minutes per day for 0–180° of *śīghra*-anomaly for Jupiter: **Left:** The *Siddhāntaśiromaṇi* formula (green) and the *Karaṇakutūhala* approximation to it (blue). **Right:** The *Siddhāntaśiromaṇi* formula compared to a sample of the values tabulated in *Brahmatulyasāraṇī* MS S45 ff. 12v–13r (red dots) and their reconstruction (purple line).

The "four procedures" somewhat vaguely alluded to in closing are the alternating *manda*- and *śīghra*-corrections specified in *Karaṇakutūhala* 2.14 (see note 4), following *Siddhāntaśiromaṇi* 2.34cd–36ab [Śāstrī 1866, 52]. Namely, the mean position is to be adjusted first by the appropriate *manda*-equation and then by the *śīghra*-equation corresponding to the *manda*-adjusted position, after which the process is repeated. In the case of Mars alone, each equation should be halved when first applied, but not thereafter. The *Siddhāntaśiromaṇi* prescribes iteration of the alternating corrections for all star-planets until their longitudes are fixed.

## Verse Ten: Correction for Mars' *manda*-Apogee

bhaumāśukendrasya padasya jāta-
gamyasya bhāgāḥ phalavat phalaṃ ca
kulīra[na]krādigate svakendre
hīnādhikaṃ spaṣṭam asṛṅmṛdūccam || 10 ||

The degrees of the past [or] future [part, whichever is smaller,] of the quadrant of the *śīghra*-anomaly of Mars are like [the argument of an] equation [in the table of *manda*-apogee correction for Mars]. And the [corresponding] equation, when its own [*śīghra*-] anomaly is in Cancer or Capricorn, is [respectively] subtracted or added [to make] the *manda*-apogee of Mars accurate.

### *Verse Analysis*

Meter: *upajāti.*

The manuscript has *maṃdasya* instead of *padasya*, suggesting that a scribe at some point misread *ma* for *pa* and then added an *anusvāra* to obtain a familiar term at the expense of the meter (and the sense). We have supplied the first syllable of the word *nakra* (or perhaps it was originally *makra* in error for the better-known synonym *makara* "crocodile, sea-monster"?) for the zodiacal sign Capricorn.

### *Technical Analysis*

As we reconstruct it, the *Brahmatulyasāraṇī* verse is largely borrowed from the corresponding *Karaṇakutūhala* rule [Plofker forthcoming, ch. 2].[5] It amounts to multiplying the arc between the *śīghra*-anomaly of Mars and the closest integer multiple of 90° by the scale factor $\frac{3}{20}$ or 9 arcminutes per degree. The result is applied to displace the longitude of Mars' *manda*-apogee backwards or forwards in the ecliptic, depending on whether the planet is in the half-circle of anomaly centered on opposition or in the one centered on conjunction, respectively. At its conjunction, opposition or quadrature there is no correction to the *manda*-apogee, while the correction is maximum $(0;9^\circ \times 45 = 6;45^\circ)$ when the anomaly is an odd multiple of 45°.

Qualitatively, this adjustment has the effect of moving the *manda*-apogee towards the planet at the octants around conjunction, which decreases the speed of its motion, and away from the planet at the octants around opposition, which increases its speed (or strictly speaking slows down its retrograde motion). Bhāskara evidently derived the rule from an algorithm in the *Siddhāntaśiromaṇi* in which the scale factor $\left(\dfrac{6;40}{\mathrm{Sin}\,45}\right)$ is applied to the Sine of the past or future arc of the quadrant of *śīghra*-anomaly. This normalizes the absolute value of the *manda*-apogee correction term to a maximum of $6;40^\circ$ at the octants of *śīghra*-anomaly, rather than $6;45^\circ$ as in the *Karaṇakutūhala*/*Brahmatulyasāraṇī* version.

Bhāskara's commentary in the *Siddhāntaśiromaṇi* says of this correction merely

---

[5] *Karaṇakutūhala* 2.5 [Mishra 1991, 20; Rao and Uma 2008, S17] (compare *Siddhāntaśiromaṇi* 2.24–25 [Śāstrī 1866, 46–47]):

> bhaumāśukendre padayātagamya-
> svalpasya liptā khakhavedabhaktāḥ ||
> labdhāṃśakaiḥ karkimṛgādikendre
> hīnānvitaṃ spaṣṭam asṛṅmṛdūccam ||5||

