# The "Latitudines breves" and Late Medieval University Teaching 

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#### Abstract

While the new logico-mathematical approach to natural philosophy initiated by the socalled Oxford calculators around the third decade of the 14th century spread rapidly to the continent and was carried on at several European universities, there arose in the second half of the same century another way of dealing with the same questions that received considerable interest. Based on geometry rather than logic and philosophy of language, this new development sought to use geometrical notions and figures to express the key ideas employed by the calculators to analyze motion and quantify qualities-for example, the notions of "uniformity" and "disformity." Two texts especially advanced this new geometrical turn: Nicole Oresme's De configurationibus and the shorter Tractatus de latitudinibus formarum, whose authorship was not completely established before now. The latter text was the point of departure for the development of a new discipline, the "scientia de latitudinibus formarum," which obtained full scientific status as a core component of university teaching, along with similar disciplines. This new science was intensively cultivated especially at the University of Vienna. Here even shorter texts were produced for the sake of teaching - epitomes of the original text, which we may call "Latitudes breves." This paper provides an edition of three different of such epitomes, presents their contents and discusses the context of their production and development against the background of late medieval university teaching. A detailed description of the manuscripts is included.


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## I General Introduction and Status quaestionis

In the first decades of the 14th century, a new approach to the treatment of motion and quality - both of which were key factors in the (late medieval) Aristotelian analysis of natural phenomena-was adopted by the so-called Oxford calculators. This new approach was characteristically more quantitative than ontological-definitional and stimulated, as a natural epiphenomenon, questions connected increasingly to mathematical issues and methods. Although the traditional questions about the nature of the main physical magnitudes of space, time, matter and, especially, motion-What is motion? To which of the categories does motion belong if it does not constitute a category by itself? Is time something real that exists outside the soul?-continued to survive for centuries within the corpus of Aristotelian commentary, these questions were supplemented in the 14 th century by other, sometimes competing questions that were, from the outset, posed in a completely different way: How much time will elapse in the course of a motion over a particular distance? What is the velocity of a motion at a particular instant of time? In what sense can we say that a natural process begins or ends? And, more generally, how is this or that thing to be "measured"? However, such questions never touched on the problems of empirical methodology nor were they even intended to facilitate the search for an empirical method of measurement. Instead, their aim was to pose and analyse problems in a way that was as purely abstract as possible.

During the first generation of Oxford calculators-and at Merton College, in particular-logic and the philosophy of language were the twin motivations for raising new questions as well as the instruments for solving or, at least, for clarifying, them. The collections of sophismata - that is, logical puzzles and exercisescomposed by Richard Kilvington and William Heytesbury contain perhaps the best examples of this approach. Although not the only works of their kind, these collections - especially Heytesbury's Regule - were extremely popular, at least until the 16th century, both at British universities and on the Continent, where they were eagerly copied, reworked and commented on at different educational centers. ${ }^{1}$

Although the calculators' new approach to the treatment of motion and quality soon found an echo and rapidly gained supporters who were able and willing to continue it by repeating more or less the same topics and problems, by trying to supplement those topics and problems, by correcting them at a few points or by introducing a few not entirely unimportant shifts of perspective, an essentially new approach emerged around the beginning of the second half of the 14th century. This new "internal approach" consisted in privileging the use of geometry for solving-as Oresme would have said-"those matters that some other people seem to perceive in a confused way" and "express obscurely." ${ }^{2}$ Of course, the transformation proposed for dealing with these questions was in principle a methodological one, even if it is true that Oresme himself may have desired a more than purely methodological change and even if the only thing that remained at the end was, in fact, a methodological trivialization of the original, more profound insights. Nonetheless, it is certain that the representation of motions and qualities by means of geometry was one of the goals that Oresme unambiguously pursued (Maier 1952, 289-353; 1968, 89-109).

To a great extent, this transformation was only an internal development, whose aim was to find a more satisfactory way to present the same problems discussed within the sophismata literature and the logical-philosophical approach used by many of the first calculators. The quantification of all processes in the natural world, and - although less studied to date - in human behavior, as well continued

[^0]unabated. This was not the critical "humanism" of Bruni or Vives, who fought fiercely against everything that contained even a trace of calculationes, sophismata, incipit et desinit, maxima et minima or the like. However, the opposition that it entailed between a logical or "sophismatical" approach, on the one hand, and a geometrical one, on the other, generated from the very outset a demand for new concepts. There is some evidence that this opposition increased over time until, at last, logic finally prevailed. ${ }^{3}$ However, geometrization was able to resist and confront logic for a period of perhaps two hundred years. It provided a practical alternative that not only survived but, as long as it could find and retain a connection to university education, even expanded in scope. How did this happen? The aim of this paper is to provide one answer to this question - not, I would hasten to add, the only answer, but one that is important and well documented, although not yet generally known.

The study of the late medieval method of representing qualities and motions geometrically ("geometrization" for short) has focused to date primarily on two main texts: the treatises De configurationibus (DC) and De latitudinibus formarum (LF). The first text was composed by Nicole Oresme and the second is now generally attributed - on the basis of Maier's study - to Jacobus de Sancto Martino. ${ }^{4}$ However we define the relationship between these two texts-and this is still an open question in my opinion - it has always seemed beyond doubt: (1) that they are the key texts for the geometrization approach and (2) that the second text, LF, was disseminated much more widely than the first one, DC, and much more widely, in fact, than any other text dealing with the doctrine. As I have already reported, there are around 50 extant copies of LF and hence considerably more than was previously supposed. In addition, CD was never printed, whereas LF was printed four times (Di Liscia 2010, 421-28).

However, to obtain a more accurate knowledge of the expansive tendency of the geometrization approach in late medieval philosophy - spearheaded above all by LF - a number of shorter texts must also be considered. These texts, which I have

[^1]edited and will discuss here, are the so-called latitudines breves (LB), a title that is justified in terms of texts' contents as well as the historical documents relating to them. Although in fact independent of one another, these three texts (LB 1, LB 2 and LB 3) are all very similar. They contain many passages and sentences taken from the original LF. In a word, the manuscripts transmitting the three texts cannot be classified as copies of LF. The facts that I will present below will indicate why this is true. Nor is there any justification for regarding LB 1, LB 2 and LB 3 as "bad copies" of an ideal, original, unique LB. It is much more likely that all three LB texts are directly derived from, but not identical with, LF. It is also very possible that there are more LB texts in existence than the ones we currently know.

It is a notable phenomenon in the history of ideas - or rather in the history of mathematics and of the philosophy linked to it- that the diffusion of the doctrine of the geometrization of qualities and motions concerns almost exclusively the treatise LF and no other work of a similar nature. It is not obvious why this should be so. However, it can be understood by considering some aspects of late medieval university education.

## II The Latitude of Forms at the University of Cologne

The University of Cologne was for a certain time, if not the first, at least one of the major universities in the German-speaking world. In terms of the eclectic philosophical life of the 15th century, it was unsurpassed by other universities. Within this context, instruction in the doctrine of the latitude of forms is well documented at Cologne around the end of the 14th century.

For the training of arts students at Cologne, the following texts were prescribed: Aristotle's Physica, De caelo, De generatione et corruptione, Metheora and Parva naturalia; Sacrobosco's De sphaera; the Theorica planetarum; some books of Euclid's Elements (the first three certainly); Pecham's Perspectiva communis; and "some treatise on the proportions, and some on the latitude of forms, one on music and one on arithmetic." ${ }^{5}$ Apparently, this instruction according to "some treatise"

[^2]on the latitude of forms is a very special feature that distinguishes Cologne from the teaching at Heidelberg, its "parallel university," although the context in which the doctrine of the latitude of forms was taught at both universities was generally the same. Heidelberg seems to have emphasized the fact that within the mathematical disciplines teaching should be based on "a complete text." ${ }^{6}$ The expression "aliquem de latitudinibus formarum" certainly refers to texts derived from the Tractatus de latitudinibus formarum (LF) of Jacobus de Sancto Martino. But why was this text not explicitly mentioned? Ignorance of the author's name - which we may in this instance assume - cannot have played a decisive role since the author of the Theorica planetarum is not identified either. ${ }^{7}$ The statutes are perhaps not as clear as we would wish. However, one cannot escape the problem simply by reinterpreting them again and again. ${ }^{8}$ In any case, it seems clear that the situation at the time they were written (1398) looked like this: teachers were not required to lecture on a specific text but "on the latitude of forms" as a topic or doctrine. This must also have been true in other disciplines such as arithmetic, astronomy, optics and music - that is, those disciplines that at that time were generally classified by the Aristotelian tradition as middle sciences (scientiae mediae).

However, interest in the latitude of forms seems to have gradually declined at the Cologne Faculty of Arts in the course of the 15th century. Given the important role that certain philosophical trends played during this period in guiding the university's ideological orientation-especially in determining the contents of its curricula-it is likely that the dominant Albertist and Neo-Thomist currents promoted the previous positive attitude toward mathematized physics, including the doctrine of the latitude of forms, less forcefully. ${ }^{9}$

This view is supported, for example, by the Positiones of Henry of Gorkum (and other Cologne masters), which provides evidence as to the type of material used at the university to teach natural philosophy. ${ }^{10}$

[^3]Henry comments on a positio in the second book of Aristotle's Physics concerning the relationship between mathematics and physics: Are these essentially physical or mathematical? The Thomist-oriented reading of this book provided a good opportunity not only to mention but also to discuss in detail the latitude of forms, which, as one of the scientia media, belonged-as we have seen-to the arts curriculum. Therefore, it is all the more surprising that the doctrine is not mentioned at all, although several others are: the traditional middle sciences and some other, very new ones. ${ }^{11}$ These views are only reported: pro affirmative is provided on the authority of St. Thomas (and on that of the Aristotelian text!); pro negative is stated to be the opinion of Averroes. These are the two views that had laid the basis for discussion in the 14th and 15th centuries. The Cologne Positiones could have gone further than it did-a strong indication that interest in the latitude of forms was weakening at the University of Cologne. Such a bias is ultimately consistent with a distancing from the mathematically oriented physics of the calculators and other Parisian physicists close to Buridan. The calculators' typical fields and "analytical languages" were treated in only a few places and then only superficially. ${ }^{12}$ Thus, a discussion of maxima et minima occurs in connection with a passage in the first book of the Physics concerned with the limit ad maximum and ad minimum in the entia naturalia. It is obvious that the commentator here is aware of the calculators' discussions and terminology but considers the matter too unimportant for further consideration. A similar attitude can be found in relation to the infinite, a very convenient area for calculations and, occasionally, for the latitude of forms. However, the comment in this case is limited to the discussion of whether the infinite can be understood cathegorematice or syncathegorematice. Basically, all the mathematical content has been omitted and replaced by semantic discussions. In the end, it is clear where all these discussions belong: "as it is clear for all those propositions

[^4]'the line is infinite,' 'the motion is infinite,' 'infinite are the parts in the continuum,' 'there were infinite humans,' 'Socrates can carry an infinite weight,' 'infinite <things〉 are finite <things〉.' And because the exposition of these propositions concerns logic, I will postpone them for the treatment of the exponibilia." ${ }^{13}$ It remains to be examined in more detail how far the scholars at the Cologne Faculty of Art went in their study of the latitude of forms and whether physical texts-mainly commentaries on the Aristotelian Physics-were produced that could perhaps reveal a more serious concern with the mathematical methods and approaches stemming from the calculators. However, it appears that the geometrization phase had ended by around the beginning of the 16th century and that, following a somewhat superficial engagement with mathematics, the topics considered in that phase were reassigned to logic and the philosophy of language. ${ }^{14}$

## III The Latitude of Forms at the University of Vienna

At the University of Cologne, interest in the doctrine of the latitude of forms-which had been considerable in the university's early days-gradually declined in the 15th century, perhaps under the influence of the Albertists and Neo-Thomists. However, it did not abate at the University of Vienna, where the so-called middle sciences-of which the scientia de latitudinibus formarum was one-were experiencing a true flowering at that time. ${ }^{15}$

As we will see in the next sections, there is convincing evidence that the manuscripts containing the three texts LB $1-3$ stem from the University of Vienna, where the doctrine of the latitude of forms was received and integrated into the curriculum at an early date. Interest in the doctrine must have continued almost

[^5]without interruption at least until 1515, when the text of LF was printed in Vienna. Hence, my aim in this chapter is to provide historical evidence for the acceptance of the doctrine of the latitude of forms at the University of Vienna and thus a proper historical background for the emergence of the texts LB $1-3$ as visible products of a then existing scientific discipline, the scientia de latitudinibus formarum.

## III. 1 Henry of Langenstein

How could such a doctrine have reached Vienna if - as we may assume - it did not originate there? The first step may have been taken by the students or colleagues of Jean Buridan who left Paris to help found and develop universities in Germany (Gabriel 1974). We know for certain that the major figures of Marsilius of Inghen and Albert of Saxony were very active in disseminating the new philosophical trends that had gained the upper hand in Paris. As Markowski (1988) has correctly pointed out, "Buridanism" is the key word here. Incorporating many of the new ideas on logic and natural philosophy in Aristotlian commentaries and in special treatises as well, the works of these masters soon reached many centers of study, such as Prague, Cracow, Vienna, Erfurt, Heidelberg, Cologne, Leipzig, Rostock and Greifswald, which were either located within the German Empire or academically connected with it. The printing of their works and the circulation of an impressive amount of manuscripts indicate that their influence also extended to Italy. ${ }^{16}$ Nevertheless, neither Marsilius nor Albert seems to have taken serious notice of the latitude of forms or of Oresme's configurationes doctrine. ${ }^{17}$

The situation regarding Henry of Langenstein (d. 1397), another member of Buridan's circle, is completely different. Henry, who was highly esteemed at the University of Vienna, did not write commentaries on Aristotle, but composed short, original, specialized works on natural philosophy as well as a copious commentary on the Bible. ${ }^{18}$ There can be no doubt that he was aware of the doctrine of the geometrization of motions and qualities, which he called the scientia de latitudinibus.

[^6]By this phrase, he evidently did not mean Oresme's treatise on the configurationes but another famous text-namely, LF. Perhaps he was the first or, in any case, one of the first to bring the teaching of the latitude of forms to Vienna and introduce it into the curriculum. It is also certain that, following his appointment at Vienna in 1384, he was active - along with Henry Totting von Oyta-in reforming the curriculum, that he was quite familiar with the latitude of forms and that he included this doctrine among the middle sciences in his Arbor scientiarum, alongside astronomy, music, perspective, the science of weights and-interestingly—"the subtle calculations." ${ }^{19}$ Besides this clear statement, there are two passages in Henry's De reductione effectuum in which he shows his familiarity with the ars of the latitude of forms, one of which, however, is not mentioned by Clagett. ${ }^{20}$ Henry of Langenstein may very well be the person who introduced the doctrine of the latitude of forms at Vienna: he went to Vienna after studying at Paris, he was active in the organization of the curriculum in Vienna, he himself was very interested in the latitude of forms - and certainly not only as a mere method of representation-and his works provide clear evidence that he knew this set of ideas, which he characterized as a scientific discipline or doctrine. ${ }^{21}$

## III. 2 The Statutes of 1389

Be that as it may, the inclusion of the latitude of forms in the curriculum alongside the other middle sciences and the Aristotelian commentaries is readily apparent from the Statutes of the Faculty of Arts of 1389 - that is, some ten years before the University of Cologne. These statutes have been known, at the very latest, ever since the publication of Kink's Geschichte der Wiener Universität (1854). They were printed again in Lhotsky's standard work on the Vienna Faculty of Arts (1965).

[^7]To obtain their licentia in artibus, students were required to read a treatise on proportions and one on the latitude of forms, a book about music and a book about arithmetic. ${ }^{22}$ A further passage on the libri ordinarie legendi-which, as we shall see below, is indeed very similiar to the list that I found in Ms. Melk, Stiftsbibliothek, 901 - contains the complete title De latitudinibus formarum and states that two grossi are to be payed for lectures on it. The context is quite similar, but it is interesting to note that the so-called Proportiones longe Bradwardini is the textbook proposed for the study of proportions. Therefore, we may assume that the statutes are referring here to lectures on the complete original treatise of LF as well. ${ }^{23}$

On Clagett's dating of LF, this text cannot be the one meant here since it had not yet been produced. But what other text could the statutes be referring to? Certainly not to Oresme's De configurationibus. Moreover, it is extremely unlikely that the statutes - or anyone else - would refer to Oresme's Euclid commentary in these terms, and it is highly unlikely that many treatises on proportions, on music, on arithmetic and, as we have seen in the statutes of the Cologne faculty, on the latitude of forms were in use at that time. What, then, is the explanation? There is a quite simple and easily verifiable answer. We have only to recall that there is no reason to think that the original treatise LF was not read in Vienna and that wherever we find the expression latitudines formarum it is very likely that some version of LF was known. ${ }^{24}$ The text, as LB, obviously existed earlier and formed the basis for abstracts, excerpts and summaries. In addition, as we have already

[^8]seen, there is unmistakabe evidence in the same document, in which LF is included among the libri ordinari, that it was studied and analysed in full at the Vienna Faculty of Arts.

Thus, starting around 1389, the use of the original treatise LF was so successful that its subsequent inclusion in the teaching curriculum led naturally to the production of several shorter versions: our latitudines breves. A similar phenomenon may be observed in connection with the other middle sciences and even in the case of the Aristotelian commentaries-as for instance in the Questiones Wienneses, which are transmitted in the manuscript $\mathbf{W i}$ along with LB 2 (for the description, see section IV. 2 below). What the statutes did prescribe at this point was that pro licentia in artibus students should use some of these texts, which were in turn edited versions of LF made for the classroom (and we know from another passage in the statutes that the text LF was, indeed, taught). Of these abbreviated texts, only three (LB $1-3)$ have thus far been identified, but it is likely that we will find more as research progresses. ${ }^{25}$ It is difficult to say how much time passed between the composition of LF and its "normalization" at the University of Vienna, since we have no precise indication of the origins of LF itself. However, since its author was not a member of the university staff, we can be sure that this change did not happen overnight. A certain amount of time must have elapsed before a text could become a standard "handbook" to be read, commented on and abbreviated at university teaching level.

## III. 3 Proportions, the "Sister" Discipline of the Latitude of Forms

Before continuing with our survey of the incorporation of the latitude of forms into the university curriculum, it will be useful to make some general remarks on the science of proportions. This discipline deserves special attention because it, too, was considered a "new middle science" and because its textual background is similar to that of the latitude of forms.

