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An Arabic Introduction to Euclidean Geometry in Didactic Verse

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Abstract

We present an Arabic geometrical treatise in didactic verse (*manẓūma* or *urjūza*) composed by the prolific 11th/17th century Shīʿa legal scholar and expert in *ḥadīth* (sayings of the Prophet Muḥammad), al-Ḥurr al-ʿĀmilī. The treatise is a rhymed rendition of the widely read introduction to Euclidean geometry, *Ashkāl al-taʾsīs*, composed by the 6th/13th century mathematician and logician, Shams al-Dīn al-Samarqandī. Although the use of didactic verse was not uncommon in late medieval Arabic education, within the mathematical sciences the technique was applied more frequently to arithmetic or algebra than to geometry. By its nature, versification requires condensation and the distillation of the essence of the subject. Thus these didactic verses can reveal what the author considered to be the most essential features of al-Samarqandī's treatise and of Euclidean geometry.

I Introduction

The use of rhymed prose/didactic verse for broadly pedagogical purposes has a long history. Instruction in a wide range of topics has been presented in versified form. Since these rhymed works made no claim to be literary or artistic, van Gelder (1995, 117) has suggested the term "didactic verse" rather than "didactic poem" to designate the genre—a suggestion which I shall follow in this paper.

Examples of didactic verses may be found in many ancient literary traditions (Schuler and Fitch 1983; Harder 2007). Prior to the medieval period, didactic verse was instructional primarily in a generic sense, such as providing basic instruction in social morals—although one can point to exceptions such as Hesiod's versified description of weather phenomena in *Works and Days*, *De rerum natura* of Lucretius, and the *Georgics* of Vergil.

Didactic verse also developed into an important genre within the Arabic-Islamic milieu, although it has seen relatively little study. Examples can be found already in the pre-Islamic period, but the genre grew in importance after the ʿAbbāsid period. The rise of didactic verse in Arabic parallels the development of the *madrasa* as an instrument of the state. At this time, there was no universal curriculum—each *madrasa* had its own curriculum, specified by the patron who established the insti-

tution. With the rise of formal curricula, didactic verse was used more intentionally to provide instruction relating to specific topics (van Gelder 2007).

Instruction in the religious sciences was almost always paramount in the medieval *madrasa*, although the "rational sciences" such as mathematics were also found in some curricula. Of course, the mention of the "rational sciences" in the foundation documents of a *madrasa* does not necessarily mean that such instruction was either thorough or consistently provided. But the view that the mathematical sciences almost completely vanished from the curriculum by the ninth/fifteenth century has been shown to be too simplistic (Brentjes 2008a). Moreover, *madrasa* instructors sometimes offered courses in mathematics outside the formal institutional structure as well.

Whether instruction in the rational sciences was provided inside or outside the *madrasa*, the focus of these courses was typically textual, rather than disciplinary—the student would read a particular treatise with his teacher, who would then certify his competence to teach the text. The purpose for studying mathematics was often closely tied with a student's desire for political or legal appointment within society. Thus many students who attended instruction in mathematical sciences might not have been interested in mathematical knowledge for its own sake but only as a means to a personal end. In this context, such genres as the *fawā'id* ("advantages" of a discipline or topic) which presented only the highlights of a text and *manẓūmāt* which presented those essentials using didactic verse became ever more popular (Brentjes 2014, 99).

Nearly three decades ago, Galal Shawqi (1984) published a preliminary survey of Arabic mathematical works written in the form of didactic verse (called either *manẓūma* or *urjūza*). Since that time, there have been few studies of the genre by historians of the mathematical sciences.

In this paper, I introduce an example of didactic verse on geometrical science that was apparently unknown to Shawqi when he compiled his survey. The treatise presents a rhymed version of the popular introduction to geometry, *Ashkāl al-ta'sīs*, by Shams al-Dīn Muḥammad ibn Ashraf al-Ḥusaynī al-Samarqandī (died 701/1302).[1] Such versified statements were often composed for use in a pedagogical context in order to help students memorize/recall essential information. Within mathematical sciences, the use of *manẓūma* is more often associated with teaching arithmetical or algebraic procedures than with geometrical topics.[2]

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[1] Sobieroj (2016, 7) suggests a tripartite division of the genre of "didactic verse": (1) versifications of a prose tract, (2) "poems" not based on a specific prose text, and (3) abridgments of longer didactic "poems." The *Manẓūmat Ashkāl al-ta'sīs* clearly falls within the first of Sobieroj's divisions.

[2] Only three of the 28 mathematical treatises in didactic verse whose titles are mentioned by Shawqi deal with geometrical topics. And these three deal with more practical topics such as surveying and area measurements (*misāḥa*)—usually in combination with instruction on calculation (*ḥisāb*).

II Background: *Ashkāl al-ta'sīs*

Since the *manẓūma* claims to present in didactic verse the contents of the *Ashkāl al-ta'sīs*, it is useful to pause and review some of the essential characteristics of al-Samarqandī's treatise. The treatise is itself a condensed and compact introduction to geometrical science.[3] In it we find most of the definitions and axioms from book I of Euclid's *Elements* (although none of Euclid's "common notions" were included) and 35 geometrical propositions—29 propositions are taken from book I, five from book II, and the first proposition from book VI of the *Elements* is included as well.[4]

II.1 Mathematical Contents

The treatise opens with a set of definitions. Most are based on the definitions of book I of the *Elements*. There are two insertions: the first definition of book XI (the definition of a solid) is inserted after Euclid's first two definitions, and a definition of multiplication of line segments which appears in the definitions of book II in some parts of the Arabic transmission.[5] A number of definitions are also omitted, such as definitions of different kinds of triangles and definitions related to components of circles—although Euclid's definitions of classes of quadrilaterals are included. These definitions are followed by six axioms or preliminary assumptions. This collection of axioms does not include Euclid's infamous Parallel Postulate (*Elements* I, Postulate 5) because al-Samarqandī believed he had a mathematical demonstration for this principle, which he gives in his third proposition. Euclid's common notions of mathematics are omitted from *Ashkāl al-ta'sīs*.