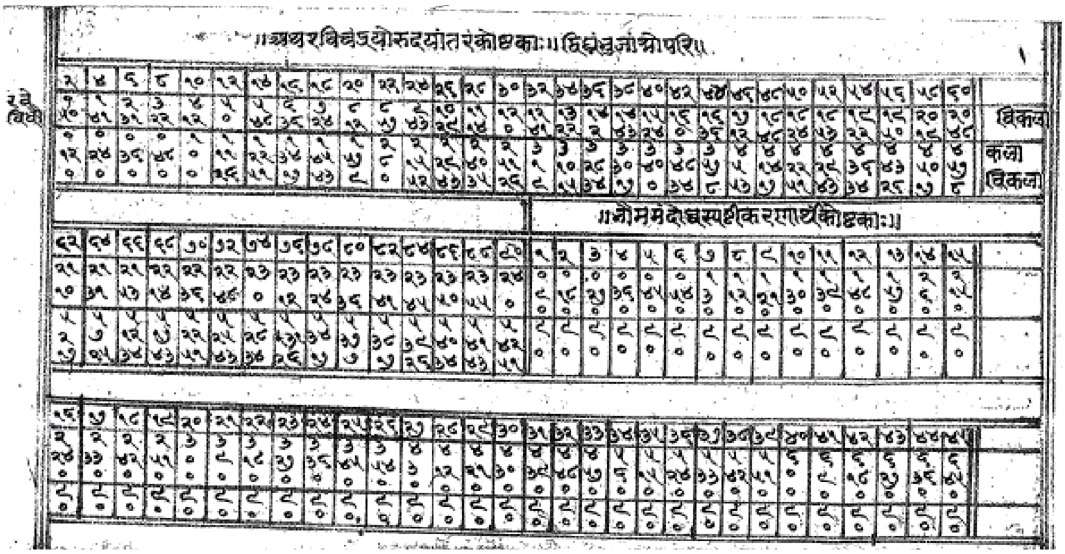

**Figure 11**: The table of the apogee-longitude correction for Mars for argument values 0–45°, beginning in the center of the page (MS S45 f. 7v).

*atrāgama eva pramāṇam*, i.e., it is the quantity prescribed in the received tradition. Its origin and efficacy are discussed in more detail in Duke [2005]. Figure 11 shows its tabulated values as they appear in MS S45.

## Colophon

iti brahmatulyasāraṇīślokāḥ ||

Thus, the ślokas of the *Brahmatulyasāraṇī*.

## Post-Colophon: Astrological Procedure

candrarāśau kalāṃ sarve dvādaśair bhāgam āharet ||
yātrodvāhe śubhe kārye candrāvasthāḥ parityajet ||1||

pravāsanaṣṭāmṛtatājayākhyā
hāsyāratikrīḍitasuptabhuktāḥ ||
jarāhvayāḥ kampitasusthitaṃ ca
meṣādimukhyā himagor avasthāḥ ||1||

In every zodiacal sign of the moon, one should divide the minute [and?] degree [? of longitude] by twelve. [The remainder gives the number of the lunar *avasthā* or "status."] When a journey or marriage is to be made auspicious, the *avasthā* of the moon should be disregarded.

[Those] called journey, loss, immortality, victory; laughter, delight, play, sleep, eating; [that] called old age, and trembling [and] comfort; are the states (*avasthās*) of the moon starting from the beginning of Aries.

The word *avasthā* is a technical term in Sanskrit astrology meaning "status" or "situation"; the *avasthās* are thought to predict certain events or activities indicated by their names. Our author seems to derive the *avasthā* number by the following method, of which variant forms are still practiced in modern popular *jyotish* or Indian astrology. Namely, the numbers of the degree and minute of the zodiacal sign occupied by the moon are arithmetically combined in some fashion not entirely clear from the rule as stated, and the resulting integer is divided by 12. Evidently it is the remainder from this division that designates the number of the *avasthā*.

Another list of twelve lunar *avasthās*, mostly with quite different names, is supplied in the astrological work *Phaladīpikā* of Mantreśvara (ca. fifteenth century or later?).[6] Also unlike our present author, Mantreśvara assigns the lunar *avasthā* at the desired moment of nativity, query, etc., according to the integer part of the quotient from dividing the moon's position in the cycle of constellations (interpreted in the units *vighaṭikās* of which there are 3600 in the cycle) by 300.

## Scribal Signature

iti likhitaṃ malūkacandreṇa ||

Thus, (this text) was written by Malūkacandra.

## Acknowledgements

We gratefully acknowledge the libraries that kindly supplied images of their manuscripts of the *Brahmatulyasāraṇī*: Columbia University's Smith Indic collection, and the Bhandarkar Oriental Research Society in Pune. We are especially grateful to Anuj Misra, our doctoral student at the University of Canterbury, for his help in obtaining the manuscript copy from the latter institution. We thank Martin Gansten for his very helpful elucidation of the post-colophon astrological procedure and its relation to the *Phaladīpikā*. The developers and maintainers of

[6] See [Pingree 1981, 92]. Chapter 4, verse 16 states [Ojha 1969, 82]:

> ātmasthānātpravāso mahitanṛpahito dāsatāprāṇahānir
> bhūpālatvaṃ svavaṃśocittaguṇanirato roga āsthānavattvam ||
> bhītiḥ kṣadbādhitatvaṃ yuvatipariṇayo ramyaśayyānuṣaktir
> mṛṣṭāśitvaṃ ca gītā iti niyamavaśātsadbhir indor avasthā ||16||