For the study of the doctrine proportions, which was not pure mathematics but a mathematical doctrine intimately connected with natural philosophy (and hence a middle science as well), a wider spectrum of treatises was available. The first of these was Thomas Bradwardine's then famous treatise De proportionibus velocita-

[^9]tum. This text was followed by many other reworked treatises, which, while exhibiting varying features or emphases, were all ultimately based on Bradwardine's. One of the more influencial works of this kind was Albert of Saxony's Tractatus proportionum. Despite the fact that this treatise was apparently read all over the continent and hence in Vienna as well, the documents at Vienna University often mention another text called Proportiones breves (Clagett 1959, 481-94). This text was often considered a separate work of Bradwardine's. However, it is, in fact, merely a summary of his original De proportionibus velocitatum. Furthermore, since it constituted a fundamental discipline-defining text, it was sometimes abbreviated, like the Proportiones breves, and sometimes commented on in detail. In fact, there is a text on proportions with the title Tractatus longus preserved in two Viennesse manuscripts (ÖNB 4951, ff. 260v-271v and 4953, ff. 19r-36r). I think that this text must be the same as the proportiones longe mentioned in the statutes. The situation seems to have been similar in the case of other mathematical disciplines such as the Algorismus and the Arithmetica, which included partially related material. ${ }^{26}$ The Proportiones breves seems to have been quite a popular text. It was finally printed in Vienna in 1515 along with the text of LF (without any geometrical figures) and thus provided material that could be used for teaching both disciplines at the Faculty of Arts. The situation and the phenomenon were the same in both cases: a longer, more detailed and theoretically more complex treatise was replaced by a shorter version for teaching purposes - Bradwardine's treatise by the Proportiones breves and LF by the LBs. However, there was a significant difference: whereas a text named Proportiones breves, which was used and circulated as such for the sake of covering the content of the proportions doctrine, did in fact exist, there was no text called Latitudines breves. Here, apparently, no short version of the original LF ever acquired the status of a "standard" summary, as the Proportiones breves did in the case of Bradwardine's treatise. Instead, there were a number of different texts, all of them in common use at the Vienna Faculty of Arts. As a result, we are justified in using the appellation Latitudines breves, borrowing it from the parallel situation described above, as long as we keep in mind that the two situations also differ in a certain respect.

## III. 4 The Acta of the Faculty of Arts: 1385-1416

Among the first records of the Vienna Arts Faculty (1385-1416) relevant for our inquiry, we find considerable evidence regarding the process of "normalizing" or "standardizing" the doctrine of the latitude of forms by including it in the university

[^10]curriculum. The first Acta has been well edited by Paul Uiblein. For the years before, however, the research situation is unfortunately less favorable. Nevertheless, we will see that there is some precise evidence clearly showing the survival of the latitude of forms in Vienna until at least the beginning of the 16 th century. ${ }^{27}$

By the way, the incorporation of the doctrine of the latitude of forms into the Vienna curriculum was not an isolated phenomenon but one accompanied by the acceptance of the new logic and of other new scientific developments in late medieval literature (Kren 1987). ${ }^{28}$ The Acta includes the same books mentioned in the statues. Hence, the background is basically the same as at the University of Cologne. In natural philosophy, the Aristotelian libri naturales - in the usual order beginning with the Physics, followed by De caelo and so on-were obligatory. In geometry, several books of the Elements were read. In arithmetic, the Arithmetica speculativa by Jean de Muris was generally the basic text. The Acta also shows that a great deal of attention was paid at Vienna to the middle sciences. These included astronomy, perspective and, of course, proportions and the latitude of forms. The study of astronomy was based on the Theorica planetarum and Sacrobosco's De sphere, while that of perspective was based on texts by Pecham and Witelo, although it is likely that only some minor parts of the latter's monumental work were studied. For logic, students were required to attend courses not only on the Aristotelian texts but also on the parva logicalia, which included quite a large dose of insolubiia, obligationes and consequentiae.

A closer look at the Acta for the years 1385-1416 shows which magistri taught what, when and on the basis of what text. For the doctrine of the latitude of forms, the results are as follows: ${ }^{29}$

| Name | Date |
| :--- | :--- |
| Nycolaus Dinckelpuel $^{1}$ | Sept. 1, 1391 |

[^11][^12]| Name | Date |
| :--- | :--- |
| Iohannes Flueck $^{2}$ | Sept. 1, 1392 |
| Petrus de Pulka $^{3}$ | Sept. 1, 1394 |
| Bernhardus Werbardus $^{4}$ | Sept. 17, 1395 |
| Michael Suchenchacz $^{5}$ | Sept. 1, 1396 |
| Ulricus de Patavia $^{6}$ | Nov. 10, 1398 |
| Fridericus de Patavia $^{7}$ | Nov. 31, 1399 |
| Vitus de Praga (Student) $^{8}$ | Jan. 2, 1401 |

Henry Totting of Oyta, he later became Dean of the Faculty of Arts and of the Faculty of Theology as well. During 1405-1406, he served as Rector of the University. He participated in the Council of Constance (1414-1418) and was engaged in several political affairs involving the Holy See. He composed over sixteen works, primarily on religious and theological questions (see Aschbach (1865, 430-40) and especially Madre (1965)).
2 ". . . Petrus de Walze 5 libros Euclidis . . . magister Leonardus Obligatoria et Insolubilia, magister Petrus de Pulka Arismetricam et Proportiones, magister Fridericus Drossendorf Perspectivam, magister Iohannes Flueck Latitudines formarum . . Petrus de Rutlinga Consequencias" (Uiblein (1968, 79), my italics).

3 "... Magister Mathias de Walse Obligatoria et Insolubilia . . . magister Nicolaus Dinkilspuhel Speram materialem . . . magister Petrus de Pulka Latitudines formarum" (Uiblein (1968, 106), my italics). According to Aschbach (1865, 424-28), Petrus de Pulka lectured on almost all subjects but later focused on theology. See also Uiblein (1966, 95-107).

4 "... Cholomanno de Nova Villa Consequentiae, magistro Bernhardo Werbardi Latitudines formarum, magistro Iacobo Ettenhaym Proportiones breves..." (Uiblein (1968, 122), my italics).
5 "...magister Wilhelmus Püschinger Supposiciones, Ampliaciones et Appellaciones et Restrictiones, magister Iacolbus de Ettenh(eim) Obligatoria et Insolubilia ... magister Nicolaus Ungelter Consequencias Marsilii ... magister Waltherus (Walter von Lenzburg) Proportiones ... magister Michael Suchenschacz Latitudines formarum..." (Uiblein (1968, 138), my italics). According to Aschbach (1865, 416-18), Michael Suchenschatz focused on theological matters and followed the doctrines of his master Henry of Langenstein.
6 ". . . magister Ulricus de Patavia latitudines formarum . . . magister Marquardus de Wila Obligatoria" (Uiblein $(1968,165)$, my italics).

7 "... Magister Cunradus de Rotenburg Proportiones breves Wragbardin ....Hermannus de Waltsee Perspectivam Communem Allacen ... Ihoannes de Winpina Euclidem, magister Iohannes Gossolt Consequencias magistri Marsilii . . . magister Fridericus de Patavia Latitudines formarum..." (Uiblein (1968, 171), my italics).
8 ". . . baccalarii volentes temptari pro licencia (...) $9^{\circ}$ Ihoannes Rechberger defecit in Rethorica, Obligatoribus et Insolubilia, 4 libris Thopicorum, in Perspectiva, Musica, Proporcionibus, in exercicio 2di Posteriorum, sed actu stetit 3ii De anima et in parte 5ti Ethycorum. $10^{\circ}$ Vitus de Praga defecit in $4^{\circ}$ Methaphysice, Latitudinibus formarum et in partem anni..." (Uiblein (1968, 187-8), my italics).

| Name | Date |
| :--- | :--- |
| Ihoannes de Rechperb ${ }^{9}$ | Sept. 1, 1402 |
| Iohannes Fühlt ${ }^{10}$ | Sept. 1, 1414 |
| Thomas Öder de Aspach $^{11}$ | Sept. 1, 1415 |

As this list shows, a regular concern with the doctrine of the latitude of forms is well documented at the University of Vienna between the years 1391 and 1415. The magistri of the Arts Faculty lectured on different subjects every semester. There was no specialist for the latitude of forms or even for mathematics. Every master had to be able to teach in all areas, from the Aristotelian ethics to the proportions and the latitude of forms. Hence, any of the individuals named may have been the "author" - that is, compiler or abbreviator-of such summary texts as LB 1-3. Perhaps these texts were conceived as "anonymous" summaries that the teacher should and could use in the classroom without concern for questions of authorship. The subject matter of the lectures thus had a more abstract character since it was more independent of one particular text composed by one well-known reputed author.

## III. 5 The Wiener Artistenregister at the Faculty of Arts: 14161447

For the years after 1416, we unfortunately do not have an edition as excellent as the one produced by Uiblein for the first period. However, there are a large number of supplementary materials that attest to the study of the latitude of forms at the faculty. By examining the Wiener Artistenregister 1416-1447 (2007), in particular, we can link some names to activities related to the teaching of the latitude of forms, but these links are too weak to establish the authorship of any LB. Looking at the register, it is evident that the doctrine was taught almost every year during the years in question. ${ }^{30}$

[^13][^14]| Name | Year | No． | Page |
| :--- | :--- | :--- | :--- |
| Johannes Flechtner | 1416 | 3291 | 3 |
| Johannes de Bamberga | 1417 | 3448 | 8 |
| Geor〈g？ ius Forster | 1418 | 3678 | 14 |
| Johannes de Ulma | 1421 | 4062 | 26 |
| Cunradus Langestat | 1423 | 4459 | 38 |
| Johannes de Tytmanyn | 1424 | 4655 | 44 |
| Jacobus de Schirling | 1425 | 4809 | 48 |
| Erhardus de Langniczen | 1425 | 4832 | 49 |
| Lucas de Prun | 1426 | 5010 | 54 |
| Johannes Sachs | 1427 | 5151 | 58 |
| Andreas Santperg | 1428 | 5371 | 64 |
| Conradus de Landsperg | 1429 | 5574 | 69 |
| Wolfgangus de Chuttefeld | 1430 | 5575 | 74 |
| Johannes Slitpacher | 1430 | 5579 | 74 |
| Joeronimus de Öting | 1431 | 5958 | 81 |
| Johannes Raetelchofer | 1432 | 6194 | 87 |
| Uldaricus Langmantel | 1432 | 6202 | 87 |
| Johannes Raetelchofer | 1433 | 6363 | 93 |
| Leonhardus Halstat | 1435 | 6645 | 101 |
| Bero 〈Magni〉 | 1437 | 6856 | 108 |
| Tidmannus de Kalmaria | 1438 | 7049 | 114 |
| Johannes Permater | 1439 | 7278 | 121 |
| Jodocus Esslinga | 1441 | 7886 | 136 |
| Johannes Echtertinger | 1441 | 7891 | 136 |
| Thomas de Cisterstorff | 1443 | 8460 | 151 |
| Sigismundo de Lengenveld | 1443 | 8469 | 152 |
| Johannes de Lansperig | 1445 | 8912 | 164 |
|  |  |  |  |

Some of these masters－such as the Swedes Tidemannus Vesthof（＝Tidmannus de Kalmaria）${ }^{31}$ and Bero Magni de Ludosia（from Lödöse）${ }^{32}$ —have been studied

[^15]extensively. For most of them, however, no further information is yet available.

## III. 6 Friedrich of Essen-Essenbach's Quaestiones quodlibetales

A group of Questiones quodlibetales from 1447 contained in the manuscript Mnthe manuscript that contains LB 3-provides yet further evidence for the survival of the doctrine of forms between 1416 and 1447. This evidence is also remarkable for the fact that the statutes of 1389 include an entire section devoted to disputations de quolibeto, for which, of course, no specific titles or disciplines are stated, and hence nothing directly concerning the doctrine of the latitude of forms can be found (Lhotsky 1965, 255-57).

These questions were composed by a doctor of theology named Friedrich of EssenEschenbach, to whom I shall return later (see the description of the manuscript, section IV.3. below, text 7 , ff. $13 \mathrm{v}-56 \mathrm{v}$ ). As is usual in such cases, the text integrates a variety of topics. The complete collection comprises forty questions, all of which include the name of the master who defended a particular thesis under it (I have added a number for each question). Some of the questions - such as No. 2-concern purely logical matters: utrum descensus sit consequentia formalis (ff. 14r-16v, Mathias de Weinsperger) and No. 33: utrum sillogismus expositorius sit bonus et formaliter consequentia (ff. $45 \mathrm{v}-46 \mathrm{v}$, Clemens de Pfarzach). Others are about ethics. These include No. 16: utrum omnis delectatio sit moraliter bona (ff. 28v-29r, Bartholomaeus de Syracusa) and No. 23: utrum voluntas humana possit sine habitu actum moraliter bonum elicere (ff. 35v-36r, Thomas de Wuldendorff). Still others discuss topics concerning the soul-for example, No. 1: utrum possit probari ratione evidenti animan humanam post hominis mortem perveniare (ff. 13r-14r, Johannes de Camanig) and No. 11: utrum tota anima sit in toto corpore animato (ff. $24 \mathrm{r}-25 \mathrm{r}$, Johannes de Peregriens). There is a question specifically about astrology (No. 10, $\mathrm{ff} .24 \mathrm{r}-\mathrm{v}$ ) and one about medicine (No. 22 , ff. $34 \mathrm{v}-35 \mathrm{r}$ ). Question 12 deals with the well-known physico-mathematical problem of the sphere touching a plane: utrum sicut corpus durum tangit durum, ita sphericum tangat planum sibi super positum in punto (ff. $25 \mathrm{r}-26 \mathrm{r}$, Nicolaus de Huttendorff). For us, No. 30, which concerns the reception of fourteenth-century natural philosophy is especially relevant: utrum non suscipere magis et minus similiter in habere contrarium insit subiectum ex natura rei vel secundum dici (ff. 43r-v, Paulus de Bamberga). No. 40, the final, somewhat longer question, discusses Burley's theory of intension and remission of forms: utrum ad hoc quod aliqua qualitas intendatur, requiratur quod gradus precedens et sequens maneant simul (ff. 41r-65v, Lukas de Stherduus). There is also a brief question by a magister Philipus Mauritius of Vienna that runs: utrum omnem latitudinem formarum inesse eidem materie subiective sit rationibus phisicis demonstrabile (ff. $45 \mathrm{r}-$
ascribed to Nicholas Oresme but now considered to be the work of Jacobus de Sancto Martino" (p. 94, f. 20).
v）．This last question exhibits a close acquaintance with the doctrine of the latitude of forms，enabling us to infere that this discipline was still being studied at Vienna and that，since the disputation took place at the Faculty of Theology，it was con－ sidered useful enough to be applied to other subjects．A quotation in extenso may illustrate the kind of problems to which the doctrine of the latitude of forms was then linked：${ }^{33}$

## Whether it is demonstrable by physical reasons that each latitude of forms is present in

 the matter subjectively．Furthermore，a latitude is an intensively divisible quality，as for instance a quality of the first or of the second kind and continuous local motion．A uniform latitude is

[^16]Notandum quod inest materie subiecte quod inheret materie tamquam forma suo subiecto．Item ratio phisica non est aliquid nisi［f．45v］argumentum phisicum．Item illud dicitur est demonstrabile ex rationibus phisicis quod pertinet ad causas vel demonstratur ex phisica．

Secundus articulus，conclusio 〈prima〉：quamvis latitudo forme insit solum materie prime，non tamen omnino，ut patet de speciebus intentionalibus．Secunda conclusio：omnem latitudinem for－ marum inest materie probatur subiective ex rationibus phisicis est persuasibile．Patet 〈per〉auc－ toritatem Philosophi primo Phisicorum，ubi dicit materie est attributum subiecti quod fit．Etiam probatur ratione，quia videmus quod ex straminibus attributum continetur in materia ignis，igitur $\langle e t c\rangle$ ．Tertia 〈conclusio〉：propositum responditur：sicut omnem latitudinem formarum inest ma－ terie prime est ex rationibus phisicis persuasibile，ita etiam ex rationibus phisicis demonstrabile， quia solum divisibile extensive inheret materie prime．Probatur：omne quod est eductum，deponi－ tur materie id est materie prime．Sed sic est de latitudine forme，etc．Maior est vera loquendo naturaliter．Consequenter responsio habetur ex secunda propositione ad questionem affirmativa， etc．
one that is of the same degree throughout and must be understood as follows：it is a latitude that in itself（secundum se）and according to any divisible part of it is of the same intensity throughout．A difform latitude is one that is not of the same degree throughout and can be varied in infinite ways．Precisely uniform is such 〈a latitude〉 that is equal between the excesses of the degrees．

Furthermore，a figure is described as follows：it is a magnitude enclosed by a border or by borders and for this term（＝ly or＂li＂according to this copyist）＂magnitude＂is to be considered a substance．A rectilinear figure is a 〈figure〉 that is contained by straight lines alone．But a curvilinear is a 〈figure〉 that is contained by one straight line and one curve．Furthermore，of the figures，one is plane，and this is the figure that is measured both in longitude and in latitude by a straight line，one is curved，and this is one that either according to the longitude or according to the latitude is measured by a curved line．

It is to be noticed that an angle is the drawn connector of two lines．A right angle is the angle that is drawn by two straight lines falling perpendicularly one to the other．An acute angle is an angle less than a right angle．A difform latitude that is called＂uniform difform＂varies in another way－namely，when（inquantum）it 〈in its variation〉 retains a proportion of equality．

It is to be noticed that what inheres in the matter as the form 〈inheres〉 in its subject is present in the matter subjectively．Furthermore，a physical reason is nothing but a physical argument．Furthermore，that which belongs to the causes or which is demonstrated from physics is said to be demonstrable from physical reasons．

Second article，first conclusion：even if the latitude of a form is present only in the first matter，however，not in all cases，as it is evident in the intentional species． Second conclusion：it is persuadable that it can be proved from physical reasons that the latitude of the forms is present in the matter subjectively．This is evident on the authority of the Philosopher in the first book of the Physics，where he says that to the matter belongs an attribute of the subject which it $\langle=$ the matter〉 constitutes． This is also proved by reason，since we see that the attribute of straws is contained in the matter of the fire，and therefore etc．Third conclusion：the proposed 〈question〉 is answered：as it is persuadable from physical reasons that every latitude of forms is present in the first matter，it is likewise demonstrable from physical reasons because only that which is divisible in extension inheres in the first matter．This can be proved： everything educed sets matter aside－that is，first matter．But the situation is the same regarding the latitude of the form，etc．The major 〈premise〉 is truth speaking naturally （＝physically）．Consequently，there is an affirmative answer to the question by taking the second proposition．

As it can be seen，the content of this question is relevant for our survey．It discusses the problem of whether it is possible or not to physically verify the existence of a latitude on（or in）the matter of the body which has the quality（that is，in
the subjectum). This question is ultimately answered in the affirmative. For us, the fact that the author is aware not only of LF but also of at least one of the LBs is noteworthy. In addition to the standard definitions of latitudo uniformis, difformis and so on, the question also contains the definition of "angle." This definition can only be assumed for the original text of LF, but it was explicitly provided as a supplement in LB 2 and LB 3. I am convinced that further investigation of such collections of questiones will uncover more examples. ${ }^{34}$ It is noteworthy that the copyist of this text expressly states the date and place where this question was discussed: "anno domini 1447 (...) in aula artistarum in alma universitate wiennense" (see the description below, section IV.3).