Following the definitions and axioms, we find a collection of 35 propositions, all but one of which is present in the *Elements*. The sole exception is the already-mentioned Proposition 3, which is an attempt to demonstrate Euclid's Parallel Postulate. The following table correlates each of the propositions in the *Ashkāl al-ta'sīs* with the corresponding proposition in the *Elements*.

Commentators and editors of the *Elements* have sometimes divided Euclid's propositions into theorems and problems. Theorems were statements of relations existing among geometrical entities, which were then proven to be correct using previously established relations as well as the definitions and axioms. Problems, on the other hand, focus on constructions. The reader is first told how to carry out

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[3] De Young (2001) has translated al-Samarqandī's treatise into English and has discussed some features of its mathematical content.

[4] Al-Samarqandī's third proposition is an attempt to demonstrate Euclid's Parallel Postulate. It is the only proposition that is not taken from the historical Euclidean corpus.

[5] For example, we find a form of this definition included in Ibn Sīnā's *Kitāb al-Shifā'* (Ibn Sīnā, 1977, 69) and in al-Abharī's *Iṣlāḥ Uqlīdis* (Dublin, Chester Beatty Library, ms. arab. 3424, f. 14b).

1= I, 13	8 = I, 8	15 = I, 22	22 = I, 34	29 = I, 43
2 = I, 14	9 = I, 11	16 = I, 23	23 = I, 35	30 = I, 47
3 = Post 3	10 = I, 12	17 = I, 26	24 = I, 36	31 = II, 1
4 = I, 4	11 = I, 15	18 = I, 27+28	25 = I, 37	32 = II, 2
5 = I, 24+25	12 = I, 16	19 = I, 29	26 = I, 38	33 = II, 4
6 = I, 5	13 = I, 18	20 = I, 32	27 = I, 41	34 = II, 5
7 = I, 6	14 = I, 19	21 = I, 33	28 = VI, 1	35 = II, 6

Table 1: Table showing correspondence between the propositions of *Ashkāl al-ta'sīs* and the propositions of Euclid's *Elements*.

a specific construction, after which Euclid demonstrates that these techniques will necessarily produce the desired result. Only four problems or constructions were included in the *Ashkāl al-ta'sīs*: the construction of a perpendicular line (Propositions 9 and 10, = *Elements* I, 11 and 12), the construction of a triangle from three given lines (Proposition 15, = *Elements* I, 22), and the construction of an angle equal to a given angle (Proposition 16, = *Elements* I, 23).

Each proposition includes a condensed demonstration. Apart from the proposed proof for Proposition 3 (= Euclid's Parallel Postulate), these condensed demonstrations follow the general structure given long before by Euclid. The condensation of the demonstrations comes in the form of omission of many of the intermediate or repetitive steps in Euclid's demonstrations, as well as omitting the repetition of Euclid's conclusion, and similar techniques.

Each demonstration in al-Samarqandī's treatise is accompanied by a diagram that illustrates the geometrical content of the proposition. These diagrams follow the standard conventions of diagram construction used throughout the Euclidean transmission. Euclidean diagrams consist of two essential components: the geometrical elements themselves—points, lines, surfaces, etc.—and the labels that identify the points and hence any geometrical element defined by the points.[6]

II.2 Pedagogical Popularity

Al-Samaqandī's treatise continued to be read and copied until the nineteenth century, as is evidenced in the multitude of surviving manuscript copies. The suc-

[6] Some Euclidean manuscripts, in both primary and secondary transmissions, also contain numerals inserted into some diagrams, presumably as a specific illustration of the mathematical relation being demonstrated (De Young 2012c). When included, these numerical elements are most frequently found in books V and books VII–IX, sometimes also in book X, and rarely in book II and book VI. Since these numerical elements do not appear in all Euclidean manuscripts, they are apparently not an essential component of Euclidean diagrams. No manuscript of the *Ashkāl al-ta'sīs* that I have inspected has included numerical elements within its diagrams.

cinctness of the treatise has invited repeated commentary and explication.[7] The most popular of these commentaries was penned in Arabic by the Ottoman mathematician Ṣalāḥ al-Dīn Mūsā ibn Muḥammad (died about 844/1440), who is often denominated by modern historians using his title, Qāḍīzāde al-Rūmī.[8] It was completed in 815/1412 and presented to Ulugh Beg, probably with the aim of gaining the patronage of the ruler.[9] This commentary became at least as popular as al-Samarqandī's original treatise, judging from the surviving numbers of manuscripts.

Historians, in their discussions of this commentary, have focused primarily on the "demonstration" of the Parallel Postulate that it contains. This "demonstration" was incorrectly attributed to al-Samarqandī by earlier investigators, even though Qāḍīzāde al-Rūmī had explicitly stated that he was including the "demonstration" of Athīr al-Dīn al-Abharī. Al-Rūmī had included this "demonstration" because the *Ashkāl al-ta'sīs* did not contain a formal or mathematically rigorous demonstration of Euclid's postulate (Jaouiche 1986, 16–17). Al-Samarqandī's demonstration of Euclid's postulate is rather philosophical, relying on a variant of Zeno's paradox of the tortoise and the hare.[10]

One reason for the longevity of the treatise appears to have been its role in education. As already noted, the curriculum of the *madrasa* changed from place to place and from time to time. However, we have some specific descriptions of curricula in the Ottoman Empire. For example, a report entitled *Kevâkib-i seb'a* (literally, *The Seven Planets*), which was prepared in 1742, describes education in the mathematical sciences in Ottoman *madrasas* (Ihsanoğlu 2004, 14):

> Geometry and arithmetic are easily apprehensible subjects, and because they do not require much deep thought are not studied as separate subjects. They are taken up along with [the discussion of *kelâm* (Islamic scholastic philosophy)]. There is a book titled *Eskâl-i Te'sis* in geometry that they would read at the *iktisar* level. Following that, they would read Euclid with its proofs at the *istiksa* level.