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# Appendix: Edition of the *Brahmatulyasāraṇī* verses

| ओं श्रीगणेशाय नमः ॥ f. 6r S29

नत्वा वल्लभनन्दनं तदनुगोपालांह्रिपद्मद्वयं
ज्ञात्वा श्रीगुरुवाक्यतो ह्यहर्निशं [दृष्ट्वा] द्युमेवाधुना ॥
सिद्धान्तेषु यथोक्तखेचरविधि[भ्य]ः स्पष्टकोष्टं मुहु-
र्मध्यस्पष्टविभागतो ग्रहगणात्कुर्वे दिनौघादहम् ॥१॥

कृत्वादौ करणोक्तवासरगणं शिष्टैः सुहृष्टात्मभि-
र्भाज्यं खाग्नि ३० मितैरवाप्तकमिदं सूर्यै १२ र्विभाज्यं पुनः ॥
लब्धं विंशति २० भिर्भजेदथ चतुःशेषाङ्कसंज्ञा ध्रुवं
अङ्कास्ते मिलिताः स्वकोष्टकगता लङ्कानगर्यां खगाः ॥२॥

मध्याः स्वदेशीयखगा भवेयु-
र्देशान्तरेणाब्द x[x]x राम- ॥
बीजेन युक्ता गणकैस्ततश्च | f. 6v S29
स्पष्टाः क्रियन्ते फलयुग्मकेन ॥३॥

केन्द्रस्य दोरंशमितिश्च कोष्टे
भुक्तं तदग्रं परभोग्यकं च ॥
कलादिकं तद्द्विवराहतं तु
षष्ट्युद्धृतं भुक्तकमानकेन ॥४॥

3 °वाधुना] °वाद्युना S29 4 कोष्टं] कोष्टो S29 7 °वाप्तक°] °वात्मक°S29 र्विं°] वि° S29
10–11 °युदेशान्तरेणाप्रभथराम S29 14 कोष्टे] कोष्टं S29 15 °ग्रं परभोग्यकं] °ग्रपरभोग्यंकं S29

युक्तं भवेन्मन्दफलं ग्रहाणां
स्वर्णं क्रमान्मेषतुलादिकेन्द्रे ॥
ग्रहस्य भुक्तिर्विवराहतं च
षष्ट्युद्धृतं केन्द्रवशाद्धनर्णम् ॥ ५ ॥

ग्रहोणमुच्चं च फलं रसाधिकं
चेत्सूर्यतः शोध्य लवादिकं कृतम् ॥
भागाङ्कसंख्यागतकोष्टकं तयोः
कलादिकं शेषं विवराहतं तत् ॥ ६ ॥

षष्ट्या विभक्तं स्वमृणं च भोग्या-
त्कार्यं विहीनाधि[क]तत्क्रमेण ॥
आदौ हि मन्दार्ध[x]केन तस्मा-
त्समग्रं [x x x] पुनः पुनश्च ॥ ७ ॥

द्राक्केन्द्रभुक्तिर्विवरेण निघ्ना
षष्ट्युद्धृतं स्वं च फलस्य वृद्धौ ॥
ह्रास ऋणं मन्दगतेर्ग्रहाणां
कृतामिति स्यात् स्फुटखेटभुक्तिः ॥ ८ ॥

यदा न शुद्धा तु विलोमशोध्या
शेषेषु वक्राभवतीह भुक्तिः ॥

5 ग्रहोण◦] ग्रहोन◦ S29 13 ◦भुक्तिर्वि◦] ◦भुक्तिवि◦ S29 15 ह्रास ऋणं] ह्रासो ऋणं S29 मन्दगतेर्ग्र◦] मन्दार्द्धगतेग्र◦ S29

भौमादिकाः कर्मचतुष्टयेन
कुजस्तु यावत्स्थिरतामुपेति ॥ ९ ॥

भौमाशुकेन्द्रस्य पदस्य जात-
गम्यस्य भागाः फलवत्फलं च ॥
कुलीर[न]क्रादिगते स्वकेन्द्रे
हीनाधिकं स्पष्टमसृङ्मृदूच्चम् ॥ १० ॥

इति ब्रह्मतुल्यसारणीश्लोकाः ॥

चन्द्रराशौ कलां सर्वे द्वादशैर्भागमाहरेत् ॥
यात्रोद्वाहे शुभे कार्ये चन्द्रावस्थाः परित्यजेत् ॥ १ ॥

प्रवासनष्टामृतताजयाख्या
हास्यारतिक्रीडितसुप्तभुक्ताः ॥
जराह्वयाः कम्पितसुस्थितं च
मेषादिमुख्या हिमगोरवस्थाः ॥ १ ॥

इति लिखितं मलूकचन्द्रेण ॥

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2 °मुपेति] °मुपैति S29 3 पदस्य] मंदस्य S29 6 °सृङ्मृदूच्चम्] °सृम्मृदुच्चं S29 10 प्रवा[१]स[२]नष्टामृ[३]ता[४]जयाख्या S29
11 हा[५]स्या[६]रतिक्रीडि[७]तसु[८]प्त[९]भुक्ता S29 12 ज[१०]राह्वया कंपि[११]तंसुस्थि[१२]तं च S29

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