## III. 7 The Influence of Humanism on the Latitude of Forms at the University of Vienna

It is a well-known fact that, from the second half of the 15 th century to the early 16th century, the Vienna Arts Faculty underwent major changes mainly due to the influence of Neoplatonism and, above all, Humanism. From the middle of the 15th century onward, students at the University of Vienna attended lectures on Cicero, Virgil and Juvenal. Attendance at a lecture on a liber in rethorica was obligatory. ${ }^{35}$ In 1451, two individuals prominent in the cultural scene of the time visited Vienna: the philosopher, theologician, historian and politician Nicholas of Cusa, and John of Capistrano, the Italian itinerant preacher who had a far-reaching influence on the life of the university. Tomas Ebendorfer (Thomas von Hasselbach), the famous historian of Austria, was perhaps the most active person in promoting the new trends at Vienna. Aneas Silvio Piccolomini, the later Pope Pio II and a leader of the humanist movement, was also connected with the university. ${ }^{36}$

The new currents in philosophy and education did not only concern poetry,

[^17]rhetoric and belles lettres - as a misunderstanding of the term "humanism" might lead one to suppose. At the University of Vienna, which maintained close contacts to Italy and its culture, the humanistic approach also included innovative ideas in scientifc fields as well (Keßler 1979). The important work of integrating philosophy, science and philology undertaken by the Greek Cardinal Bessarion (1403-1472), who arrived in Vienna in 1460, should also be mentioned in this context. Although Bessarion made major efforts to recover the scientific treasures of classical culture, it was Johannes Gmunden (1380/84-1442) and then Georg of Peuerbach (1423-1461) and Regiomontanus (1436-1476) who laid the basis for the new astronomy and the new cosmology that were to be developed a hundred years later. The extraordinary coherence of the work and cultural impact of Conrad Celtis (1459-1508) is perhaps the best example of the trends connecting science and poetry at the time. ${ }^{37}$

The humanist movement was probably the most important cultural force operating against the scholastic tradition in philosophy, science and logic. It eventually imposed its own criteria of knowledge and scientificity. Humanism combated scholastic grammar, rude Latin and the seemingly pointless concern with "the English subtilities" and their like. It fought the obligationes, the consequences and all the other logical disciplines worked out during the 14th century and ultimately pushed all the parva logica and sophismata out of the curriculum. Why should such equally abstruse disciplines as the latitudines formarum and the proportions still be tolerated? We could certainly expect that the impact of humanism would have caused the study of the calculations, sophismata and, since they were historically allied, the doctrines of proportions and the latitude of forms as well to be removed from the curriculum at German universities and, in particular, at Vienna. ${ }^{38}$

Nevertheless, the true situation seems to have been quite different. The new

[^18]trends in philosophy and science do not appear to have developed any special aversion to these two doctrines. Whatever might be the reason for this - perhaps it was because these two disciplines were considered new middle sciences - the doctrine of the latitude of forms was still a fixed, well-documented part of the curriculum at the Faculty of Arts in Vienna at that time.

The most important and impressive document in this connection is an expositional commentary on LF. This commentary is preserved in two manuscripts, one from Vienna and the other from Freiburg in Breisgau. The Vienna manuscript was written by Michael Lochmayr de Heydeck, Dean of the Faculties of Arts and Theology and Rector of the University. There is no further evidence supporting Lochmayr's authorship of the commentary, which he nonetheless could have used in his teaching. However, since he stated that he copied the text in Vienna in 1466, we may assume that the teaching of LF was still well positioned in the university curriculum in the middle of the 15th century. As we will see below when discussing the content of the texts, Lochmayr's commentary on LF is clearly linked to the content of some of the LBs. ${ }^{39}$

In the middle of the 15th century, when Johannes von Gmunden and Georg von Peuerbach were teaching and Regiomontanus was a young student of mathematics and astronomy, the University of Vienna was perhaps the most popular university in Germany. ${ }^{40}$ Regiomontanus's intellectual interests developed within a framework of natural philosophy and the middle sciences. At Vienna, he attended lectures on the Metheora, Theorica planetarum, Perspectiva communis, Tractatus de proportionibus and Proportiones breves Bragwardini, Arismetrica and Latitudines formarum - that is, in all the subjects mentioned above. Some of his masters - for example, one Georg Molitoris from Eggenburg in 1450 and a certain Stephan Molitoris from Bruck an der Leitha in 1452-are listed as having taught proportions and, more particularly, the latitude of forms. ${ }^{41}$ Whatever the central motivation for the changes promoted by the humanist movement may have been, it was clearly not directed against the doctrine of proportions, the doctrine of the latitude of forms or the other middle sciences, all of which were still included in the university curriculum. ${ }^{42}$

[^19]
## III. 8 The Wiener Artistenregister at the Faculty of Arts: 14471471

The Wiener Artistenregister provides the same sort of information for the period from 1447 to 1471 as it does for the previous years (see section III. 5 above) - namely, that the doctrine of the latitude of forms continued to be taught ${ }^{43}$.

| Name | Year | No. | Page |
| :--- | :--- | :--- | :--- |
| Conradus de Mundelham | 1448 | 9557 | 9 |
| Johannes de Echtertingen | 1449 | 9836 | 17 |
| Michael de Nissa | 1449 | 9873 | 18 |
| Thomas Mestlin | 1450 | 10195 | 25 |
| Wolffangus de Lauffen | 1451 | 10657 | 37 |
| Stephanus de Prugk | 1452 | 10960 | 44 |
| Gregorius de Kusaal | 1452 | 11033 | 46 |
| Thomas Odenhofer | 1452 | 11045 | 47 |
| Martinus Hainczel | 1454 | 11574 | 60 |
| Conradus Schorndorff | 1455 | 11902 | 68 |
| Vitus de Schaufhausen | 1456 | 12231 | 76 |
| Conradus de Mundelhaym | 1457 | 12504 | 83 |
| Paulus Tag | 1457 | 12541 | 84 |
| Stephanus Schontritt | 1458 | 12962 | 95 |
| Leonardus Räss | 1458 | 12968 | 95 |
| Johannes Stainer | 1459 | 13385 | 106 |
| Gabriel Balther | 1459 | 13418 | 107 |
| Johannes Swalb | 1460 | 13755 | 115 |
| Marcus Chezner | 1463 | 14377 | 131 |
| Thomas Wienner | 1464 | 14547 | 138 |
| Petrus Egkch | 1465 | 14772 | 144 |
| Johannes Ckanczler | 1466 | 15023 | 150 |
| Bilhelmus Plätler | 1467 | 15286 | 157 |
| Bartholomeus Tichtel | 1468 | 15493 | 164 |
| Uldaricus Nensling | 1470 | 16014 | 178 |
| Georius Haller | 1470 | 16058 | 179 |
| Rupertus Newschel | 1471 | 16494 | 190 |

As late as 1498, there is a similar record of a master Andrea lecturing on the
Koller $(1996,13)$ observes that "Vienna closed itself first of all to political developments and soon after to the cultural ones." The university's high regard for its own history could have been one reason why it lost its connection to humanism and the new cultural trends.
43 Wiener Artistenregister 1447-1471 (2007).
"latitudines fa.," an abbreviation that can only refer to the latitudines formarum. ${ }^{44}$

## III. 9 Two Further Lists of Lectures on the Latitude of Forms

To this evidence can be added two further unmistakable statements that I came across in two different manuscripts. Although only one one of the manuscripts is dated, it is quite evident that both texts are closely connected with the Vienna Faculty of Arts in about the middle of the 15 th century. The manuscript from Vienna, ÖNB 4951 (f. 163v) contains a list of the magistri and the texts to be read: ${ }^{45}$

> Physicorum: a magistro Alphonso Tanner Metaphysica: a magistro Johanne Londior
> De celo et mundo: a magistro Thoma Horn Metheorum: a magistro Gerardo Posteahnso
> De generatione et corruptione: a magistro Laurentio Asmiol
> Parva naturalia: a magistro Jacobo Ponteano
> Theoricas planetarum: a magistro Simone Laß Musicam Muris: a magistro [illegible]
> Topicorum: a magistro Christophoro Elmboc Perspectiva communis: a magistro Wolfgango [illegible] 4 or libros Euclidis: a magistro Gregorio Pzango Latitudines formarum: a magistro Jacobo Alpaz Proportiones breves: a magistro Joanne [illegible] Arismeticam: a magistro Johanne Petri

As regards the doctrine of proportions and the latitude of forms, it seems that the old curriculum was still being followed in the year 1501, which is the date of the manuscript (fol. 271 v ). ${ }^{46}$ The teacher of the latitude of forms was a certain Jacobus Alpaz. The rest of the entries in the list indicate the same background of Aristotelian texts and middle sciences, including the Proportiones breves. This is one of the manuscripts containing the Proportiones longe or Tractatus longus de proportionibus, which, along with the similar text of an Arithmetica (perhaps by Muris, but in the manuscript ascribed to Bradwardine as well), was also copied in Ms. ÖNB 4953.

A second undated list of university lectures very similar to the one mentioned above is contained in manuscript Melk, Stiftsbibliothek, 901 (362, G 22, p. 601).

[^20]This register does not contain the names of individual teachers. However, it does appear to state the number of lessons offered or, more likely, of the number of groschen (grossi) to be paid for each lesson. For lessons on the latitude of forms, the amount appears to be four. No number is provided for Topicorum or Physicorum.

## Liber audiendi ad magisterium

Lectiones
$\left.\begin{array}{c}10 \\ 6 \\ 5 \\ 2 \\ 5 \\ 5 \\ 5 \\ 4 \\ 2 \\ 2 \\ 4\end{array}\right\}\left\{\begin{array}{l}\text { Logicorum } \\ \text { Metaphysica } \\ \text { Metheorum } \\ \text { De celo et mundo } \\ \text { De generatione et corruptione } \\ \text { Perspectiva } \\ \text { Quattuor libri Euclidis } \\ \text { Theorica planetarum } \\ \text { Latitudines formarum } \\ \text { Proportiones } \\ \text { Arismetrica } \\ \text { Musica } \\ \text { Topicorum }\end{array}\right.$

## terciora

$\left.\begin{array}{l}10 \\ 10 \\ 10\end{array}\right\}\left\{\begin{array}{l}\text { Metaphysicorum } \\ \text { De anima } \\ \text { De generatione et } \\ \text { corruptione } \\ \text { Physicorum }\end{array}\right.$

## III. 10 The Printing of LF

The final printing of the treatise LF in 1515 constitutes the last link in the chain of evidence demonstrating the continuity of the latitude of forms as a subject of instruction at the University of Vienna. ${ }^{47}$ Since the treatise was directly connected with the Vienna University curriculum, the decision to print it can only be a reaffirmation of a teaching tradition that had begun over a hundred years before. The volume also included an Arithmetica communis, the Proportiones breves and two Algorismus texts. These were not the old Algorismus texts of Sacrobosco and other writers, but the Algorismus de integris by Georg von Peuerbach and de Algorismus de minuciis phisicis by Johannes von Gmunden. Georgius Tannstetter Collimitius, "Artium et Medicine doctor: et Mathematice in studio Viennensi professor ordinarius," was responsible for this edition. ${ }^{48}$

[^21]
## IV The Manuscripts of the Latitudines breves

The three texts of the Latitudines breves are transmitted in three different manuscripts, all of which appear to be associated with instruction at Vienna University. Although these manuscripts have been previously described in a general way in old catalogues, they deserve greater attention. For us, the transmission of LB in the context of other mathematical and middle sciences is especially noteworthy.

## IV. 1 Latitudines breves I (Mü)

$\mathbf{M u ̈}=$ München, Bayerische Staatsbibliothek, Clm 18985, ff. 52r-53r (text 15).

- $229 \mathrm{ff} ., 8^{\circ}$, Paper, middle of the 15 th century (3: 1439).
- Collection of: Natural philosophy (Commentaries on Aristotle), proportions, latitude of forms, astronomy, optics, geometry, arithmetics, logic, philosophy (Avicenna, Thomas Aquinas, Egidius Romanus; see Remarks below).
- Description in: Halm et al. (1878b, 228 ( $=1779$ )). This catalogue description is too general. Many texts are not distinguished or even mentioned.
- Provenience: Tegernsee 985 (see Redlich (1931, 191)).
- Hand / copyist: Except texts 9, 10 and the fragment in 7, all texts were copied by one and the same person, namely by Andreas de Roting in the year 1439 (see Text 3 and Redlich (1931, 191)).
- Remarks: First mentioned by Curtze $(1899,293)$ as "Handschriften über latitudines formarum." Indirectly mentioned (from Curtze) in Clagett (1959) under "other manuscripts" (p.398, footnote*). As can be appreciated in the tabulae contentorum, $\mathbf{M} \mathbf{u}$ is made up of two different parts that were later bound together. For the first part, the only one to be described here, carries the copying date 1439 . It contains short writings on the then current logic, mathematics and middle sciences as these were typically taught at Vienna University (Faculty of Arts). The second part, which is not relevant for us, includes metaphysical and physical writings, which at least partially reflect the intellectual controversies in the philosophical milieu at Paris around the middle of the 13th century (Avicena, Thomas Aquinas, Egidius Romanus).
Tabulae contentorum: [1] On the inside of the book board: "Attinet monasterio Tegernsee. 14830. Iste libellus attinet venerabili monasterio Sancti Quirini Regel et monasterio in Tegernsee. In hoc libello continetur varie materie philosophice, videlicet de artibus liberalibus, ut patet intuenti et de talibus instructo, videlicet: $\langle 1\rangle$ de obligationibus; $\langle 2\rangle$ de insolubilibus; $\langle 3\rangle$ de fallaciis, $\langle 4\rangle$ de appellationibus, $\langle 5\rangle$ de restrictionibus, $\langle 6\rangle$ de mathematica, $\langle 7\rangle$ de geometria, $\langle 8\rangle$ de astronomia, $\langle 9\rangle$ de arismetrica etc., et sic de similibus. Expliciunt contenta. Quedam adhuc aliqua contenta in calce." [2] On the inside of the
anterior board：＂Cod．lat．18985＂［from a recent hand］．From a contempo－ rary hand as［1］：Adhuc aliqua contenta huius libellis que non sunt signata in sequenti folio in altera parte ex inadvertentia．$\langle 1\rangle$ Tractatus Sancti Thome de Aquino de intellectu et intelligibili； $1\langle 2\rangle$ Tractatus Sancti Thome de unitate intellectus possibilis et incorruptibilis； $2\langle 3\rangle$ Tractatus eiusdem de compara－ tionibus actionibus ocultis．Item habetur etiam ibi dd．33，32； $3\langle 4\rangle$ Tractatus seu liber eiusdem de principiis et causis naturae； $4\langle 5\rangle$ Tractatus eiusdem de motu cordis； $5\langle 6\rangle$ Tractatus eiusdem de possibilitate eternitatis mundi； $6\langle 7\rangle$ Tractatus eiusdem de formis elementorum．Item de materia prima notabilia et de celo；$\langle 8\rangle$ Questiones super libros methaphysice．［Here＂ $\mathrm{N}^{\circ} 542$＂from a recent hand］$\langle 9\rangle$ In fine errores Avicenae et Algazelis sequentis Avicena．$\langle 10\rangle$ Item articuli errorum fratrum predicatorum propter quos 16 annis condemnati fuerunt et anno domini 143 ［sic！］comunioni fidem instituti［？］．
－Contents［Part 1：ff．1r－128r］：
－ff．1r：Probatio pennae．
1．ff．1r－8r：Anonymous：〈Quaestiones naturales et mathematicae〉：1：utrum omni numero sit numerus maior．Inc．：＂Magister Buridanus respondendo ad questionem notat primo quod numerus secundum Aristotelem est．．．＂； 2 （f． $3 \mathrm{r}-4 \mathrm{v}$ ）：＂Utrum in quolibet continuo．．．＂； 3 （ff． $5 \mathrm{r}-8 \mathrm{r}$ ）：＂Utrum pos－ sibile est esse magnitudinem infinitam．＂Expl．：＂．．．fuit semper linea pedalis．Ex illis patent solutiones argumenti．＂Apparently a partial copy of a commentary to the Aristotelian Physics（see for instance，but with differing content，Ms．München，BSB，Clm，19869，ff．195r－360r，Dispu－ tata Magistri Uldarici de Tubing in octo libros physicorum，f．280v：＂utrum omni numero sit numerus maior．Et numerus est duplex，scilicet numerans et numeratus．．＂）．
2．ff． $8 \mathrm{v}-22 \mathrm{v}$ ：Anonymous：$\langle$ Super librum de sophisticis elenchis Aristotelis $\rangle$ ． Inc．：＂．．Quia iuxta phylosophum $2^{\circ}$ Elenchorum ignotas virtutes vocab－ ulos de facili．．．＂Expl．：＂．．et hoc punctus sequitur fallacia rationis．＂

3．ff．23v－25v：Anonymous：〈De passionibus $\rangle$ ．Inc．：＂．．．Passio est duplex， scilicet naturalis ut passiones vegetative．．．＂Expl．：＂．．．vel adulatio vel ［？］est litigatio et discordia．＂
－26r－28v：empty．
4．ff．29r－35r：Anonymous：$\langle$ De obligationibus $\rangle$ ．Inc．：＂circa materiam obli－ gationum dubitatur primo quod sit subiectum．．．＂（29r）．Expl．：＂．．．sunt tamen concedende ex posito ut patet in casu iam posito．＂Probably ex－ tracts or reworking from Johannes Hollandrinus＇treatise De obligationibus （ed．Bos 1985）．