[7] See Sezgin (1974, 114–115) for a listing of commentaries and super-commentaries known to modern scholarship.

[8] A preliminary edition of his commentary, based on several manuscripts available in Tunis, was published by Souissi (Qāḍīzāde al-Rūmī 1984).

[9] Brentjes (2008b) has discussed political patronage and its importance for the practice of mathematical sciences in Islamic societies.

[10] His argument can be summarized as follows: if the two interior angles on the same side of an incident line are less than two right angles, the two given lines are approaching one another. (That is, the distance between the two lines is decreasing.) And it is not possible for two lines to approach one another forever without intersecting. Therefore the two lines will meet on the side where the interior angles are less than two right angles. For full translation see note 49, below.

Reports such as this suggest that the *Ashkāl al-ta'sīs* was a standard part of the curriculum in at least some *madrasas* during the last half of the eighteenth century. Fazlıoğlu (2008, 26–29) also identifies this treatise as part of the intermediate level of the curriculum. The manuscript evidence, though, suggests that it was as likely to be read in the commentary of Qāḍīzāde al-Rūmī as in its original form.

The *Ashkāl al-ta'sīs* was printed in Istanbul in 1274/1858 in the form of al-Rūmī's commentary, a further indicator of its long-standing popularity as a textbook in the Ottoman domains (De Young 2012a, 13–16). The Istanbul edition was printed with the super-commentary notes of Tāj al-Saʿīdī (11th/17th century) placed in the margins. This on-going interest also suggests that the treatise had a place in the educational curriculum.

III Author of the *Manẓūma*: al-Ḥurr al-ʿĀmilī

Muḥammad ibn Ḥassan ibn ʿAlī ibn Ḥusayn al-ʿĀmilī, who is better known by his nickname, al-Ḥurr al-ʿĀmilī, was born in the village of al-Mashghara in the Jabal ʿĀmil (now southern Lebanon) on 8 Rajab 1033 / 26 April 1624. His family included a number of influential Shīʿite Ithnāʿasharī (Twelver) scholars. His initial education was provided by his father and a maternal uncle. At about the age of 40, he left the Jabal ʿĀmil region and traveled eastward to visit Shīʿa holy sites in Iraq.[11] From there, he traveled onward to Mashhad, arriving in 1073/1662–1663, where he spent the remainder of his life as chief judge (*qāḍī al-quḍā*) and Sheikh al-Islām at the shrine of Imām ʿAlī al-Riḍā, the eighth Shīʿa Imam. He died 21 Ramaḍān, 1104 / 26 May, 1692.[12]

Known as the second of the three great Muḥammads of the seventeenth century,[13] Al-Ḥurr al-ʿĀmilī was a prolific author, a well-known expert on *ḥadīth* (sayings of the Prophet Muḥammad) whose best-known work is *Wasā'il al-Shīʿa*, a compendium of ḥadīth. Al-Ḥurr al-ʿĀmilī was also recognized as a poet. Most of his poetry was religious/devotional in character, although he also is credited with several *manẓūma*

[11] During this time period, the Safavid government had adopted an official policy of promoting Shīʿite views. As Sunnī scholars moved to more congenial areas, the ensuing vacuum in intellectual leadership drew Shīʿite scholars such as al-ʿĀmilī to Safavid Iran.

[12] The basic biographical data concerning al-Ḥurr al-ʿĀmilī are summarized by Scarcia (1997, III, 588–589) and Bar Asher (2004, XII, 478–479).

[13] The first was Muḥammad Kashānī (also known as Muḥsin al-Fayz) and the third was Muḥammad Bāqir Majlisī. Al-ʿĀmilī met Majlisī during his sojourn in Isfahān. It was through the agency of Majlisī that al-ʿĀmilī was introduced to Safavid Shāh Sulaymān I (initially installed with the name Ṣāfī II).

or *urjūza*, which are typically more didactic in character and were sometimes copied together in a single codex.[14]

IV General Attributes of the *Manẓūmat Ashkāl al-ta'sīs*

Since we have outlined above some salient features of *Ashkāl al-ta'sīs*, which is rendered into verse form in this *manẓūma*, we turn now to a consideration of how these didactic verses correspond to the prose mathematics on which they are based.

The author tells us in the short versified preamble that his primary goal in writing these didactic verses was to make it easier for students to memorize the essential elements of the *Ashkāl al-ta'sīs*. Given this apparently pedagogical motivation, it is somewhat surprising that the manuscripts available for this study contain almost no marginalia apart from a few scribal corrections to the text, despite many manuscripts having ample margin space available for comment. If the treatise was intended for use by students, one might expect to find somewhat more interaction with the text through marginal notes or explanations of the text.

According to the traditional colophon found in most manuscripts I have examined, Al-ʿĀmilī's didactic verses on geometry were completed on the last day of Jumādā II, 1056 / 11 August 1646.[15] Thus these didactic verses were composed while he was still quite young, well before he had left the Jabal ʿĀmil region. The circumstances that motivated this effort are still unknown, apart from the author's statement in the preamble that he wished to aid students in memorizing the required texts.