5．ff．35v－39r：Anonymous：$\langle$ De insolubilibus $\rangle$ ．Inc．：＂circa tractatum de
insolubilibus［here the word＇obligationibus＇has been crossed out］est no－ tandum primum quod insolubile dicitur．．．＂Expl．：＂．．．de materia in－ solubilibus．Et sic est finis insolubilium scriptorum per me Andreas de Roting sub anno domini millesimo quadrigentesimo tricesimonono in die Juliani＂（a later hand has added＂1439＂）．At f．37v Johannes Hollandrinus is mentioned：＂alia est opinio magistri Holandrini．．．＂
6．ff．39v：Anonymous：Fragment on Logic，most probably connected with the previous texts 2 and 3 （see above Tabula contentorum）．
7．ff．40r－v：Anonymous：〈Geometria〉（Fragment）：Inc．：＂Utrum super quamlibet lineam datam rectam contingat triangulum equilaterum col－ locare．Quod sic arguitur auctoritate Euclidis．．．＂Expl．：＂．．．ipse erunt equals．＂
8．ff．41r－42v：Anonymous：$\langle$ De fallaciis $\rangle$（Fragment）：Inc．：＂Fallacia elenci causatur uno modo quando．．．＂Expl．：＂．．．specialis posterius（？）con－ tradictorum requisite．＂

9．ff． $42 \mathrm{v}-45 \mathrm{v}$ ：Anonymous：$\langle$ De sensibus $\rangle$（Fragment）：Inc．＂Notandum per organum auditus intelligitur．．．＂Expl．：＂．．．vocatur augmentatione vel diminutione eiusdem．＂

10．ff．46r－v：Anonymous：〈Musica brevis〉：Inc：＂Subiectum musice est ly numerus．．．＂An abbreviation of the Musica of Jean de Muris（Falkenroth （1992），without mention of this ms．）．Expl．：＂．．．cum talibus proposition－ ibus solum consistit musica ut patet in figura＂（a graphic follows）．
11．ff．47r－v：Anonymous：〈Arithmetica brevis〉：Inc．：＂Mathematica est scientia quantitatis considerativa precise secundum quantitatis continue aut discrete aut alia habentem atributionem ad aliqua illas．．．＂Expl．： ＂．．．sunt extrema proportionis duple，igitur etc．＂At f．47v is Bradwar－ dine＇s Geometry mentioned：＂per secundam propositionem Geometrie Bragwardini $4^{t i}$ capituli＂（probably Geometria speculativa，III，chs．4， 3．43，Molland（1968，100－102）．
12．ff． $48 \mathrm{r}-\mathrm{v}$ ：Anonymous：$\langle$ Astronomia brevis $\rangle$ ．Inc．：＂mobilis et arcus qui sunt intra puncta solsticiorum．．．＂Expl．：＂．．．et illud divido per 22 et numerus．＂

13．49r－50v：Anonymous：〈Arithmetica〉（Fragment）．Inc．：＂quater 8 bis 16 sunt 32 ．Similiter quater 8 sunt $32 \ldots$ ．．．Expl．：＂．．．ut tria quinque sunt 15．＂Probably the continuation of 11.

14．51r－v：Anonymous：〈Perspectiva brevis〉．Inc．：＂Sequitur de perspec－ tiva scilicet scientia media inter geometriam et naturalem philosophiam．＂ Expl．：＂．．．modo sicut raritas et densitas considerantur sicut etiam ef－ fectus，etc．＂Extracts of brief reworking from John Pecham＇s Perspectiva communis（Lindberg 1970）．

15．ff．52r－53r：Anonymous：$\langle$ Latitudines breves $I\rangle$ ．Inc．：＂Liber latitudinum formarum qui subordinat〈tur〉 phylosophia speculative medie inter physi－ cam et geometriam．．．＂Expl．：＂Quedam econtrario incipit a non gradu et terminatur ad certum gradum ut ibi［Figur］＂Reworking from Jacobus de Sancto Martino，Tractatus de latitudinibus formarum（Smith 1954）．Ed． here V．2．1．

16．ff． $53 \mathrm{v}-55 \mathrm{v}$ ：Anonymous：〈Theorice definitiones astronomie〉．Inc．：＂Cir－ culus eccentricus est qui habet centrum．．．＂Expl．：＂．．．solum ad vacuum repletionem quare probatur mathematice．＂

17．ff．56r－65v：Anonymous：$\langle$ Astronomia brevis $\rangle$ ．Inc．：＂Sequitur de astrono－ mia．Sphera secundum Theodosium est corpus solidum．．．＂Continuation f． 58 r：＂quotiens erit iste 11454 et remarebut in residuo．＂Expl．（65r）： ＂．．．causa quia sub maiori angulo comprehenduntur．＂Perhaps a rework－ ing from Theodosio＇s De sphaera，written by the same hand as text 12 and connected with it．

18．ff． $57 \mathrm{r}-\mathrm{v}$ ：Euclides，Elements of Geometry（Fragment or reworking）：Inc．： ＂Sequitur de geometria in libris Euclidis．Punctus est．．．＂Brief introduc－ tion and some definitions from book I．Included by Folkerts（1989，51－52） under＂reworking and commentaries from the fifteenth century．＂

19．ff．57v－58r：Pseudo－Bradwardine：Arithmetica：Inc．［Rubr．：Arismetica Bragwardinis］：＂Subiectum aristmetice est numerus consideratus ut quan－ titas discreta．．．＂Expl．：＂．．．resultat utrobique（？）ut 4816．＂Although this text is factually headed＂Arismetica Bragwardini，＂it is rather a sum－ mary or a reworking of an arithmetical text that was often attributed to Bradwardine（for this problem，see Busard（1998））．

20．ff．58r－65r：Anonymous 〈Astronomia〉：Inc．：＂Quotiens erit iste 11454 et remanebunt in residuo．．．＂Expl．：＂．．causa quia secundo maiori angulo comprehendetur．＂Apparently a continuation of 12 and 16.

21．ff．65v－66v：Anonymous $\langle$ De proportionibus $\rangle$ ：Inc．：＂Proportio duobus modis sit dicta，scilicet communiter dicta et proprie dicta．＂Expl．： ＂．．scilicet 9 ad tria et 3 ad 1．＂

22．ff．67r－128r：Anonymous 〈Paulus de Mellico［？］〉：〈Quaestiones variae de ethica et philosophia naturali〉 Inc．：q． 1 （ff． $67 \mathrm{r}-72 \mathrm{r}$ ）：＂utrum velocitas motuum sequatur proportionem maioris inequalitatis potentia motoris su－ per potentiam rei mote．Quod non quia proportio est habitudo．．．＂On 70r，the Geometry of Bradwardine is mentioned：＂patet per aliquam Bra－ guardini $3^{\circ}$ tractatu geometrie sue．．＂F．72r：＂．．．hic positio est magistri Iohannis Buridani．．．＂；q． 2 （ff． $72 \mathrm{v}-124 \mathrm{v}$ ；f． 115 rv ：empty）：＂utrum vere fortis debeat et teneatur in bello potuis eligere propriam mortem quam
mortem sui principis"; q. 3 (ff. 125r-128r): "Utrum proportio velocitatum motuum debeat attendi penes proportionem potentiarum motivarum ad suas resistencias. Arguitur quod non, quia proportio velocitatum..." Expl.: "...ex sola variatione sue figure a causa ibi dicta et finitur de isto." The catalogue description Halm et al. (1878b) assumes the information on f .86 r as valid for the whole text: "quaestio disputata a 1439 feria VI. ante dominicam palmarum post combustionem castri principis auctore Paulo de Mellico." However, it is not absolutely clear if Paulus de Mellico is the author of the whole text or, what seems to be more likely, only the respondent to this one quaestio. Lhotsky $(1965,67)$ mentions a Paul von Melk, who would have been active at the Vienna Faculty of Arts around 1451. This is surely the same person.

- ff. 128v-130r: Probatio pennae and fragments on moral philosophy.
- ff. 131r-226r [Teil II]: Thomas Aquinas, Tractatus de intellectu, Tractatus de intellectu et intelligibili..., Avicenna, Tractatus de materia prima, Egidius Romanus: Errores Avicennae et Algazalis, Articuli errorum fratrum praedic. propter quos XVI annis condemnati fuerunt.


## IV. 2 Latitudines breves II (Wi)

$\mathbf{W i}=$ Wien, Österreichische Nationalbibliothek, 4784, ff. 237r-240v (text 9).

- $261 \mathrm{ff} .8^{\circ}$, Paper, middle of the 15 th century.
- Collection of: Natural philosophy (Commentaries on Aristotle), proportions, latitude of forms, astronomy, music, ethics (Commentaries on Aristotle).
- Description in: Tabula (1869, 380); texts 1-7; Markowski (1985, 252)
- Provenence: Lunael. O. 57.
- Remarks: The only mention of LF known to me is found in Markowski's description, from whom the appropriate denomination of "quaestiones wiennenses" comes (to the extent that we can assume that these texts are not only conserved in a Vienna manuscript but also reflect the teaching at the Vienna Faculty of Arts).


## - Content:

1. ff. 1r-69r: Quaestiones Wiennenses super $I-V$ libros "Ethicorum" Aristotelis. Inc.: "Circa initium libri Ethicorum in quo determinantur de quo..." Expl.: ". . patienti autem solum est malum pene."

- 69v: empty.
- ff. 70r-71r: Scheme on the moral virtues and vices.
$-71 v-72 \mathrm{v}$ : empty.

2．ff． $73 \mathrm{r}-172 \mathrm{v}$ ：Anonymous：Quaestiones Wiennenses secundum I．Buri－ danum super $I-I V, V I-X, X I I ~ l i b r o s " M e t a p h y s i c o r u m " A r i s t o t e l i s . ~ I n c .: ~$ ＂Utrum metaphysica sint sapientia et omnium habituum intelectualium perfectissima．Arguitur quod non quia sapientia est．．．＂Expl．：＂．．．est vera quia pluralitas non est ponenda sine necessitate．＂
－ff．173r－177v：empty．
3．ff．178r－201v：Anonymous：Quaestiones Wiennenses secundum I．Buri－ danum super＂De sensu et sensato．＂
4．ff．202r－208r：Anonymous：Quaestiones Wiennenses secundum I．Buri－ danum super＂De memoria et reminiscentia＂Aristotelis．
5．ff．208v－220v：Anonymous：Quaestiones Wiennenses secundum I．Buri－ danum super＂De somno et vigilia＂Aristotelis．
6．ff．221r－225：Anonymous：Quaestiones Wiennenses secundum I．Buri－ danum super＂De longitudine et brevitate vitae＂Aristotelis．
7．226r－229v：Anonymous：Quaestiones Wiennenses secundum I．Buri－ danum super＂De morte et vita＂Aristotelis．
－ff．230r－v：empty．
8．ff．231r－236v：Pseudo－Bradwardine：Proportiones breves．Inc．：＂Sequitur proportiones breves Braguardini．Dubitatur qui parti philosophie subordi－ natur illa．．．Sequitur textus．Omnis proportio aut est communiter dicta aut proprie dicta．．．＂Expl．：＂et sic terminat librum suum et sic est fi－ nis pro quo sit benedictus deus in secula seculorum amen．＂Ed．Clagett （1959，481－494）；without this manuscript．The text is incomplete（until Clagett（1959，490，l．360））and includes some additions．
9．ff．237r－240v：Anonymous：〈Latitudines formarum $I I\rangle$ ．Inc．：＂Prosequ－ untur latitudines formarum．Quia formarum latitudines multipliciter vari－ antur，etc．．．＂Expl．：＂．．．est per figuram consimiliter variata．Et vobis et nobis in perdoctus corniferus petri perquinti［？］．＂Reworking of Jacobus de Sancto Martino，Tractatus de latitudinibus formarum（Smith 1954）．Ed． here V．2．2．

10．ff．241r－244r：Jean de Muris：Musica speculative．Inc．：＂Circa inicium musice Muris nota musica est scientia．．．＂According to Falkenroth（1992， 63 ）is the version $B$ of this text．
11．ff．244v－258r：Anonymous 〈Theorica planetarum〉．Inc．：＂Circa theoricas planetarum．．．＂Expl．：＂．．．tertius quartus et sectilis etc．＂

## IV． 3 Latitudines breves III（Mn）

$\mathrm{Mn}=\mathrm{München}, \mathrm{BSB}, \mathrm{Clm} 19850$ ，ff．7r－10r（text 4）．
－ $221 \mathrm{ff} .8^{\circ}$ ，Paper，middle of the 15 th century（ $7: 1447 ; 8: 1430$ ）．
－Collection of：Natural philosophy，arithmetics，astronomy，latitude of forms， music，proportions．Quaestiones quodlibetales variae．
－Description in：Halm et al．（1878b， $278(=2211)$ ）．This catalogue offers only general information which was assumed almost unchanged in the descrip－ tion of Falkenroth（1992，p．63，under Mü3）．Markowski（1981，p．147），who characterizes the texts 1－4 as Abbreviata wiennensia，offers a better description．
－Provenence：Tegernsee 1850 （see Redlich（1931，187））．The whole codex transmits scholarly texts of the Vienna Faculty of Arts from around the middle of the century in the fields of natural philosophy，mathematics and，above all，the＂middle sciences．＂Further evidence of this，which in Markowski＇s description does not occur，is provided by the Quodlibetum of Friedrich von Eschenbach，which was delivered in Vienna in 1430 （see 5．5）．As for other， similar manuscripts from Tegernsee Abbey，there is tabula contentorum offering a general description of the content（Redlich 1931）．
－Remarks：First mentioned in Curtze $(1899,293)$ as＂Handschriften über lati－ tudines formarum．＂Indirect（from Curtze）mentioned in Clagett（1959）under ＂other manuscripts＂（p．398，Anm．＊）．Murdoch and Sylla（1978，263，n．137） on the scientiae mediae（see also here notes 11， 15 and section VI）．
－Tabula contentorum：［Cod．lat．19850］＂Iste libellus attinet venerabili monasterio sancti Quirini regis et martiris ac patroni in Tegernsee，quem obtulit Deo et Sancto Quirino pro usu fratrum magister Uldaricus de Landau ibidem professus．In quo continetur hec infra signata：〈1〉 Abbreviata super Musi－ cam Muris；$\langle 2\rangle$ Proportiones Braguardini；$\langle 3\rangle$ Arismetrica et latitudines for－ marum；$\langle 3\rangle$ cum notacionibus terminorum aliquorum ex Sphera materiali；$\langle 4\rangle$ Quodlibetum integrum cum suis questionibus variis gramaticalibus，loycalibus， et physicalibus disputatum per egregium doctorem sacre theologie magistrum Fridericum de Eschenbach ${ }^{49}$ canonicum ad sanctum Mauricium Auguste；$\langle 5\rangle$ Questiones trium librorum de celo et mundo；$\langle 6\rangle$ Questiones parvorum natu－ ralium cum suis problematibus；$\langle 7\rangle$ Questiones de memoria et reminiscentia； $\langle 8\rangle$ Questiones de somno et vigilia cum problematibus；$\langle 5\rangle$ Questiones magistri Martini Hämert astronomi multum periti duorum librorum metheororum＂（see also Markowski 1981，147）．

## －Content：

1．ff．1r－2r：〈Jean de Muris〉：〈Musica brevis〉．Inc．：＂Subiectum musice est ly numerus sonorum．Unisonus est．．．＂Expl．：＂．．．consistit in tripla proportione．＂Mentioned by Falkenroth（1992，63）．
2．ff． $2 \mathrm{r}-4 \mathrm{v}$ ：Pseudo－Bradwardine：〈Tractatus brevis proportionum〉．Inc．： ＂Circa proportiones notandum subiectum huius scientie est ly proportio．

[^22]Proportio est habitudo rerum．．．＂Expl．：＂．．．illius resistentie quam cum tota．＂（Clagett 1959，490）
3．ff．4v－7r：Anonymous：〈Arithmetica brevis〉．Inc．：＂．．．Subiectum arismetice est ly numerus．Numerus est collection．．．＂Expl．： ＂．．．dyapendente et dyapason ut patet in figura subscripta［the correspon－ dent graphics follows］．＂On the question concerning a possible＂Arith－ metics＂by Bradwardine，see Busard（1998）．
4．ff．7r－10r：Anonymous：〈Latitudines breves III $\rangle$ ．Inc．：＂Circa latitudines formarum．Notandum subiectum huius scientie．．．＂Expl．：＂．．．per figuram similiter variatam．Et hoc infinite mode variari posset．＂In Markowski＇s description（loc．cit．）als＂abbreviata wiennensia．＂Rework－ ing from Jacobus de Sancto Martino，Tractatus de latitudinibus formarum （Smith 1954）．Ed．here V．2．3．

5．f．10v：Anonymous：〈De mathematica〉（Fragment）：Inc．：＂Mathematica est scientia．．．．＂

6．ff．11r－12r：Anonymous：〈Notationes terminorum super tractatum sphaerae〉．Inc．：＂Circa speram nota quod subiectum est totum uni－ versum．．．＂Expl．：＂．．et quelibet talis pars vocatur gradus etc．＂（see Markowski 1981，99）．
7．ff． $13 \mathrm{v}-56 \mathrm{v}$（f． 49 v ：empty）：Friedrich von Essen－Eschenbach：Quodli－ betum disputatum per Fridericum de Eschenbach im Jahre 1447 in Wien cum nominibus magistrorum．Inc．：＂Incipit quodlibetum disputatum per magistrum Fridericum de Eschenbach doctorem in theologia anno domini 1447．＂The same hand that wrote the tabula contentorum writes on f .12 v ： ＂sequens quodlibetum，disputatum est per reverendum magistrum Frider－ icum de Essen Eschenbach baccalaureum formatum in theologia anno do－ mini $1447(\ldots)$ in aula artistarum in alma universitate wiennense．＂Expl．： ＂．．．non est in aliis qualitatibus intensibilibus．＂For more information，see section III． 6.

8．ff．57r－103r：Gerardus Seglaw：Quaestiones super I－IV libros de caelo et mundo Aristotelis．Inc．：＂circa inicium libri de celo et mundo sciendum quod Aristoteles．．．＂Expl．：＂Et sic est finis huius $4^{t i}$ tractatus et con－ sequenter omni librorum de celo et mundo．M〈agister〉 Gherardi Seglaw anno domini 1430．．．＂

9．ff．103v－104v：Anonymous：〈Quaestio utrum scientia naturalis sit de om－ nibus rebus〉 Inc．［Q．1］：＂．．．utrum scientia naturalis sit de omnibus re－ bus．Ad questionem arguitur quod scientia naturalis．．．＂Expl．：＂unitas scientie vel diversitas tot scientie agregate．＂At the beginning：＂in die Sancti Laurentii．＂Markowski（loc．cit．）describes these pages as＂duae
quaestiones Wiennenses super libros De celo et mundo." There is no doubt about the Viennese origin of this piece, but it is only a quaestio; the presumed second quaestio is only the answer. In the middle of the page, the copyist has highlighted "et sic est finis de celo et mundo" in red. Contrary to what Markowski assumed, this text has nothing to do with Aristotle's De caelo but rather with the classification and the scientific status of natural science, a topic that was usually discussed in the initial paragraphs of commentaries on the Physics.
10. ff. $105 \mathrm{v}-139 \mathrm{r}$ : Anonymous: 〈Quaestiones wiennenses super parva naturalia Aristotelis. Inc.: "Circa initium parvorum naturalium queritur primo de passionibus. . . ."
(a) ff. 105v-125r: Anonymous: Super de sensu et sensato.
(b) ff. 125r-130r: Anonymous: Super de memoria et reminiscentia.
(c) ff. 130v-139r (f. 137v: empty): Anonymous: Super de somno et vigilia.

Expl.: "...ex se sed per celeste. Et sic de aliis rebus, etc." See Markowski 1981, 91-93.

- ff. 139v-140v: empty.

11. ff. 141r-217r: Martinus Hemmert (Catalogus, loc. cit. "Hemmerl," Markowski, 1981, "Hammert"): Questiones in libros Meteorum. Inc.: "Utrum necesse est istum mundum inferiorem esse continuum lationibus superioribus..." Expl.: "...et tiphones descendant ad terram ex nubibus. Tertius metheorum Aristotelis." See, additionally, Ms. München, BSB, Clm 18990 (dated 1481) containing "De novo foro concepta quatuor librorum Metheororum" by the same author (Markowski 1981, 75).

## V The Texts of the Latitudines breves

My only aim in this section is to provide an edition of the texts I have designated Latitudines breves I-III. As noted above and as is sufficiently evident from a consideration of the manuscripts' description, these texts are all very closely related to one another and to other texts produced at the Vienna Faculty of Arts, as previously mentioned in the Expositio on LF and the mathematical Compendium.

## V. 1 Editorial Remarks

To a certain extent, it is more difficult to edit texts like the Latitudines breves than it is to edit "real texts" for which an ideal original version can be assumed and which may be reconstructed by philological means. We know that LB $1-3$ ultimately go back to LF. However, they are not copies of the latter but re-workings or summaries produced for teaching purposes. As a result, we cannot correct or complete them
with the help of the model LF．Moreover，if we assume that the＂writers＂or＂copy－ ists＂were probably the authors of these short texts－which is not impossible－we should try to retain all the special features they exhibit as expressions of their writ－ ers＇particular understanding of the doctrine of the latitude of forms．For this reason， when editing these texts－whose Latin is often idiosyncratic and，due to their sub－ ject matter，very technical－I have taken a compromise approach，correcting only where necessary to improve comprehensibility and making corrections that，in some cases，do not correspond to parallel passages in LF．（LB 3，for instance，fluctuates between the two correct forms super basi and super basim．）This applies particularly to the figures，which very often were not drawn by the copyist or compiler．For the sake of clarity，I have enlarged some of the figures and incorporated them into the main body of the text．In general，however，I have tried to respect each text＇s par－ ticularity．In some cases，where a figure is referred to in the text but the manuscript contains only a gap，I have used the text to reconstruct the missing figure．I have not normalized the Latin，retaining，for instance，the medieval endings que for quae and forme for formae．In a few passages，I have standardized the spelling for the sake of clarity，although I have generally preserved each text＇s orthographic peculiarities． It should also be noted that the text of LB 3 contains annotations in a second hand． These annotations are very hard to decipher．However，I have included my reading of them between the symbols $\rfloor$ ．Finally，to improve readability，I have divided the text according to LF，as set out in the previously cited edition of Thomas Smith （1954）．To facilitate the comparison of the corresponding passages in LF and Eu－ clid＇s Elements I（here E），I have added short references between square brackets at the beginning of the referred sentence．Thus，for instance，the reference［Div． $\mathrm{L}_{1}$ ］ means that the passage is a reworking version of or connected to the first division of the latitudes in LF，whereas［Def．${ }_{1}$ ］is a reference to the first definition in LF．

## V． 2 The Texts

## V．2．1 Latitudines breves I

〈Prohemium $\rangle$
$\langle\text { Sequitur }\rangle^{1}$ liber latitudinum formarum，qui subordinatur phylosophie specu－ Mü52r lative medie inter physicam et geometriam，secundum aliquos quia＂latitudo＂est terminus geometrie et＂forme＂terminus physice．Utilitas istius scientie est ad in－ telligendum multas partes physice ad alterationem，an fiat secundum intensum vel

[^23]remissum，et ad materiam tertii Physicorum．Et nota：gradualis intensio forme dici－ tur eius latitudo，et sic intensio gradualis et latitudo gradualis supponunt pro eadem et similiter latitudo forme．Non tamen sinonime，quia ly latitudo forme non est ab－ solute intensio gradualis，sed connotat relative talem intensionem esse ymaginabilem per figuram geometricam et extensam．

Et non solum forma est latitudinabilis quo ad eius intensionem sed etiam mo－ tus localis et forma substantialis quo ad fieri vel mutari．Consequenter intensio gradualis forme vocatur eius latitudo；eius vero extensio quo ad subiectum vel quo ad durationem vocatur longitudo．Extensio mensuratur per lineam in subiecto，lati－ tudo autem aut intensio per lineam perpendiculariter super eandem lineam erectam． Linea non perpendiculariter data aut alia existens non est latitudo adquisita quo ad subiectum et sit fuerit acquisita，tunc illomodo，quia alias esset latitudo sine longi－ tudine，quod est impossibile．Et sic arguitur longitudo notior est ibi quam latitudo quia facilius scimus extensionem rei quam intensionem．${ }^{2}$

## 〈 Divisiones et definitiones latitudinum ex capitulis II． 1 et II． 3 〉

［Div． $\mathrm{L}_{1}$ ］Latitudinum quedam uniformis，quedam difformis．

［Def．1］Uniformis est que est eiusdem gradus per totum，ut ibi | 2 | 2 | 2 |
| :--- | :--- | :--- |
| sunt equales |  |  | 20 gradus in omnibus partibus［Fig．1］．



Fig．1：f．52r
 ［Div．$L_{2}$ ］Latitudinum difformium quedam est secundum se totam difformis，quedam non secundum se totam difformis．

［Def．3］Prima est cuius nulla pars est precise uniformis，ut | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | ，ita quod nulla pars tot gradus 〈habet〉 sicut alia．

［Def．4］Secunda est cuius aliqua $\langle$ pars $\rangle$ est difformis，ut | 2 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | ．Dicitur secundum partes difformis，quia non valet omnes，cum due sunt uniformes．

［Div． $\mathrm{L}_{3}$ ］Latitudinum secundum se totas difformium quedam est difformiter dif－ formis，quedam uniformiter difformis．［Def．5］Latitudo uniformiter difformis est

[^24][^25]illa cuius est equalis excessus gradum inter se eque distantium，ut | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | ．［Def．${ }^{6}$ ］ Latitudo difformiter difformis est cuius non est equalis excessus graduum inter se eque distantium，ut $1 / 2|3| 4 \mid 6$ quia non est equalis excessus quattuor ad triam sicut tria ad unum，quia quattuor ad triam｜excessus est sesquitertius sed tria ad unum est sesquialter［Fig．2］．



Fig．2：f．52r
［Div． $\mathrm{L}_{4}$ ］Latitudinum uniformiter difformis quedam incipit a non gradu，id est ab infinita modica parte，et terminatur ad certum gradum．Alia incipit a certo gradu et terminatur a non gradum，ut $4|2| 1$ ．Sed nulla incipit a non gradu et terminatur ad non gradum，quia talis non est uniformiter difformis，quia in principio intenditur et in fine remittitur［Fig．3］．


Fig．3：f．52r
［Div．L5］Latitudinum difformiter difformium quedam secundum se totam difformiter difformis，quedam vero non．

［Def．7］Latitudo secundum se totam difformiter difformis est illa cuius nulla pars est precise uniformiter difformis，ut | $2\|3\|$ | $8 \mid 1$ |
| :--- | :--- | :--- | :--- | :--- | ．

［Def．8］Latitudo vero non secundum se totam difformiter difformis est illa cuius aliqua pars est uniformiter difformis，ut hic | $2\|3\|$ | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- |
| ． |  |  |

［Div．L $\mathrm{L}_{6}$ ］Latitudinum secundum se totas difformiter difformium quedam est unifor－ miter difformiter difformis，quedam difformiter difformiter difformis．
Notandum quod sicut ymaginatur quandam latitudinem in ulla sui parte variatam〈esse〉，quam vocatur difformem，ita quod si uniformiter varietur，vocatur latitudo uniformiter difformis，ita convenienter ymaginemur etiam quandam variationem latitudinis 〈esse〉 uniformem et quandam difformem；unde rursus variationem dif－ formem，quandam uniformiter difformem，quandam difformiter difformem．Et sicut uniformis variatio reddit latitudinem uniformiter difformem，sic difformis variatio 55 latitudinis reddit latitudinem difformiter difformem． ［II． $3_{\mathrm{p} 30}$ ］Latitudo uniformiter difformiter difformis est que inter excessus graduum inter se eque distantium servat eandem proportionem aliam tamen a proportione equalitatis，quia si servaret eandem proportionem inter excessus graduum eque dis－

34 post tria］trium add．Mü｜tria］correxi ex tertium Mü 38 nulla］correxi ex ulla Mü 43 nulla］correxi ex ulla Mü 47－48 uniformiter ］correxi ex difformiter Mü
tantium tunc esset latitudo uniformiter difformis．Et si nulla servaret 〈proportionem〉 uniformitas $\langle$ non potest〉 attendi in latitudine tali，et sic non esset uniformiter diffor－ miter difformis．Aliter sic dicitur：est latitudo in qua datis quibuscunque quattuor punctis equalibus extensis eque distantibus quo ad subiectum vel 〈quo ad〉 succes－ sionem quo ad tempus intensio excessus primi ad secundum habet se in eadem proportione inequalitatis ad excessum secundi ad tertium usque excessus secundi ad tertium habet se sicut ad excessum tertii ad quartum，et sic consequenter de aliis．


Fig．4：
Difformis est que non est eiusdem gradus per totum［Fig．5］：


Fig．5：
Latitudo secundum se totam difformis 〈est illa〉 cuius nulla pars est uniformis［Fig．6］． Latitudo autem non secundum se totam difformis est cuius aliqua pars est difformis et alia uniformis［Fig．7］：


Fig．6：


Fig．7：

Latitudo uniformiter difformiter est cuius equalis est excessus gradium inter se eque distantium［Fig．8］：


Fig．8：
Latitudo difformiter difformis cuius non est equalis excessus graduum inter se eque distantium［Fig．9］：

63 post tempus］successione scr．et del．Mü


Fig. 9: graduum inter se eque distantium non servat eandem proportionem aliam a proportione equalitatis, ut in isto exemplo [Fig. 10]:


Fig. 10:
Latitudo secundum se totam uniformiter difformiter difformis est que inter excessus graduum inter se eque distantium servat eandem proportionem aliam tamen a


Fig. 11:


Fig. 11: reconstruction

Item quedam latitudo incipit a non gradu et terminatur ad non gradum, ut ibi [Fig. 12]:


Fig. 12:
Quedam incipit a certo gradu et terminatur ad certum gradum, ut ibi [Fig. 13a-b]:


Fig. 13: a)


Fig. 13: b)

Quedam incipit a certo gradu et terminatur ad non gradum，ut ibi［Fig．14］：


Fig．14：
Quedam econtrario incipit a non gradu et terminatur ad certum gradum，ut ibi 85 ［Fig．15］：


Fig．15：

## V．2．2 Latitudines breves II

〈Prohemium〉
Wi237r Prosequuntur latitudines formarum：
＂Quia formarum latitudines multipliciter variantur，etc．＂${ }^{3}$
Nota：titulus huius libri est latitudo formarum et subiectum ordinatur scientie spe－ culative scilicet medie，quia subiectum eius est latitudo forme，quod est compositum ex termino naturali et mathematico．Utilitas huius liber est $\langle\mathrm{ad}\rangle$ intelligendum quodlibet motum secundum intensum et remissum et tertium Physicorum．Etiam valet ad multa sophismata de alterationibus．

Latitudo forme est gradualis intensio forme accidentalis，sed longitudo est ex－ tensio alicuius forme secundum subiectum．Et formarum quedam est divisibilis， quedam indivisibilis．Indivisibilis est ut anima．Sed divisibilium quedam sunt divis－ ibilis extensive ut forme brutorum animalium，alie sunt divisibiles intensive ut forme accidentales ipsius intellectus，ut videtur；quaedam sunt divisibiles extensive et in－ tensive，ut caliditas et frigiditas；et quaedam sunt divisibiles extensive secundum extensionem subiectum，ut caliditas，alie secundum durationem，ut motus localis．

## 〈I．1：Divisiones et definitiones latitudinum〉

Notandum quod uniformitas et difformitas est regularitas aut irregularitas forme， 15 sed latitudo est quarum intensio．

[^26]［Div．$L_{1}$ ］Et est duplex：quedam uniformis，et est $\langle i l l a\rangle$ que est eque gradus per totum seu eque intensa per totum，sed quedam difformis，et $\langle\mathrm{est}\rangle$ cuius etc．
［Div．$L_{2}$ ］Et latitudo difformis est duplex：quedam est secundum se totam difformis， difformiter difformis，et 〈illa〉 est que non equaliter excessus graduum inter，etc．
［Div．L4］Latitudinum uniformiter difformium quedam incipit a non gradu et termi－ natur ad certum gradum［Fig．1］，alia incipit a gradu et terminatur ad non gradum ［Fig．2］，alia incipit a gradu et terminatur ad gradum［Fig．3］．Et non potest dare quartum membrum．Exempla sunt figure pertractande．


Fig．1：reconstruction


Fig．2：


Fig．3：
［Div． $\mathrm{L}_{5}$ ］Item latitudinum difformiter difformium quedam est secundum se totam difformiter difformis，quedam non secundum se totam．Latitudo difformiter secun－ dum se totam est 〈illa〉 cuius nulla pars est uniformis；alia 〈est〉 difformiter difformis per totum；alia est 〈illa〉 cuius una pars est uniformis alia difformiter uniformis，ut hac［Fig．4］．


Fig．4：reconstruction
［Div．L．${ }_{6}$ ］Item latitudinum difformiter difformium secundum se totam，quedam est uniformiter difformiter difformis incomplexe，et est que sic se habet quod est equalis proportio $\langle n i s\rangle$ graduum eque distantium inter se alia tamen a proportione equali－ tatis；alia est difformiter difformiter difformis，et conversus non est eadem proportio partium eque distantium，ut hac［Fig．5］．


Fig．5：reconstruction
Et debet intelligi de distantia partium per situatam distantiam．Declarat nunc terminos et dicitur ulterius procedit．

〈II．1：De figuris geometricis〉
Auctor presuponit quid est linea，quid est figura，quid est linea，quid est linea recta， quid curva，quid est circulus ex Euclide．
Wi238r $\left[\mathrm{E}_{\text {def．}}\right]$ Punctus est cuius pars non habet positionem in continuo et est rerum \｜ indivisibilium quantitatum 〈habens〉 positionem in continuo cuius nulla pars est．
$\left[\mathrm{E}_{\text {def．2 }}\right]$ Linea est longitudo sine latitudine et profunditate ymaginata：
［ $\mathrm{E}_{\text {def．3 }}$ ］Linea recta est ab uno puncto ad aliud brevissima extensio in extremitates suas utrumque eorum recipiens．Pro opposito curva．
［ $\left.\mathrm{E}_{\text {def．14 }}\right]$ Figura est magnitudo termino vel terminis clausa．
［ $\mathrm{E}_{\text {def．19 }}$ ］Figura rectilinea patebit．
［ $\mathrm{E}_{\text {def．8 }}$ ］Angulus est duarum linearum alternus contactus．
［ $\mathrm{E}_{\text {def．10 }}$ ］Angulus rectus est angulus contentus per duas lineam quarum una cadit super aliam perpendiculariter：-
［ $\left.\mathrm{E}_{\text {def．12 }}\right]$ Angulus acutus est minor recto：
［ $\mathrm{E}_{\text {def．11 }}$ ］Obtusus est angulus maior recto：

［ $\mathrm{E}_{\mathrm{def} .1}$ ］Angulus rectilineus est qui est causatur per lineas rectas．
［II． $1_{\text {def．1 }}$ ］Item figura angularis est que habet angulum vel angulus．
［II． $1_{\text {def．2 }}$ ］Figura non angularis est figura non habens angulum sicut circulus．
［II． $1_{\text {def．3 }}$ ］Figura monoangula est que habet solum unum angulum ut：

［II． $\left.1_{\text {def．5 }}\right]\langle$ Figura $\rangle$ multiangula est que habet multos angulos：$\rangle$
［II． $1_{\text {def．4 }}$ ］ Figura $\rangle$ biangula est duorum angulorum precise，sicut exemplum：
［II． $1_{\text {def．59 }}$ ］Item figura multiangula habet sub se tot species quot sunt 〈species numerorum $\rangle$ post dualitatem ascendendo，ut triangula，quadrangula，etc．
［ $\mathrm{E}_{\text {def．20 }}$ ］Triangulus quedam dicitur ysopledrus，scilicet quando habet tria latera equalia，aliter dicitur ysothelerus，et est triangulus continetur dua latera equalia et tertium inequale，tertius est scalenus，et est triangulus continetur tria latera inequalia．
［ $\left.\mathrm{E}_{\text {def．21 }}\right]$ Item 〈triangulorum〉 quedam dicitur orthogonus，et est triangulus habens unum angulum rectum；alius amblygonus，et est triangulus habens unum angulum obtusum；et quedam dicitur exigonus，et est triangulus continens tres angulus acutus． solis lineis curvis，sicut figura constans ex duabus portionibus circuli；quedam ex linea recta et curva，ut medietas circuli；et quedam sunt medietas circuli．
$\left[\mathrm{DF}_{5}\right]$ Nota figura rectilinea est quedam solis rectis lineis continetur，sed figura curvi－ linea est que per omnes lineas curvas continetur，ut hic（ vel ymo 〈unam〉 curvam 75 et aliam rectam，ut $\triangle$ ．
［ $\mathrm{DF}_{6}$ ］Item figurarum curvilinearum quedam omnibus lineis curvis quedam aliqua