These didactic verses on geometrical science are identified in the title of the treatise either as *manẓūma* or as *urjūza*, the latter being less common. In Arabic, either term can be used to refer to a rhymed metrical work. The term *manẓūma* is from the triliteral root *n-ẓ-m*, which has as its root meaning to string pearls or to put something together or into an order, and hence to construct one's speech in a metrical or versified fashion. Since the majority of the manuscripts I have examined in this study use the term *manẓūma* in the title of these didactic verses, I shall adopt

[14] For titles of many of his writings, see the listings by Kantūrī (1912) and al-Ḥusaynī (1965–1966, I, 27–33). Al-Ḥurr al-ʿĀmilī's writings are frequently mentioned among works of émigré ʿĀmilī scholars by Abisaab (2004, 156–173). Among the versified works credited to him were: *Urjūza fī-l-zakāt*, *Manẓūma fī tārīkh al-nabī*, *Urjūza fī tārīkh al-maʿṣūmīn*, *Manẓūma fī masā'il naḥwiyya*, *Urjūza fī-l-maʿānī wa-l-bayān*, *Manẓūma fī-l-Akhlāq*, *Manẓūma fī masā'il kalāmiyya*, *Manẓūma fī ʿilm al-nujūm wa-l-falak*, as well as his *Manẓūma fī-l-handasa*.

[15] A few manuscripts state the date of completion as Rabīʿu II, although they all agree on the year.

the same designation in this paper, unless referring to a specific title that uses the term *urjūza*.

IV.1 Poetic Characteristics of the *Manẓūma*: Meter and Rhyme

Arabic poetry involves an interplay of both meter and rhyme. According to classical Arabic poetics, there were eight fundamental meters.[16] The meter is a function of the length of syllables, rather than stress as in many forms of Indo-European poetry. A syllable is considered to be "short" when a consonant is followed by a short vowel. When a voweled letter is followed either by an unvoweled consonant or by long vowel, the syllable becomes "long."

The term *urjūza* in classical Arabic denotes a poem constructed in a meter known as *rajaz*, which was generally preferred for construction of didactic verse. Each line of the verse (*bayt*, pl. *abyāt*) consists of a couplet in the form of two half-lines or hemistichs (*shaṭr*, dual *shaṭrān*), with each half-line of the couplet ending in the same rhyme. The *shatrān* or paired half-lines are often arranged in two parallel columns on the page, separated by a space and sometimes also a symbol, although some copies use only a symbol to indicate the separation between the two components of the couplet. Rajaz meter may by shown symbolically as follows:

$$- - \cup - \,|\, - - \cup - |\, - - \cup - | \qquad - - \cup - |\, - - \cup - |\, - - \cup - |$$

where the symbol − represents a long syllable and the symbol ∪ represents a short syllable.

Classical Arabic verse often used monorhyme in which each line of the poem ended in the same rhyme. Rhyme is based on the final consonant in the last word of the line (or half-line). When a long vowel precedes the final consonant, it must be included in the rhyme.[17] In many later poems, more varied, and sometimes quite complicated, rhyme patterns developed over time. In this *manẓūma*, however, the author found it easier to use a pattern of paired rhyme in which the two halves of each line rhyme with one another but not necessarily with preceding or following lines, resulting in a somewhat more open rhyme pattern: *aa–bb–cc–dd…*

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[16] For a succinct summary of different meters used in classical Arabic poetry, see Wright (1971, II, 362).

[17] The long vowels *yā'* and *wāw* are sometimes treated as interchangeable in the rhyme pattern but the long vowel *alif* is not and must always be matched with a corresponding *alif* in the rhyming word. When reciting traditional poems—and all traditional poems were meant to be recited aloud—the short vowel attached to the last consonant may sometimes be dropped. And in some contexts the *kasra* and *ḍamma* attached to the final consonant may be interchanged without altering the rhyme, but the *fatḥa* may never be interchanged with the others.

IV.2 Codicological Characteristics of the *Manẓūma*

The *manẓūma* consists of approximately 165 *abyāt* or rhyming couplets. The number of couplets varies slightly from one manuscript to another because some have alternative formulations for the preamble and the colophon sections. There are 150 couplets that summarize the geometrical content of the *Ashkāl al-ta'sīs*. 30 couplets are devoted to the initial definitions and postulates. The remaining 120 are devoted to the versification of al-Samarqandī's 35 propositions.

In general, we can divided the manuscripts containing the *manẓūma* into two families—those that were copied together with other treatises in didactic verse composed by al-ʿĀmilī and those in which the didactic verses on geometry were combined in various ways with other treatises. Most codices of the first variety are recognizable because the didactic verses on geometry do not have a separate title page or formal *basmala* (pious invocation of Allah that begins almost every Arabic book or treatise) but rather are introduced only by a title line, often in red. In such codices, the *manẓūma fī-l-handasa* usually occupies the last position and is typically preceded by a *manẓūma* devoted to *zakāt*, or alms-giving. The fact that the *manẓūma* is often found as part of a collection copied in a specific order implies that the copyists regarded these four rhymed treatises as a unit. Since we lack full cataloging information for some manuscripts, we cannot be certain that each of these collections contains precisely the same four treatises found in Cairo, Dār al-Kutub, Magāmīʿ 846. But what we can know from the evidence in the scans provided is that in each case the *manẓūma* on geometry is preceded by al-ʿĀmilī's didactic verses on alms-giving (*zakāt*).

In other manuscripts, this *manẓūma* is treated as a free-standing or independent composition. That is, the treatise begins with a separate title (sometimes a separate title page, sometimes only a title panel or title statement on the first folio of the treatise), followed by a *basmala*. In most cases, since we have not had access to the remainder of the codex, it is difficult to ascertain with complete certainty whether the codex contained other *manẓūmāt* by al-ʿĀmilī. In the absence of any contrary evidence, I have presumed that these apparently free-standing copies are not part of collections of didactic verse.

Red ink is used relatively sparingly in the manuscripts that I have examined. Red ink is used for the title line of the *manẓūma* in several composite manuscripts. And in a small number of manuscripts, red overlining is used to indicate the beginning of each new definition or proposition. In some manuscripts, red ink is also used in signs that signal the separation of the two couplets of each line, although such techniques also are not universally employed. The majority of manuscripts, however, are written completely in black ink.