[^27]recta alia curva continentur vel curvis．
$\left[\mathrm{DF}_{7}\right]$ Item quedam figura dicitur plana et est cuius longitudo et altitudo mensuratur
linea recta；alia est curva，cuius longitudo vel latitudo mensuratur linea curva，et ideo differentia est inter figuram curvam et curvilineam，quia circulus est linea curvilinea et tamen non curva．

## 〈II．2：Suppositiones〉

Prima suppositio：omnia que secundum aliquam proportionem habent se ad invicem participant rationem quantitatis．
$2^{a}$ suppositio：omne quod graduali excessu excedit aliquid vel exceditur ab aliquoper modum quantitatis est ymaginandum．
$3^{a}$ suppositio：excessus gradualis est latitudo forme vel intensio forme．
$4^{a}$ ：omne quod excessu graduali excedit aliud vel exceditur ab alio habet latitudinem gradualem．Patet de albedine ut quattuor ad duo．
$5^{a}$ ：omne quod secundum dimensionem aliquam est quantum，tale secundum illam dimensionem potest excedere vel excedi ab alio．
$6^{t a}$ ：omne quod secundum plures dimensiones est quantum，talem etiam potest excedere aliud｜｜secundum tales dimensiones vel excedi ab alio．
$7^{m a}$ ：omne quod excedit aliquod vel excedere potest vel exceditur ab alio oportet quod sit quantum vel ymaginari sicut quantum．
$8^{v a}$ ：omne quod solum secundum extensionem suarum partium excedit vel exceditur in proportio ymaginandum est solam unam habere dimensionem tanquam linea sive longitudo．
$9^{n a}$ ：extensio forme ymaginanda est per lineam rectam．Intensio vero per figuram planam super rectam lineam consurgentem．
$10^{m a}$ ：cuilibet puncto in linea recta super quam figura plana collocatur correspondet propria latitudo in tali figura．
$11^{m a}$ ：quilibet punctus in extensione propriam habet intensionem．
$12^{m a}$ ：cuilibet puncto in extensione propria intensio sibi correspondens attendenda 105 〈est〉 per lineam rectam super datum punctum perpendiculariter erectam．
$13^{m a}$ suppositio：forme permanentes vel etiam forme que ymaginatur sicut perma－ nentes habent extensionem suam secundum extensionem subiecti．Sed forme succes－ sive habent extensionem suam secundum extensionem durationis vel etiam tempus， licet tam iste quam ille utreque possint habere extensionem 〈secundum subiectum〉．

## 〈III．3：Propositiones〉

110 Illis premissis ponit auctor propositiones：
Prima propositio：omnis latitudo cuiuslibet forme ymaginanda est per figuram

[^28]planam super rectam lineam consurgentem．Et probatio eius cuiuslibet potest omitti，quia aliter conclusio erit falsa．
$2^{a}$ propositio：nulla latitudo forme est ymaginanda convenienter ex omnibus lineis
curvis．｜Patet ex precedenti，cum talis non consurgit super lineam rectam．
$3^{a}$ propositio：nulla latitudo forme ymaginanda est per modum circuli，quia talis forma oportet unam figuram curvam．
$4^{\text {ta }}$ propositio：nulla latitudo ymaginanda est per figuram sine angulis，quia talis oportet quod sit figura circularis，que non habet angulum．
$5^{t a}$ propositio：nulla latitudo ymaginanda est per figuram monoangularem，quia
nulla talis situata est super lineam rectam．
$6^{\text {ta }}$ ：omnis latitudo ymaginanda est per figuram plurium angulorum；patet ex quarta et quinta．
$7^{\text {ma }}$ propositio：nulla latitudo ymaginanda est per figuram super lineam rectam consurgentem per angulum obtusum sive maiorem recto，quia aliter sequeretur quod intensio forme esset sine extensione，quod ita est impossibile．
$8^{v a}$ ：nulla latitudo forme mensuranda est secundum portionem circuli maiorem semi－ circulo．
$9^{a}$ ：omnis latitudo uniformis incipit a certo gradu et terminatur ad certum gradum． $10^{m a}$ ：omnis latitudo incipiens a non gradu est difformis，patet ex diffinitione．
$11^{m a}$ ：omnis latitudo sive uniformis vel difformis incipiens a certo gradu est ymagi－ nanda per unam figuram incipientem ab angulo recto．
12：omnis latitudo terminata ad certum gradum ymaginanda est per figuram desinentem ad angulum rectum．
$13^{m a}$ ：omnis latitudo incipiens a non gradu ymaginanda est per figuram 〈incipi－ entem）ab angulo acuto：


14：omnis latitudo terminata ad non gradum ymaginatur 〈per figuram terminatam〉 ad angulum acutum；et est conversa $13^{e}$ ．
15：omnis latitudo incipiens uniformiter difformiter a non gradu ymaginanda est per figuram incipientem ab angulo rectilineo et acuto．
Wi240r 16：omnis latitudo terminata uniformiter \｜difformiter ad non gradum ymaginanda est per figuram 〈terminatam〉 ad angulum acutum et rectilineum．
$17^{m a}$ ：omnis latitudo difformiter difformiter incipiens a non gradu ymaginanda est per figuram incipientem ab angulo acuto ascendentem per lineam curvam．
$18^{v a}$ ：omnis latitudo terminata difformiter difformiter ad non gradum ymaginanda
est per lineam curvam descendentem ad angulum acutum．
$19^{m a}$ ：omnis latitudo uniformis per totum ymaginanda est per figuram quadrangu－ lam rectilineam（！）．
$20^{m a}$ ：nulla latitudo in aliqua parte sui difformis，quantumcunque sit uniformis in fine et in principio，ymaginanda est per figuram quadrangulam rectangulam．

127 ante nulla］quod add．Wi｜portionem］correxi ex proportionem Wi｜circuli］post maiorem trans．Wi 137 ymaginatur］correxi ex terminatur Wi
$21^{m a}$ ：omnis latitudo uniformiter difformis incipiens a non gradu ymaginanda est per figuram rectilineam incipientem ab angulo acuto et terminatam ad angulum rectum．
$22^{a}$ ：omnis latitudo uniformiter difformis incipiens a certo gradu et terminata ad non gradum ymaginanda est per figuram rectam et terminata ad figuram vel angulum acutum．
$23^{a}$ ：Omnis latitudo uniformiter difformis incipiens a certo gradu et terminata ad certum gradum ymaginanda est per quadrangulam figuram cuius duo anguli super basi sunt recti．Reliquorum angulorum unius est acutus，alter obtusus．
$24^{m a}$ ：nulla latitudo incipiens a non gradu et terminata ad non gradum est uniformis neque uniformiter difformis，licet possit habere partes uniformiter difformes｜aut uniformes．
25：omnis latitudo incipiens uniformiter difformiter a non gradu et terminata uni－ formiter difformiter ad non gradum ymaginanda est per figuram super cuius basi in utroque termino est angulus acutus．
26：nulla latitudo secundum se totam difformiter difformis est per figuram rectilin－ eam ymaginanda．
27：omnis latitudo secundum se totam difformiter difformis est ymaginanda per figuram cuius altitudo terminatur per lineam curvam vel curvas．
28：omnis latitudo cuius aliqua pars est secundum se totam difformiter difformis et aliqua non，ymaginanda est per figuram cuius aliqua pars 〈altitudinis sue terminata est〉 per lineam curvam．
29：omnis latitudo uniformiter difformiter difformis incipiens a non gradu terminatur ad certum gradum．
$30^{m a}$ ：omnis latitudo uniformiter difformiter difformis incipiens a non gradu ymag－ inanda est per figuram triangulam habentem super basim unum angulum rectum， reliquos vero curvilineos et acutos．
$31^{\text {ma }}$ et ultima：omnis latitudo cuiuscunque forme qualitercunque variata ymagi－ nanda est per figuram consimiliter variatam．

## V．2．3 Latitudines breves III

## 〈Prohemium〉

Circa latitudines formarum．
Notandum $\langle$ est quod〉 subiectum huius scientie est ly latitudo forme．Et ista scientia est media inter mathematicam et naturalem，ut patet ex subiecto．Latitudo forme est intensio forme intensibile．Formarum quedam est indivisibile ut anima intel－

165 termino］correxi ex terminorum Wi 179 variatam］correxi ex variata Wi｜post variatam］ Et vobis et nobis in perdoctus corniferus petri perquinti $a d d$ ．Wi
lectiva，quedam divisibile．Divisibilium quedam sunt divisibiles extensive ut forme elementorum，quedam intensive ut accidentia．Intensio forme in proposito dicitur latitudo et eius extensio ratione subiecti dicitur longitudo．

## 〈I．1：Divisiones et definitiones latitudinum〉

［Div．L．${ }_{1}$ ］Latitudinum alia $\langle$ est〉 uniformis alia difformis．
Mn7v $\quad$ Uniformis est latitudo que est eiusdem gradus in intensione per totum：$\square$
〈Latitudo〉 Difformis 〈est〉 que non est eiusdem gradus per totum：

［Div． $\mathrm{L}_{2}$ ］Latitudinum difformium quedam est secundum se totam difformis， quedam autem non．Latitudo secundum se totam difformis est 〈illa〉 cuius nulla pars est uniformis．Latitudo difformis non secundum se totam est 〈illa〉 cuius aliqua pars est uniformis．
［Div．L3］Latitudinum difformium secundum se totas quedam est uniformiter dif－ formis，cuius est equalis excessus graduum inter se eque distantium sicut prima que uniformiter variatur in suis partibus，alia difformiter difformis cuius non est equalis excessus．
［Div． $\mathrm{L}_{4}$ ］Latitudinum uniformiter difformium quedam incipit a non gradu et termi－ natur ad gradum，quedam incipit a gradu et terminatur ad non gradum，quedam incipit a gradu et terminatur ad gradum．Notandum gradus est infinita modica pars intensionis．
［Div．L5］Latitudinum difformiter difformium quedam est secundum se totam diffor－ miter difformis，quedam non．Latitudo secundum se totam difformiter difformis est illa cuius nulla pars est uniformis aut uniformiter difformis sed latitudo non secun－ dum se totam difformiter difformis dicitur 〈illa〉 cuius aliqua pars est uniformis vel uniformiter difformis．
［Div．L $\mathrm{L}_{6}$ ］Latitudinum secundum se totas difformiter difformium quedam est uni－ formiter difformiter difformis，sicut illa que inter excessus graduum inter se eque
Mn8r distantium servat eandem proportionem alia tamen a proportione equalitatis；｜｜alia est difformiter difformiter difformis，sicut 〈est〉 que inter excessus graduum eque distantium non servat eadem proportionem．

## 〈II．1．De geometricis figuris〉

［ $\left.\mathrm{E}_{\text {def．} 1}\right]$ Notandum：punctus est cuius pars non habet positionem in continuo．
［ $\mathrm{E}_{\text {def．2 }}$ ］Linea est longitudo ymaginata sine latitudine et profunditate，cuius extrema sunt duo puncta．
［ $\mathrm{E}_{\text {def．3 }}$ ］Linea recta est ab uno puncto ad alium brevissima extensio in extremitates suas utrumque 〈eorum〉 accipiens．Per oppositum curva．
［ $\left.\mathrm{E}_{\text {def．14 }}\right]$ Figura est magnitudo termino vel terminis clausa．
［ $\mathrm{DF}_{5}$ ］Figura rectilinea est que omnibus rectis lineis clauditur．Curvilinea figura est
$\left[\mathrm{DF}_{7}\right]$ Figura plana est cuius tanta longitudo quam latitudo mensuratur linea recta． Per oppositum curva．
［ $\mathrm{E}_{\text {def．} 8}$ ］Angulus est duarum linearum alternus contactus et aplicatio earum non directa．
$45\left[\mathrm{E}_{\text {def．10 }}\right]$ Angulus rectus est angulus contentus per lineas rectas perpendiculariter super se invicem cadentes．
［ $\left.\mathrm{E}_{\text {def．12 }}\right]$ Angulus acutus est angulus recto minor．［ $\mathrm{E}_{\text {def．11 }}$ ］Obtusus est angulus recto maior．
［ $\mathrm{DF}_{1}$ ］Figurarum quedam angularis，quedam non angularis．Angularis est que habet tudo quam latitudo mensuratur linea recta，curva figura dicitur cuius longitudo vel latitudo mensuratur linea curva．

## 〈II．2．Suppositiones〉

Prima suppositio：omnia que ad invicem se habent secundum aliquam proportionem sunt quantitas，quia proportio est habitudo rerum ad invicem in quantitate．
Secunda suppositio：omne quod gradualiter excedit vel ab eo exceditur est，ymagi－ nandum per modum quantitatis continue．「Intelligendo figuram debet esse extensi－ vam et intensivam in quolibet excessu graduali．Per excessum gradualem intelligendo excessus intensivus」．
Tertia suppositio：excessus gradualis et latitudo gradualis vel intensio forme idem angulum vel angulos．Non angularis que nullum angulum habet．
$\left[\mathrm{DF}_{2}\right]$ Figurarum angularium quedam monoangulare，quedam plurium angulorum． Monoangulare que solum unum angulum habent ut illa O et continetur una linea curva．［DF3］Figurarum plurium angulorum，quedam biangule，quedam multian－ gule．Biangula est que est duorum angulorum precise，et talis｜aut duabus curvis lorum quam duorum．Et talium quedam est triangula quedam quadrangula et sic〈in〉 infinitum．
［ $\mathrm{DF}_{4}$ ］Figurarum biangularium quedam solis lineis curvis continetur，quedam curva et recta，ut semicirculus vel portio circuli，quedam constituitur ex arcu et costa，etc．
$\left[\mathrm{DF}_{5}\right]$ Figurarum multiangularium quedam rectilinee，quedam curviliniee．Rectilinea dicitur que solis continetur rectis lineis，ut quadratum．Curvilinea que omnibus lineis curvis vel curva et recta continetur．
［ $\mathrm{DF}_{6}$ ］Figurarum curvilinearum quedam omnibus curvis continetur，quedam aliqua recta et alique curva vel aliquibus curvis．
$65\left[\mathrm{DF}_{7}\right]$ Figurarum quedam est plana，alia curva．Plana dicitur cuius tanta longi－ sunt 「et verum est quod illi termini summuntur pro eodem de ascencu et descensu 70 ab eo］in marg．quibuscumque gradibus add．Mn 71 ante quantitatis］qualitatis scr．et del． Mn
propositionis．
Quarta：omne quod excessu graduali exceditur aliud vel exceditur ab eodem habet latitudinem gradualem．
Quinta：omne quod secundum aliquam dimensionem quantum est，hoc secundum eandem 〈dimensionem〉 excedit aliud aut excedi potest ab alio，etc．「Notandum〈quod〉 quedam est latitudo perfectibilis sicut hominis et asini，et non est proprie latitudo．


Fig．1：in margine
Mn9r｜｜$\langle 6\rangle$ Alia suppositio：extensio forme ymaginanda est per lineam rectam；intensio vero per figuram planam secundum rectam lineam consurgentem．
$\langle 7\rangle$ Alia suppositio：Cuilibet puncto in linea recta super quam figuram planam collocatur correspondet propria latitudo intensionis in data figura，ideo quilibet tale punctus in extensione propriam habet intensionem．
$\langle 8\rangle$ Alia suppositio：Cuiuslibet talis punctus in extensione propria intensio sibi corre－ spondens ymaginanda est per lineam rectam perpendiculariter erectam super punc－ tum datum．
$\langle 9\rangle$ Ultima suppositio：forme permanens ymaginatur extensive secundum exten－ sionem sui subiectum，ut caliditas，sed successive ymaginatur extensionem habere secundum durationem successionis．

## 〈II．3．Propositiones－conclusiones〉

Conclusio prima：omnis latitudo cuiuscumque forme est ymaginanda per figuram planam super rectam lineam consurgentem．Et sic nulla est ymaginanda per figuram circuli aut figuram monoangule aut omnibus curvis lineis contenta aut per figuram monoangulam．「ly＂cuiuslibet＂vel＂cuiuscumque＂hic superfluit et debet obmitti， quia tale conclusio intelligitur」．Omnis talis est ymaginanda per figuram plurium angulorum．
$2^{a}$ conclusio：Nulla latitudo est ymaginanda per figuram planam super lineam rec－ tam consurgentem per angulum obtusum sicut maiorem recto．Aliomodo erit in－ tensio forme sine extensione．「Quia non supponitur pro aliqua specie huius que est latitudo」．Corrolarium：nulla latitudo ymaginanda 〈est〉 per maiorem portionem circuli．
$3^{a}$ conclusio：omnis latitudo uniformis incipit a certo gradu et terminatur ad gradum：


Fig. 2: in margine
$\mathrm{Q}^{t a}$ conclusio: omnis latitudo incipiens a non gradu est difformis:


Fig. 3: in margine
$\lceil\langle$ Alia conclusio $\rangle$ : nulla latitudo ymaginanda est per figuram monoangulam $O$. Omnis latitudo forme ymaginanda est per figuram plurium angulorum $\square \square$. $5^{\text {ta }}\langle$ conclusio〉: omnis latitudo sive uniformis sive difformis incipiens a certo gradu est ymaginanda per figuram incipientem ab angulo recto, et omnis terminatur ad certum gradum est ymaginanda per figuram | que terminatur ad angulum rectum.


Fig. 4: in margine


Fig. 5: in margine


Fig. 5: reconstruction
$6^{t a}$ conclusio: omnis latitudo incipiens a non gradu ymaginanda est per figuram incipientem ab angulo acuto et omnis terminata ad non gradum ymaginanda est per figuram que terminatur ad angulum acutum. $7^{m a}$ : omnis latitudo incipiens uniformiter difformiter a non gradu est ymaginanda per figuram incipientem ab angulo rectilineo et acuto et omnis terminata uniformiter difformiter ad non gradum ymaginanda est per figuram que terminatur ad angulum rectilineum acutum.
$8^{v a}$ : omnis latitudo incipiens difformiter difformiter a non gradu ymaginanda est per figuram incipientem ab angulo acuto curvilineo et omnis terminata difformiter difformiter ad non gradum est ymaginanda per figuram terminata ad angulum acutum curvilineo.
$9^{n a}$ : Omnis latitudo uniformis per totum ymaginanda est per figuram quadrangulam rectilineam.
$10^{m a}$ conclusio: Nulla latitudo in aliqua sui parte difformis est ymaginanda per figuram quadrangulam rectilineam quantumcumque uniformis in principio et in fine. $11^{\text {ma }}$ : Omnis latitudo uniformiter difformis incipiens a non gradu et terminata ad gradum ymaginanda est per figuram rectilineam incipientem ab angulo acuto et
terminata ad angulum rectum. Et omnis uniformiter difformis incipiens a gradu et terminata ad non gradum est ymaginanda per figuram rectilineam incipientem $a b$ angulo recto et terminata ad angulum acutum.

## Mn10r

 $12^{m a}$ : Omnis latitudo uniformiter difformis incipiens a \|| gradu et terminata ad gradum est ymaginanda per figuram quadrangulam cuius duo anguli super basim constituti sunt recti. Reliquorum vero duorum unus est acutus alter obtusus.$13^{m a}$ : Nulla latitudo incipiens a non gradu et terminata ad non gradum est uniformis aut difformiter difformis, licet tamen posset habere pars uniformis aut uniformiter difformis.
$14^{m a}$ conclusio: Omnis latitudo incipiens a non gradu uniformiter difformiter et terminata uniformiter difformiter ad non gradum est ymaginanda per figuram incipientem ab angulo acuto rectilineo et ad angulum rectilineum terminatam. Variatio tamen in medio infinite modis potest fieri.
$15^{m a}$ : Nulla latitudo secundum se totam difformiter difformis ymaginanda est per figuram rectilineam sed omnis talis ymaginanda est per figuram cuius altitudo terminatur per unam lineam curva sive per lineas curvas.
$16^{m a}$ : Omnis latitudo uniformiter difformiter difformis incipiens a non gradu ymaginanda est per angulum habentem super basim unum angulum rectum et rectilineum et alios duos acutos et curvilineos, quia omnis talis terminatur ad certum gradum. Alter non est unifomiter difformiter difformis.
$\left\langle 17^{m a}\right\rangle$ Ultima concluditur quod omnis latitudo cuiuscumque forme qualitercumque sit variatio ymaginanda est per figuram similiter variatam. Et hoc infinite mode variari potest.

## APPENDIX TO LATITUDINES BREVES I

Further figures which occur in $\mathbf{M} \ddot{\mathbf{u}}(\mathrm{f} .51 \mathrm{v})$ and were not included in the edition.


[^29]

Figure 1: Top: München, BSB, clm 18985 (f. 53r), with some examples of latitudes and their representations according to LB I. Bottom: München, BSB, clm 19850 (f. 9r, top), showing the passage of LB III on the sixth and seventh conclusions.


Figure 2: Vienna, ÖNB, 4784 (f. 237r), including the introduction of LB II on the latitude of forms as a middle science.

## VI Remarks About the Content of the Texts

The late medieval method of representing qualities and motions, including of course their variations, has often been explained and discussed in the specialist literature. After some anticipatory indications in a quaestio by Giovanni da Casale, the main ideas were set out in Oresme's brilliant work De configurationibus and in Jacobus de Sancto Martino's more modest treatise De latitudinibus formarum. ${ }^{50}$

As a system of representation, the main ideas are easily comprehensible and quite useful. The aim of the whole enterprise is less to gain new knowledge than to clearly rework and communicate information already available, whether this is to be accepted or called into question (as in the case of the sophismata). To achieve this aim, both Oresme and Jacobus de Sancto Martino-following a line of thought that goes back to the so-called metaphysics of light of Grosseteste, Bacon, Pecham and Witelo-underline the role of geometry. ${ }^{51}$ Since the systems differ in certain special and not unimportant ways and since Oresme's method is undoubtedly the more exact, more universal and, in general, much better reasoned and founded of the two, it is possible either that Jacobus took up Oresme's method and changed it for the worse or, alternatively, that Oresme improved a method that already existed. ${ }^{52}$

The system rests on an orthogonal representation of physical factors, which is sometimes said to be a precursor of the modern Cartesian coordinate system:

[^30]a horizontal line of "longitude" (longitudo) is used to represent the length of a body (subiectum) in which a quality or a "form"-for instance, a temperatureinheres. This quality exhibits "intensities" (intensiones), which are represented as lines drawn perpendicularly on the horizontal "longitude" line. The unit of measurement used in the representation can be freely chosen, but the procedure must always be proportional. For example, if we represent intensities A and B by the lines C and D , respectively, and if $\mathrm{A}=2 \mathrm{~B}$, then C must be twice as long as D . The perpendicular lines that illustrate the distribution of intensities on the body that is represented by the longitude line, are called "latitudes" (latitudines). Here, LF deviates from Oresme's DC in an important respect. Oresme distinguishes very carefully between terms that are physical in meaning (or metaphysical, which is not relevant in this context) and those that are merely geometrical. He never uses the notion of latitude to mean anything physical. For Oresme, a latitudo is a purely geometrical term that designates the line representing an intensio - that is, the physical intensity of a quality. For the author of LF, on the other hand, there is no sharp difference in meaning between latitudo and intensio. Hence, he speaks of the "latitude of forms," an expression that, strictly speaking, has no meaning for Oresme and one that he tried, moreover, to banish from his doctrine. There is, however, a common point which is decisive for both (but which only Oresme has really exploited in all its possibilities): the longitude line can represent not only the distribution of intensities / latitudes on a body but also the behaviour (variation or not) in time of a quality or form. Thus, the horizontal line can represent the flow of time in a given direction, while the lines drawn perpendicular to it can stand for the intensities of a motion, the velocities (velocitas, but without vectorial connotation) at each point of time. As a result, the method makes it easy to represent notions that are otherwise quite difficult to explain: a "uniform" (uniformis) distribution of a form or quality in a body can be represented by a rectangular figure, because all the lines representing the intensities are equally high. The same can be said of a change or a motion that takes place regularly in time. In this special case, the perpendicular lines represent the velocities at each instant of time. Variations in the height of the latitudes show that the represented quality or motion is difform or "irregular" (difformis). Furthermore, this variation or varied distribution can take place either regularly or irregularly, so that we then have a representation of something that is either uniformiter difformis or difformiter difformis. The treatise LF also contains yet a further classification into uniformiter difformiter difformis and difformiter difformiter difformis that is superfluous in Oresme's method. The two systems differ in another crucial way: Oresme does his best to make clear that the complete figure represents the total quantity of the quality under consideration. In the special case of motion, the quantity of the quality is to be identified with the total amount of "acquired perfection" (perfectio acquisita), which is, in the still more specific case of local motion, the distance traversed. This is a fundamental notion for all of Oresme's beautiful
analyses relating to the "mean speed theorem" and the "infinites series" in the third book of DC. By contrast, LF does not include any important considerations of this kind, but only a confusing terminology: the surface of the figure that, according to Oresme, represents the quantity of a quality (quantitas qualitatis), LF also calls latitudo. ${ }^{53}$

While many aspects of and historical questions concerning the context and the place of the Latitudines breves in medieval teaching remain to be investigated, it is still possible to make a few remarks about the content of the texts themselves. On the basis of the description of the university background set out above, it would be more accurate to refer to a compiler of such texts rather than to an author in the strict sense.

All three texts represent a heavily abbreviated version of the original Tractatus de latitudinibus formarum (LF). LB 1 omits all of chapter II. 1 on geometrical figures (Smith 1954, 7-10). The compiler proceeded in much the same way as the compiler of the above-mentioned Compendium introductorium: he combined the definitions and divisions of the initial part of the treatise with propositions from chapter II.3. In addition, he introduced a few small geometrical figures that, contradicting the general aim of the original treatise, do not have an illustrative function but contain numbers that indicate, in turn, the magnitude of the different degrees in the different parts of a latitude (here understood as a synonym for "quality"). Thus, a uniform latitude is assigned to the number 222 to indicate regularity in all its parts, whereas a uniform-difformly distributed latitude is indicated by the number 1234, for instance. Only after these explanations have been provided are the "right figures" with their corresponding denominations given (some of them, which are senseless or confusing, are reproduced in an appendix at the end of the text).

The texts of LB 2 and LB 3 are more closely related to LF. Both offer a more generous selection of sentences from LF and consider all the treatise's chapters. These sentences have clearly been taken from an original LF text, for they include the original enumeration of the propositions (even if these are sometimes wrong). In both texts, the compilers' efforts to bring the doctrine of the latitude of forms close to geometry are evident, since the passages of LB 2 and LB 3 concerned with the geometrical figures in chapter II. 1 of LF, which in the latter text contain only definitions of geometrical objects, introduce short definitions for these notions that

[^31]do not occur in LF. ${ }^{54}$ Thus, whereas the author of LF has only referred summarily to Euclid and dealt with the division of the figures - which is understandable since this is all that is immediately relevant for his purpose - the compilers of LB 2 and LB 3 have taken up the passage again and supplemented it with the corresponding definitions from the Elements. This is an interesting feature that we also find in the Expositio to LF and in the Compedium. ${ }^{55}$

In addition to these details, the most important aspect of the content to be noted occurs in the three forewords, which are very similar, but not identical. These represent the clearest evidence that LB 1-3 are immediately connected to the Expositio in LF, since this text opens with a long discussion of the same topic-that is, of how and to what extent this "science," the doctrine of the latitude of forms, can be characterized as a middle science.

LB 1 refers to the "book" on the latitude of forms, whose science is to be classified between physics and geometry. This is a "middle science," as is evident from the combination of terms in the title of the treatise: latitudo is a geometrical term; forma, by contrast, a physical one. In addition, this discipline is useful for understanding the parts of philosophy that deal with motion and the intension and remission of qualities. In his introduction, the compiler indicates that the expressions latitudo forme and intensio gradualis (forme) refer to the same objects. The first expression means that the intensities can be represented by geometrical figures. Hence, the main purpose of the discipline is to provide the elements for building up such figures. The compiler of LB 1 may have read Oresme's DC or, at least, have known its doctrine indirectly, since he states the principle of orthogonality for the representation of qualities and the epistemic principle according to which longitudes and extensions are "better known" (notior) than latitudes and intensities.

LB 2 also characterizes the latitude of forms as a middle science. When considering the doctrine's status, the compiler states clearly that the notion of subiectumthat is, of the concrete research subject - is fundamental. And this subject is, he adds, both physical and mathematical in nature. LB 2 also shows many points of contact with the Expositio. For example, it includes the remark that the latitude of forms is useful for understanding certain doctrines of the Physics (Book III) as well as sophisms on qualitative motion. LB 2 also offers a brief classification of forms. It states, for instance, that forms are either divisible or, like the soul, indivisible. Divisible forms, in turn, are either (1) extensive like the forms of animals, (2) intensive like the intellect or (3) both extensive and intensive like the forms of heat and

[^32]coldness. Furthermore, some forms, it says, are divisible according to the extension of their bearers or-like local motion-according to their duration in time.

LB 3 keeps very close to LB 2. The forewords to both works are more or less the same, although that of LB 3 is shorter. The compiler of the latter text has also included a brief classification of the forms, adding that it is evident-from a consideration of the doctrine's subiectum - that the latitude of forms is a middle science. In fact, LB 3 has long been the only text known to characterize the doctrine of the latitude of forms as a science of this kind.

## VII Concluding Remarks

Since this has been a quite extensive and intricate historical survey, I would now like to summarize the most important claims discussed in this paper. These claims are as follows:

1. From the end of the 14 th century until at least the first decades of the 16 th, the doctrine, science or "art" of the latitude of forms was the leading corpus of ideas covering the field of what we could characterize as an effort to geometricize physics. ${ }^{56}$
2. This doctrine constituted a closed system of knowledge that had the status of a scientific discipline like physics, optics and ethics.
3. Apparently, the discipline was taught systematically at many German universities, and it seems to have been especially important-along with other middle sciences - at the Faculty of Arts of the University of Vienna.
4. At least initially, the lectures in Vienna were probably based on the original text of LF. As was often the case, this text was commented on and occasionally summarized in order to communicate its central ideas more simply. It was soon perceived that two of the sciences taught, the science of proportions and the science of latitudes, were indeed two new middle sciences, not mentioned by Aristotle or included in the tradition of Aristotelian commentary until the second half of the 14th century.
5. This teaching activity led to the emergence of short versions of these and similar middle science texts: the Proportiones breves for the science of proportions and what I have called the latitudines breves (LB 1-3) for the science of the latitude of forms.
6. The LBs were a product of the University of Vienna: a) We possess a huge amount of information (records, etc.) about the acceptance and teaching of this doctrine at the Vienna Faculty of Arts. Early lectures on the doctrine

[^33]at Vienna cast at least some doubt on Clagett's late dating of LF; b) The connection between the forewords of all three LBs with one another and with the Expositio preserved in a Vienna manuscript copied by the Vienna magister, Dean and Rector Michael Lochmayr provides a direct and undeniable link to Vienna University; c) All the manuscripts originated in Vienna and are dated or datable to the period in which LF and/or LB were taught.
7. Even if it were possible to relate LF or the LBs to the list of teachers of the discipline, none of these individuals could be identified as the author of LB. Probably, there was no such author but only a compiler who (correctly) thought that he was doing nothing original in summarizing the text of LF for classroom use.
8. Consequently, the content of all the texts is very poor. It seems clear from the LBs that (as Anneliese Maier has already pointed out) the doctrine never reached the deep philosophical insight of Oresme and that it was increasingly reduced to a methodological doctrine of geometrical representation.

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## Abbreviations

$\mathrm{DC}=$ Tractatus de configurationibus qualitatum et motuum, ed. Clagett (1968)

LF $=$ Tractatus de latitudinibus formarum, Smith (1954)
$\mathrm{LB}=$ Latitudines breves (ed. in this contribution)

BSB $=$ Bayerische Staatsbibliothek
ÖNB $=$ Österreichische Nationalbibliothek

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[^0]:    ${ }^{1}$ The literature on the Oxford calculators is now quite extensive. For an introduction to Heytesbury and Kilvington, see Longeway (2010) and Jung (2011). For further information, see Sylla (1982) with additional references. A good general introduction can be found in Wilson (1960), focusing on Heytesbury, and Hallamaa (2005, 25-51), including a partial edition of Roger Roseth's Letura super Sententias. For Richard Swineshead and his seminal Liber calculationum, see Murdoch and Sylla (1976). For a recent discussion about the fate of medieval mechanics with particular emphasis on the Oxford calculators, see Jung (2004). By the way, I see no reason to continue writing Regulae since, as is well known, medieval Latin regularly uses Regule. In this paper, I will "normalize" Latin terms only in those few instances in which a misunderstanding could arise.
    ${ }^{2}$ Oresme, De configurationibus qualitatum et motuum, ed. and trans. Clagett (1968, 159).

[^1]:    ${ }^{3}$ See Di Liscia (2007a) about Vives' criticism of the calculators' approach, and Di Liscia (2007b) with special reference to the tradition of the latitude of forms.
    4 Ed. Smith (1954, 371-2). For the purposes of this paper, I assume without further discussion Anneliese Maier's thesis that the author of LF is Jacobus de Sancto Martino, also called Jacobus de Neapoli. Jacobus de Neapoli was certainly the author of a treatise on the "perfection of the species," which had a large impact on the philosophy of the late 14 th, the 15 th and the beginning of the 16 th centuries. As part of my current project, "Die Geometrisierung der Metaphysik im Spätmittelalter: Jacobus de Neapoli und die Tradition De perfectione specierum" (Deutsche Forschungsgemeinschaft), I am preparing a critical edition of this text based on all known manuscripts (for a list and description of these eleven manuscripts, see Di Liscia (2010, 428-432)) as well as several other related texts.

[^2]:    5 "Item statuimus quod Baccalaureus temptandus debet audivisse libros infrascriptos: primo spectantes ad gradum Baccalauriatus in artibus. Item talis debet audivisse ultra illos in aliquibus scolis publicis alicujus Universitatis in qua protunc fuerunt quinque Regentes magistri in artibus, libros infrascriptos: Physicorum ex toto; de celo et mundo; de generatione et corruptione; Metheorum; parva naturalia quo ad quatuor libros; de senso et sensato; de longitudine et brevitate; spheram mundi; theoricas planetarum; tres libros Euclides; Perspectivam communem; aliquem tractatum de proportionibus, et aliquem de latitudinibus formarum, et aliquem in musica; et aliquem in arithmetica" (Statuta (1398) Facultatis Artium, in: Bianco (1855a, 59-74), this passage on (68). De anima is not mentioned in this list by the way. It should also be noted that a previous paragraph states that the parva logicalia were also prescribed: "Supponentes (= Suppositiones!), Ampliationes,

[^3]:    Appellationes, Consequentias, Obligatoria, et Insolubilia" (64).
    ${ }^{6}$ Something that, for instance, the author of the Quaestiones heidelbergenses does in fact provide.
    For the the teaching of the latitude of forms at Heidelberg, see Di Liscia (2010, 103-172), including an edition of this text.
    ${ }^{7}$ On the authorship and transmission of the Theorica planetarum see Pedersen (1981).
    8 For a recent study on these statutes see Brincken (1989).
    9 On the different philosophical currents of 15 th century Germany with special attention to the development in Cologne, see Weiler (1962, 56-83), Meerseman (1933/35) (1933, on the Cologne Albertism) and the excellent discussion by Hoenen $(1993,1995)$. All this material, however, focuses on the theological and philosophical discussions of the time but include few comments about the mathematical disciplines.
    10 Henry of Gorkum Positiones ad opponendum et respondendum (...) circa libros physicorum (...) Aristotelis, 1508. For Henry of Gorkum (= Heinrich de Gorychem, von Gorkum, ca. 1378-

[^4]:    1431) see Weiler (1962), on this text p. 87. In his paper on the Cologne Faculty of Arts, Meuthen (1989) emphasizes the important role of Henry of Gorkum since 1494 as "einer der maßgeblichen Universitätsbeamten" (p. 374). Also his followers Gerhard ter Stege von 's-Heerenbergh (1431-1480) and Lambertus de Monte (d. 1433) should have been very influential in the university affairs.
    ${ }^{11}$ This is one of the usual questions related to the determination of whether the scientia de latitudinibus formarum is or is not a middle science. See below, footenote 15. This brief text refers in passing to the problem of subordination and mentions many more middle sciences such as the art of navigation and even of organ construction, assigning the first to astrology and the second to music: "rationabile est condere predictas scientias medias esse ceteris principaliores, patet per Aristotelem qui de illis tribus [perspectiva, musica, astrologia are meant here] solum facit mentionem per hoc inveniens quod sunt magis principales. Sed possunt poni quedam alie illis subalternate sicut ars navalis que sulbaternatur astrologiae et ars factiva organorum que subalternatur musice et dicuntur minus principales" (f. ${\mathrm{b} 3 r^{r}}^{r}$ a).
    ${ }^{12}$ For the seminal category of "analytical languages," see Murdoch (1982).
[^5]:    13 "...sicut patet de illis propositionibus 'infinita est linea,' 'infinitus est motus,' 'infinite partes sunt in continuo,' 'infinite fuerunt homines,' 'infinitum pondus Sortes potest portare,' 'infinita sunt finita.' Et quia expositio illarum propositionum concernit logicam, ideo eas reservo tractavi de exponibilibus," (Positiones (1508), f.c1va) Exponibilia were the subject matter of several special logical treatises. See Kretzmann (1982).
    14 This phenomenon seems to have been more general, as can be observed in other texts as well. See the text edited and discussed in Di Liscia (2007a).
    15 That the scientia de latitudinibus formarum was considered a middle science has often been suspected but only recently established by historical investigation. Since this matter is the subject of another specific study, I will refer to it here only insofar as it is related to the LBs. Such reference is, in any case, unavoidable since the categorization of the "science of the latitude of forms" as a middle science occurs in all three LB texts and seems, moreover, to be a specific feature of its treatment at the University of Vienna. For a more detailed account of the latitude of forms as a middle science, see Di Liscia (Forthcoming). It is quite possible and, indeed, even likely that the Vienna model was transmitted to other universities-including the Unversity of Cologne itself.