IV.3 Illustrative Figures in the *Manẓūma*

None of the manuscripts of the *manẓūma* contain geometrical diagrams. This lack of diagrams may be somewhat surprising to modern readers who expect to find diagrams in texts or textbooks discussing Euclidean geometry. Moreover, since the *Ashkāl al-ta'sīs*, of which the *manẓūma* is a summary, contains traditional Euclidean diagrams to accompany its demonstrations, the lack of diagrams in this versified summary is unexpected.

But two manuscripts (Tehran, Majlis Shūrā 4816/1 and Majlis Shūrā 10253/2) contain illustrative figures placed in the margins.[18] These illustrations appear somewhat crudely drawn, as though sketched without the aid of drawing instruments. These figures are different from the typical diagrams that one might expect to find in a medieval mathematical manuscript devoted to geometry. In Euclidean geometrical diagrams, the points needed to identify lines, surfaces, etc., are given individual letter labels—usually following a specific pattern (Netz 1998b, 34–35). What we find in these two manuscripts are not traditional diagrams but rather illustrative examples of the figures under discussion in the definitions and propositions.[19]

The collections of illustrative figures found in the margins of these two manuscripts are nearly identical. This observation suggests that they may derive from a common ancestor. One cannot have been copied directly from the other, however, because the labeling of the figures illustrating the definitions is less complete in Majlis Shūrā 4816/1, and the figure for Proposition 17 is omitted from Majlis Shūrā 4816/1. Moreover, the figure illustrating Proposition 23 has been incorrectly constructed in Majlis Shūrā 10253/2, while it is correctly constructed in Majlis Shūrā 4816/1. In addition, the figures illustrating Propositions 31, 32, and 33 have each been duplicated in the margin of Majlis Shūrā 10253/2. Finally, the figure illustrating Proposition 35 has been omitted from Majlis Shūrā 4816/1. These variations make it improbable that the illustrative figures have been copied from one manuscript directly to the other.

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[18] These two manuscripts also share a number of textual variants, suggesting a possible genetic relation between them.

[19] Two papyrus fragments, once thought to be the earliest remnants of Euclid's text (De Young 2009, 370, n. 54), are now considered to represent remnants of study texts. They contain enunciations along with symbolic unlabeled figures, perhaps intended for use by students who were expected to learn early theorems of the *Elements* in order to use them in understanding the logic of its structure (Sidoli 2015, 392–393).

IV.4 Mathematical Content of the *Manẓūma*: Definitions and Axioms

The definitions and axioms are stated in 30 couplets. They are, with one important exception, arranged in the same order as the definitions and axioms in *Ashkāl al-ta'sīs*, although the diction has repeatedly been altered to meet the demands of the meter and rhyme structure. These changes are obvious already when we compare the opening definitions of book I (point, line, surface, and solid). Short hortatory phrases exhorting the (student) reader to learn the material well are inserted at frequent intervals, the particular phrase being chosen so as to complete the rhyme scheme and fill out one of the hemistichs. The presence of such interjections also suggests the pedagogical intent of the author.

Comparing these initial definitions to the same definitions as formulated by al-Samarqandī, in Table 2, we can also see that, in addition to altering the diction, al-ʿĀmilī has also rearranged their contents. He has first defined the three geometrical objects: lines, surfaces, solids. Following these, he has defined their termini. This arrangment differs both from that found in the *Ashkāl al-ta'sīs* and from Euclid's arrangement. It appears to be the personal decision of the versifier. Thus his goal seems to go beyond merely reproducing the content of al-Samarqandī's mathematical treatise in metrical form. Nor is it the case that the rearrangement was required for the versification. Rather, it seems to be guided more by philosophical considerations and by pedagogical concerns.

These initial definitions are the only ones that were substantially altered by al-ʿĀmilī. The remainder of the definitions follow the order and content of the definitions in the *Ashkāl al-ta'sīs*. The diction, however, has been modified in order to accommodate the requirements of the verse form.

In al-Samarqandī's treatise, the definitions are followed by six axioms or postulates. Al-ʿĀmilī has omitted the first three of these axioms. These three omitted axioms are important for Euclid's construction problems and also provide the foundation for physical construction of geometrical diagrams: (a) to draw a straight line from one point to another point, (b) to extend a straight line segment in a straight line, and (c) to draw a circle of given radius about any point. The rationale for this omission is still unclear since the author makes no statement concerning these axioms. Perhaps he considered them so self-evident that no statement of them was deemed necessary.

IV.5 Mathematical Content of the *Manẓūma*: Propositions

Following the definitions and axioms, the author introduces the enunciations of the 35 propositions found in *Ashkāl al-ta'sīs*. These propositions are placed in the same order as that found in the *Ashkāl al-ta'sīs*. The omission of al-Samarqandī's proof sketches which were included in *Ashkāl al-ta'sīs*—with the exception of Proposition

النقطة هي شيء ذو وضع غير منقسم. والخط طول بلا عرض ونهايته النقطة. والسطح ما له طول وعرض فقط ونهايته الخط. والجسم ما له طول وعرض وعمق ونهايته السطح.	النقطة التي عدت كما علم * شيا له وضع وليس ينقسم والخط ما كان له طول بلا * عرض ولا عمق فحيث حصلا طول وعرض فهو سطح ومتى * كان الجميع فهو جسم ثبتا والجسم قد ذكرته استطرادا * ولم يكن بذاته مرادا ونقطة نهاية الخط كما * خط نهاية لسطح فاعلما وآخر الجسم هو السطح . . .
The point is something having position without [being] divisible. The line [is] length without width and its terminus is a point. The surface is that which has length and width only and its terminus is a line. The solid is that which has length and width and height and its terminus is a surface.	The point is that which is considered, as is known, something possessing position and is not divisible. The line is that which has length but no width and no height. Now, where length and width occur, there exists a surface. And when they all occur, a solid [is] established. And indeed I have mentioned the solid [as] a digression; it is not [something] intended, on account of its essence. And a point is the terminus of the line just as a line is the terminus of the surface, as you should know. And the end of the solid is the surface ...