[^6]:    16 Even if partially outdated, Clagett's chapter on "The reception and spread of the English and French physics" in his classic work The Science of Mechanics in the Middle Ages provides a valuable introduction to this vast field of research (Clagett 1959, 629-82).
    17 Against Pierre Duhem, who was of the opinion that Albert and Marsilius knew the Oresmian doctrine and employed it in their works, Anneliese Maier (1952, 356-58) maintained a weaker thesis: Marsilius and, above all, Albert surely knew the configurationes doctrine but applied it only infrequently. In my opinion, Maier's arguments do not convincingly demonstrate that Marsilius or Albert were acquainted with Oresme's configurationes. By the way, there is no scientific reason at all for attributing LF (or LB) to Albert, as Aschbach $(1865,355)$ and others have supposed. See also the more up-to-date and reliable studies by Sarnowsyk (1989, 47), Berger's Bibliographie and (Berger 2000). For Marsilus as the first rector of the University of Heidelberg, see Miethke (1992). ${ }^{18}$ For bibliographical information, see Hohmann (1976).

[^7]:    19 "Tamen praecipue hoc faciunt istae duae scientiae, scilicet geometria et arithmetica; et hoc cum aliis scientiis quae pullulant ex eis, scilicet astronomia, musica, perspectiva, scientia de ponderibus,〈et scientia〉de formarum latitudinibus et subtilibus calculationibus..." from Steneck (1975, 254, f. 30). For Henry of Lagenstein and Henry of Oyta within the framework of the philosophical discussions in Vienna, see Shank (1988) (with special attention to logic and theology).
    20 See Clagett $(1968,116$, f. 7 ) and Di Liscia (2010, 8, f. 18). The expression ars latitudinum is also used by other authors such as Johannes Rucherat de Wesalia. See Di Liscia (2015, 15).
    21 Clagett's attempt to explain the strong evidence of Henry's acquaintance with LF by an even more unlikely knowledge of Oresme's Euclid commentary is an unacceptable distortion of the documentary evidence and cannot be understood unless one accepts his later dating of LF. However, LF's date is by no means certain, so that Clagett's argument runs the risk of being circular. Furthermore, Clagett has overlooked many of the observations made by Anneliese Maier and distorted other evidence in order to preserve his own reconstruction, which, on this point at least, is hardly sustainable. I cannot deal here with Clagett's opinions in full. For a more detailed account of this matter, see Di Liscia (2010, 6-9).

[^8]:    22 "Item baccallarius presentandus por licentia in artibus ad temptamen debet audivisse omnes libros spectantes ad gradum baccallariatus in artibus . . libros infrascriptos: Meteora, Parva naturalia communiter legi consueta, Theoricas planetarum, Quinque libros Euclidis, Perspectivam commune, aliquem tratatum de proportionibus et aliquem de latitudinibus formarum, aliquem librum de musica et aliquem in arithmetica..." Lhotsky (1965, 243) (my emphasis). For the textual history of the statutes, see Lhotsky's remark on p. 223).
    23 "Titulus XXIC. Libri ordinarie legendi cum ipsorum collecta (...) Primus igitur librorum sit physicorum, de quo toto dantur novem grossi. Metaphysica (...). De generatione et corruptione (...). De anima (...). Libri parvorum naturarum (...). Politicorum (...). Ethicorum (...). Yconomicorum (...). Boethius de consolation philosophie (...). De quinque libris Euclidis(...). Theorica plentarum (...). De perspectiva communi (...). Sphera (...). Proportiones longe Bragwardini tres grossi. De latitudinibus formarum duo grossi. Summa naturalium Alberti (...). De veteri arte (...). De tractatibus Petri Hispani (...). Priorum (...). Posteriorum (...). Topicorum (...). Elenchorum (...)." Lhotsky (1965, 252-3). See also Section III. 3 below.
    ${ }^{24}$ From the fact that the statutes of 1389 mention "a text" on the latitude of forms, Schöner, who tacitly assumes Clagett's dating of LF, draws the conclusion that "the work by Jacobus de Sancto Martino, which had just been written at that time, presumably cannot be meant here" (1994, 36, f. 60). I disagree completely with this opinion and the approach on which it is based. My reasons are these. First, Anneliese Maier has dated LF some twenty years earlier, and her hypothesis-which is in my opinion more accurate-deserves at least as much attention as Clagett's view. Second, and

[^9]:    more importantly, this conclusion is not supported by the facts. And the main fact is, as Schöner and others well know, that the statutes mention "aliquem de latitudinibus formarum." If LF is not the text meant here (of course, "aliquem" refers not only LF, but also to LB, which is based on it), then what text is? Clagett's dating of LF offers neither a solid basis for research nor a starting point for reasonable speculation. On the contrary, if the statement in the statutes is accepted as fact and the text in question is the one referred to under the title De latitudinibus formarum, then the conclusion can only be that Clagett's dating is not correct.
    25 As a matter of fact, a section on the latitude of forms included in a mathematical Compendium of the same time and context must also be numbered among them; see Di Liscia (2010, 357-414).

[^10]:    26 The general information already available in Aschbach (1865, 424 ff .). For the mathematical context, I refer to the unfortunately little-known but well-conducted research by Günther (1887, 198 ff .). In any case, both manuscripts also contain a text on arithmetic with features similar to the Proportiones longe: ÖNB 4951, ff. 273r-304v and 4953, ff. 36r-61v.

[^11]:    1 ". . . magister Iohannes Graser Proporciones breves . . . magister Nycolaus Dinckelpuel Latitudines formarum" (Uiblein (1968, 69), my italics). Nikolaus von Dinkelsbühl (in German, c. 1360-1433) was an outstanding figure at Vienna University. Having studied under Henry of Langestein and

[^12]:    ${ }^{27}$ For the years 1385-1416, see Uiblein (1968). The subsequent years, which are not yet edited, are preserved in the Archive of the University of Vienna: AFA II (years 1416-1446) Cod. PH 7; AFA III (years 1441-1497) Cod. PH 8; AFA III (1497-1559) Cod. PH 9). I am grateful to Thomas Maisel, the Director of the Archive of the University of Vienna for this information.
    28 Lorenz (1985) gives a comparative overview of the different disciplines as set out in the statues of Prague, Cracow, Vienna, Heidelberg, Cologne, Erfurt, Leipzig and Leuven.
    ${ }^{29}$ For the interrelationship with other scientific disciplines, see Kren (1987). Kren (p. 324) points out that the lectures on latitudines formarum provide "an especially striking example of a junior teaching monopoly".

[^13]:    9 "... Cunradous de Oenelspach Proporciones Bragwardin ... magister Ihoannes de Rechperb Latitudines..." (Uiblein (1968, 210), my italics).
    10 "... Magister Iohannes de Gmunden Perspectivam .... magister Ulricus de Egenburga Proporciones, magister Ihaonnes Fühlt Latitudines formarum, magister Thomas Öder de Aspach Musicam Muris..." (Uiblein (1968, 430), my italics).
    11 "...magister Thomas de Aspach latitudines formarum, magister Tilmannus Obligatoria Marsilii. . ." (Uiblein (1968, 453), my italics).

[^14]:    30 Wiener Artistenregister 1416-1447 (2007). To simplify the system of quotations, I have included in the table the page number and a number for each name rather than unnecessarily provide many dozens of footnotes.

[^15]:    31 According to the research conducted by Ferm（2011），Vesthof was teaching the Parva naturalia in the year 1437／38 and the Oeconomica in the academic years 1439 and 1443．In the academic year 1438／39，he lectured on the latitude of forms．Ferm＇s assessment requires two corrections：Oresme was not the author of LF，and this text has nothing to do with＂the dimensions of physical bodies＂ （Ferm 2011，65）．
    32 According to the research done by Kihlman（2011a，97），Bero lectured on the latitude of forms in 1437 （see the list above）．Unfortunately，the Latitudines formarum is no longer in Bero＇s library， which was bequeathed to the Cathedral of Skara in Sweden（Kihlman 2011b，127）．It must be noted that Kihlman correctly remarks（in the same book）that De latitudinibus formarum was＂formerly

[^16]:    ${ }^{33}$ Item latitudo est qualitas divisibilis intensive，ut est qualitas prima vel secunda $\langle$ et〉 motus localis continuus．Uniformis latitudo 〈est〉 que est eiusdem gradus per totum et debet sic intelligi：est latitudo que secundum se et quamlibet sui partem divisibilem est equalis intensionis per totum． Difformis latitudo est que non est eiusdem gradus per totum，et potest infinitis modis variari．Precise uniformiter est talis 〈que est〉 equalis inter excessus graduum．

    Item figura sic notificatur：est magnitudo termino vel terminis clausa et pro li magnitudo con－ sideratur substantia．Figura rectilinea est que omnibus lineiis rectis continetur．Sed curvilinea $\langle e s\rangle$ que continetur aliqua recta et［et：correxi ex vel Mn］aliqua curva．Item figurarum quedam 〈est〉 plana，et est figura que tam longitudine quam latitudine mensuratur linea recta；〈et〉 quedam est ［est：correxi ex linea Mn］curva，et est $\langle i l l a\rangle$ in qua longitudo vel quam latitudo mensuratur linea curva．

    Notandum quod angulus est duarum linearum pertractus contactus．Angulus rectus est angulus tractus per duas lineas rectas perpendiculariter super se cadens．Angulus acutus est angulus minor recto．Alio modo latitudo difformis potest variari inquantum dicitur uniformiter difformis inquantum servat eandem proportionem equalitatis．

[^17]:    34 Thus, for instance, the Vienna manuscript ÖNB 4907 (ff. 1r-370v) contains a set of Positiones et quaestiones e Logica, Metaphysica et Physica Aristotelis intermixtis prolusionibus in occasione actuum academicorum habitis in universitate Viennensi, among which the three following questions are included: 1) f. 1r: Utrum totalis latitudo uniformiter difformis debeat coincidere gradui medio (only the title is conserved, the disputatio itself is however lacking); 2) ff. 161v-162v: Utrum in latitudine entium universi quelibet species superior sit in infinitum perfectior essentialiter quam species inferiores, and 3) f. 240v-241r: Utrum omnis latitudo uniformis sit ex eisdem gradibus per totum. I thank Edit Anna Lukacs for informing me about these questions.
    ${ }^{35}$ By the way, the Vienna manuscript ÖNB 4953 that contains LF also contains a large number of texts on rhetoric: ff. 186r-197r Tractatus de conscribendis epistolis; ff. 198r-200v: Tractatus de coloribus videlicet figuris artis rhetoricae; ff. 203r-208v De arte dictandi tractatus by magister "Iupiter" or "Iovis." For this master, see Lhotsky (1965, 78).
    36 See Lhotsky (1965, 119-63) as well as an edition of Aneas Silvius's disputation at Vienna in 1445 on pp. 263-73.

[^18]:    ${ }^{37}$ Celtis spent the last years of his life in Vienna, where he established the Collegium poetarum et mathematicorum, a center for the study of both poetry and mathematics. For a description of the situation at Vienna University immediately before Celtis, see Lhotsky (1965, 165-205). Of course, Celtis's influence extended to cultural milieus outside Vienna as well as to other academic institutions such as the University of Ingolstadt. Schöner (1994) provides an excellent study of Celtis's influence.
    38 Training in sophismata was a central element in the continuation of the calculatores tradition, as Edith Sylla $(1985,1986,1989)$ has pointed out with considerable emphasis. In fact, such training is mentioned expressis verbis in the statutes of 1389 (see Lhotsky (1965, 254-5)). However, it should be noted that the relationship of the sophimata to the latitude of forms was by no means the same as it was to the proportions doctrine. Proportions and the latitude of forms were both "middle sciences" -partly mathematical, partly physical-each dealing mathematically with a different area of natural philosophy. But the latitude of forms, which emerged in the field of logic and philosophy of language and could occasionally be extended to other fields, was to a great extent the solution to or at least the instrument for solving certain sophismata.

[^19]:    39 Although they were written in very different ways, both manuscripts contain the complete text of LF with one and the same commentary-something Clagett (1968, 102-3) evidently failed to recognize. For a complete edition and commentary on this text, including a new report on Michael Lochmair, see Di Liscia (2010, 224-46).
    40 According to Uiblein $(1980,395)$, the University of Vienna was at this time purely in terms of numbers in its heyday.
    41 Uiblein (1980, 399-400). On Johannes von Gmunden's activities at the University of Vienna, see Uiblein (1988).
    42 Or was the scholastic tradition still so strong that it could resist the reforming tendencies of a humanism that ultimately did not manage to prevail? Referring back to Lhotsky (1965, 165),

[^20]:    44 Wiener Artistenregister 1497-1555 (2007), see 22156; p. 50.
    45 The handwriting is scarcely legible, so all the names cannot be clearly read.
    46 The description in the catalogue assumes this date for the whole manuscript (see Tabula (1869, 444)).

[^21]:    47 See bibliography LF, Vienna 1515, and for more information, Di Liscia (2010, 428)).
    48 For Georgius Tannstetter, see Graf-Stuhlhofer (1996).

[^22]:    49 ＇Eschumbach＇in Markowski，see Text 7 below．

[^23]:    1 I have added 〈sequitur〉 to make clear that LB I could be a part of a more comprehensive text． （See f．51r：Sequitur de perspectiva ．．．，f．56r：sequitur de astronomia ．．．）

[^24]:    2 Nicole Oresme，$D C$ ，I．3：＂Verumptamen quia extensio est manifestior et palpabilior，ut ita lo－ quitur，et prior cognitione quo ad nos quam sit intensio．．＂（Clagett 1968，172，ll．15－19）．

[^25]:    6 ante latitudo］forma scr．et del．Mü 15 post adquista］si vero oportet add．Mü

[^26]:    ${ }^{3}$ This is the beginning of $L F$（Smith 1954，1）．

[^27]:    56 rectilineus］correxi ex rectiatus Wi 65 triangulus］correxi ex angulus $\mathbf{W i}$

[^28]:    79 figura］correxi ex linea Wi｜post plana］figura add．Wi 100 ante super］seu add．Wi

[^29]:    139 terminata] correxi ex terminatur $\mathbf{M n}$

[^30]:    50 For an outstanding explanation and discussion of the many details involved, see Maier (1968, 76-109), Maier (1952, 289-353), Clagett (1959, 331-417) and, of course, Clagett's introduction to his edition of Oresme's DC (1968, 3-141).
    51 For instance in chapter III.v of DC, where Oresme says: "Therefore, in order to have measures and ratios of qualities and velocities one must have recourse to geometry" ("...recurrendum ad geometriam", trans. Clagett (1968, 404, ll. 7-8)). Such a claim is directly expressed in the very first sentence of LF: "Because the latitudes of forms are varied in many ways, this multiplicity is discerned with difficulty unless reference is made to geometric figures" (". . . nisi ad figuras geometricas consideration referatur," trans. Smith (1954, 2)). In his questiones on Euclid's Elements, Oresme mentions Grosseteste and Witelo when discussing the use of lines for representing physical magnitudes (for partial edition and translation, see Clagett 1968, 536, ll. 10-16).
    52 This is not the place for an in-depth discussion of this problem, which is closely connected with the question of the two texts' relative dates. Even A. Maier (1952, 369-71), who maintained that LF is earlier, believed that this text was produced after DC and hence that its author had intended to simplify Oresme's ideas. However, there is no strong evidence for this view in the text of LF itself. Moreover, there is one important indication for the opposite view, which I also consider the more likely (see below on the notion of latitudo). By the way, the method of representation is mostly two-dimensional but, as Oresme plainly states, it could also be applied in three dimensions. In that case, the represented entity would have two dimensions and the representation one more. See DC, Clagett (1968, 270-2).

[^31]:    ${ }^{53}$ Why should the author of LF introduce such mistakes and confusions just because he is shortening DC? By the way, there is a passage in DC where Oresme directly criticises this use of the term latitudo (see DC, Clagett 1968, 172-173). Of course, there may be other texts that use the notion of latitude in the way criticised by Oresme. For an excellent survey of the concept of latitude in the Oxford calculators as it relates to Oresme, see Sylla (1973). The best survey of the ontology of intensities is provided by Maier (1968, 3-73).

[^32]:    ${ }^{54}$ LF II.1: "Diffinitiones enim suppono ex primo Euclidis, videlicet, quid est linea, quid figura, quid angulus; item que linea recta, que curva; que figura plana, que curva, quis angulus rectus, quis obtusus, quis acutus, etc." (Smith 1954, 7).
    ${ }^{55}$ Furthermore, LB 1 and LB 2 include a definition of "point" that does not go back to Euclid (Di Liscia 2010, 401-404).

[^33]:    ${ }^{56}$ As far as this physics was based on qualities and motions. For an in-depth discussion of this issue, see Sylla (1971).