Table 2: Arabic text in the right column is from the *manẓūma* of al-ʿĀmilī, quoted from Mashhad, Murtadawī Library, ms. 2612/12, p. 253. The comparison Arabic text in the left column is from al-Samarqandī's *Ashkāl al-taʾsīs*—quoted from Tehran, Majlis Shūrā 6878, f. 3b. The English translation of each version follows below.

3 (an attempt to give a rather philosophical "demonstrate" or justification for Euclid's Parallel Postulate)—is perhaps the most striking mathematical feature of the *manẓūma*. These proof sketches are the part of the treatise that many a mathematician would probably consider to be among the most important. Their omission raises important questions about how the *Ashkāl al-ta'sīs* was actually used in educating students for positions of leadership in the Muslim community.

On the other hand, it is curious that "extraneous" or non-geometrical facts, such as the observation that al-Samarqandī's Proposition 5 (= *Elements* I, 5) is known as "al-Ma'mūnī" because the caliph al-Ma'mūn took a special delight in it, are included in the *manẓūma*. If the versification represents a distillation of the most essential information for students to memorize and remember, why is this editorial addition included? It suggests that the memorization mentioned by the author was perhaps actually a rote memorization for the sake of passing an examination, not a memorization of mathematical material that was considered intrinsically valuable for the student to know.

Another possible interpretation of the omission of demonstrations in the *manẓūma* is that the versification was intended to help students remember the mathematical content of each proposition. From the memorized enunciations of each proposition, the student may have been expected to be able to re-create the essential elements of the demonstrations. Such an expectation would suggest that the primary interest of the author was the *Ashkāl al-ta'sīs* and its mathematical content—perhaps as an exemplification of a particular set of logical skills and deductive arguments.

Whatever students or users were intended to learn through memorizing this *manẓūma*, it seems that it was not what modern readers might have considered to be the most important mathematical content of Euclid. Possibly the versifier's intent was to provide readers who had already studied and mastered the mathematical content of *Ashkāl al-ta'sīs* a kind of intellectual scaffolding to help them to recall the details of the treatise in their later life when acting as leaders in the community.

V Bibliographic Essay

For this project, I have examined eleven manuscripts containing the *manẓūma* of al-ʿĀmilī. In this section, I give a brief physical and bibliographical description of each manuscript based on scans provided by various libraries in Iran and Egypt, with a special emphasis on the sections which reproduce the *manẓūma*. In some cases, I have had access to the entire manuscript, but in other cases, I have been able to see only scans of the *manẓūma* itself.

Tehran, Majlis Shūrā, ms. 11/4

The treatise was copied in large, precise, elegant scribal *naskhī*. The inclusion of most of the diacritic points as well as many vowel signs, and the absence of significant marginalia, suggests that this might have been a presentation copy rather than a working manuscript. The copyist has included guide words at the end of each folio to ease the reader to the next folio. There are no figures to illustrate the text. The upper edge of the manuscript appears to have sustained some water damage, although this does not affect the text itself.

The codex, according to the modern cataloging note, contains four treatises, of which the *manẓūma* is the fourth. It occupies pages 143–156 of the codex.[20] The *manẓūma* does not have an independent title page but immediately follows the preceding treatise, which is also in the form of a *manẓūma*. The title of the *manẓūma*—written in red ink, the only occurrence of color in the *manẓūma* apart from the initial word (*yaqūlu*)—occupies the last line of page 142.

The *manẓūma* preceding our *manẓūma* in the codex appears to have been copied by the same scribe—the entire codex may well have been copied by the same person. The last lines of the previous *manẓūma* can be readily identified—they belong to the third of the four *manẓūmāt* attributed to al-Ḥurr al-ʿĀmilī in Cairo, Dār al-Kutub, Magāmīʿ 864. These facts suggest that the two manuscripts may well contain the same collection of *manẓūmāt*.[21]

The colophon gives the date of completion as the last day of Jumādā II, 1056/1646. Neither the name of the copyist nor the date of copying is mentioned.

Tehran, Majlis Shūrā, ms. 4816/1

The title page attributes the versification to Muḥammad ibn al-Ḥassan al-Ḥurr al-ʿĀmilī. A rhymed colophon at the end of the treatise gives the date of completion as Monday, the last day of Rabīʿu II, 1056 AH (1646 CE), suggesting that the author must have been quite young at the time of its composition. The dating differs somewhat from the majority of manuscripts, which give the date of completion as the last day of Jumādā II. All manuscript colophons agree on the year 1056 AH, however, so the different month is not of great consequence in terms of situating the treatise within the timeline of the life of al-ʿĀmilī.

The treatise occupies folios 3b–8a of the manuscript, with usually 17 lines per page. The text is written in a bold *naskhī* using black ink. Most diacritic points are indicated in the manuscript and occasionally vowels are also indicated, especially when the copyist wants to emphasize use of the passive construction. A number of

[20] The modern cataloger has used pagination rather than foliation to describe the manuscript.

[21] This supposition is supported by the modern cataloging notes, which tell us that the four treatises in the codex were *arājīz* composed by al-ʿĀmilī.

corrections/alterations have been noted in the margins or between the lines, apparently in another hand or at least using a different pen with a finer point. Guide words are included at the end of most folios to ease the reader into the next folio.

The text is not written in parallel justified columns, as is often done in many manuscripts containing this *manẓūma*. Instead, the two hemistichs of each couplet are separated by a small circular sign in red ink. Red ink is also used for overlining to indicate the first word of each definition as well as over the number (written out verbally) that identifies each proposition so that the reader can quickly locate any particular definition or proposition in the text.

The manuscript contains small illustrative sketches, typically placed in the outer margins of each folio, giving a kind of visual picture formally of the definitions and propositions—a feature shared with Tehran, Majlis Shūrā 10253/2. These illustrations are drawn in black, but most are then outlined or highlighted using red ink. The sketches appear somewhat crudely drawn, without recourse to drawing instruments, and those illustrations accompanying the propositions differ from traditional geometrical diagrams in that they have no labeled points. Sketches illustrating geometric entities defined in the treatise often have verbal labels. A few propositions do not have an illustration—there is no identifiable illustration to accompany Proposition 30 (the so-called Pythagorean Theorem, = *Elements* I, 47), for example.

The *urjūza* is followed in the codex by the *Ashkāl al-ta'sīs*, apparently copied in the same or very similar hand. The colophon at the end of this treatise gives the date of copying as the evening of 19 Dhū al-Ḥijja, 1054 AH (16 February 1645 CE). This date raises some serious historiographical issues. The fact that the two treatises appear to be in the same hand suggests that the date of copying should apply to both the *manẓūma* and the treatise that follows it. But if this is true, then the date of completion given in the colophon of the *manẓūma* must be incorrect. One possible explanation is that since each document begins on a new folio, rather than following immediately on one another, the two documents may have been copied by the same copyist but at different times and were only later joined into a single codex.

This is only one puzzle concerning this manuscript. It also contains a variant and somewhat expanded form of the preamble, as well as a variant of the typical colophon from which two lines have been omitted. The manuscript shares these, as well as many textual variants, with Qom, Masjid Aʿẓam, 764/3, suggesting that there is a possible genetic connection between the two.

Tehran, Majlis Shūrā, ms. 10253/2

The treatise occupies folios 8a–12b of the codex. The text is arranged in two parallel justified columns with a space separating the two hemistichs in each couplet. There are twelve lines per page, surrounded by ample margins. It is written in a clear scribal *naskhī* using black ink and includes most diacritic points. There are no

marginal corrections, but the treatise does include guide words, placed in the lower margin, to ease the transition to the next folio.

The final folio, which should contain the last fourteen couplets of the *manẓūma* and the colophon, is missing from the scans provided by the library so that information on the date of copying is now inaccessible to us. The presence of the guide word placed at the end of folio 12 suggests that the remainder of the text was at one time present in this manuscript.

This copy—like Tehran, Majlis Shūrā 4816/1—includes small sketches in black ink to illustrate some definitions and propositions of the treatise. These sketches are placed at irregular intervals in the outer margin of each leaf, approximately opposite the definition or proposition that the figure illustrates. The figures are typically rather small and appear to be drawn without the use of drawing aids. Some figures, especially those illustrating definitions, include a verbal label. Figures illustrating propositions lack the letter labels that identify points and lines as found in traditional Euclidean diagrams. The figures, although similar in construction to those in Majlis Shūrā 4816/1, are fewer in number and lack any highlighting in red.

Tehran, Dāʾiratu-l-maʿārif, ms. 833/3

The *manẓūma* occupies pages 71–74 of the codex.[22] The text, written in black ink using a rather small, crabbed *naskhī*, is arranged in two parallel columns, separated by a blank space, with 23 lines per page. Most diacritic points have been included and sometimes even vowels are explicitly indicated—especially when the versification requires use of an unusual grammatical form. There are no guide-words to smooth the transition to the next folio. There are no marginalia or illustrative figures despite the ample marginal area of the pages. The paper of the manuscript appears to be quite brittle and there has been some damage to the upper margin, often extending into the first line or two of the text.

The final page has four columns, presumably in an effort to conserve paper and avoid opening a new folio. The copyist wrote the two left-hand column first. At the bottom of the page, he continued in the left-hand margin, then in the upper margin (parts of which have now been lost). Only then did the copyist add the pair of right-hand columns. And again there was insufficient room, so he continued into the bottom margin (with the text inverted relative to the remainder of the page) and finally ended in the lower portion of the right-hand margin. Parts of the concluding hemistichs have been lost following damage repair to the lower right-hand corner of the leaf. This repair has obscured part of the date of the composition of the manuscript. One can quite easily read that its composition was completed in the year [10]56 AH. The month, however, is no longer visible. Since the copyist has reproduced many of the variant readings from Cairo, Dār al-Kutub, Magāmīʿ 846/4,

[22] The codex has been paginated rather than foliated by the modern cataloger.

it is tempting to speculate that the text originally read Jumādā II, but there simply is no evidence that we can use to decide the issue.

The modern cataloging record indicates that this *manẓūma* is the third and last treatise in the manuscript.[23]

Tehran, Millī, ms. 4818

The treatise is the final part of the codex and occupies folios 34a–38b.[24] Written in a neat scribal *naskhī*, the text is arranged in two parallel columns with 17 lines per page. The two hemistichs of each couplet are separated by a blank space in which the copyist has place a pair of red dots more or less like a semi-colon (;) except that the lower dot is often extended into a short line and curves upward at the end. There are guide words at the end of each folio to ease the transition to the next folio. There is no marginalia in this copy of the *manẓūma* and no illustrative figures have been included.

The codex appears to have been exposed to damp at some point, although the staining does not usually interfere with reading the text. The ink in the first line or two of each page seems to be flaking off the page, perhaps as a result of the exposure to dampness, making it sometimes difficult to discern precisely what the copyist wrote. Some pages exhibit a similar flaking of ink in the last line or two as well.

The first fourteen lines of folio 34a contain the end of the third of the four *manẓūmāt* by al-ʿĀmilī that are sometimes combined in a single treatise, as for example, Cairo, Dār al-Kutub, Magāmīʿ 846. The treatise is now incomplete. The last six lines, which should appear on folio 39a, the lines corresponding to the colophon, are not included in the scans provided by the library, although the guide-word that should introduce the next folio is present, suggesting that the missing section was originally copied by the scribe. Consequently, we now lack information about the date of copying which would once have been contained in the colophon.

Qom, Masjid Aʿẓam, ms. 764/3

The treatise occupies folios 138b–142a of the codex. The date of composition is given in the colophon as the last day of Rabīʿu II, 1056 AH.

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[23] The other treatises are identified as (1) *Tashrīḥ al-Aflāk* by Bahāʾ al-Dīn al-ʿĀmilī (953–1031 AH/1547–1622 CE) (Hashemipour 2007; Shawky 1976, 12) and (2) *al-Ṣafīḥa* of Ibn Hamad (?)—apparently a treatise on the tympanum of the astrolabe. I have been unable to identify the author of this second treatise, although it might be Ibn al-Bannāʾ since he wrote a popular study entitled *al-Ṣafīḥa* (Rosenfeld and Ihsanoğlu 2003, 350).

[24] I infer that this *manẓūma* may be the last in the codex from the presence of the library identification stamp on folio 38b.

The text, written in a cramped and somewhat awkward *naskhī*, includes most diacritic points. The copyist has not been consistent in the number of lines per page, varying from 17 to 21. The first hemistich of each couplet is typically followed by one or more small red circles. The use of multiple circles at the end of a few lines seems to function as a kind of space filler when a line occupies substantially shorter space than the preceding or following line. Red has also been used to highlight the word "*wa-baʿd*" which signals the end of the introductory preamble. Because of the use of red circles to separate the hemistichs, there was no effort made to produce justified columns on the page.

On folios 138b–139a, the copyist has placed three hemistichs in each line, rather than the customary two.[25] In the remainder of the treatise, he has placed only two hemistichs in each line. But because he wrote the first 37 lines with three hemistichs, there was one hemistich left over. Thus each "*bayt*" or rhymed couplet in the remainder of the treatise is broken between two lines. This has the effect of breaking the visual pattern of rhyme that one would expect to find in each line.

The copyist has followed tradition and included guide words to ease the passage from one folio to the next. I have been able to examine only the scans of the treatise itself and so I cannot say what material may have preceded it. There are no illustrative figures to support the text.

Qom, Āyatallāh Marʿashī Najafī, ms. 2786

The treatise occupies folios 1b–6a. The paper appears modern, with a blue-gray tint. The opening author-title panel and the *basmala* panel immediately below it are enclosed within double-line scalloped borders. The first page has suffered some damage, so that nearly half the author-title panel is now missing. The text has been written entirely in black ink. With the exception of the first folio, the copyist has supplied guide words to ease the transition to the next folio. The text is written in a large neat scribal *naskhī* hand and includes most diacritic points. The text is arranged in two parallel columns, with a gap indicating the break between the two hemistichs, and with thirteen or fourteen lines per page. Occasionally, when a line occupies somewhat less space than usual, the copyist inserts a place-holder (usually a circular sign strongly resembling the letter *ha*). This symbol is usually placed at the end of the line, but in at least one case (f. 4b, l. 6), it appears at the beginning of the line. There are no diagrams to illustrate the text.

After line 15, the copyist omits the next 34 lines. We may suspect that one of two possible scenarios to explain this lapse: either the manuscript from which the text was copied was missing one folio—17 lines per page seems a fairly common architectural feature—or the copyist accidentally turned an extra folio during the

[25] Sobieroj (2016, 8) found the use of three columns to be rare in the examples of Arabic didactic verse that he described.

copying process. The latter possibility is not completely unknown (De Young 2012b, 273), although I suspect that it is the less likely scenario. At the end of folio 2, the copyist has written the correct guide word (*ḍilʿāhu*), but then has omitted what should have been the first line of folio 3 from the text.

The text area of each page has been delimited from the margins by lines which often appear somewhat clumsily constructed—the lines are not completely straight nor do they often meet neatly in a right angle. It appears that these boundary lines were constructed before the text was written, since the last letters of the final word must sometimes be raised in order to avoid transgressing the established boundary lines. This boundary delineates a fairly wide margin on all sides. There is one marginal correction on folio 2b where an inadvertently omitted line has been placed (inverted) in the margin. Apart from this, the margins are empty except for the last page, where we find the last line of the colophon, along with two fairly extensive marginal notes, each appearing to be in a different hand.

The variant readings found in this manuscript are often identical to those in Tehran, Majlis Shūrā 11/4, suggesting a possible genetic linkage. There are several omissions from the text in addition to that already mentioned.

Qom, Islamic Heritage Revival Center, Urmawī Collection, ms. 4384/9

The treatise is, as implied in the cataloging, the ninth in the collection and occupies the last five folios in the codex. The paper is apparently becoming brittle and is crumbling away at the lower margin but there has been no damage to the text itself. The treatise is written in a clear scribal *naskhī* with most diacritic points inserted. The text is arranged in two parallel columns separated by a blank space with eighteen lines per page. There are guide-words included at the end of each folio to ease the transition to the next folio. There is no marginalia and no illustrative figures have been included in this treatise.[26]

The preceding *manẓūma* has been copied in the same hand as this *manẓūma*, which raises the possibility that the entire codex may be the work of a single scribe. This earlier *manẓūma* is the same as that which preceded the *manẓūma* on geometry in Tehran, Majlis Shūrā, ms. 11/4 and Majlis Shūrā, ms. 10253/2, which suggests that this codex also may well have contained all four *manẓūmāt* also found in Cairo, Dār al-Kutub, Magāmīʿ 846.

[26] The lack of any marginalia, the neatly justified columns, as well as the use of red ink for the title line all suggest that this manuscript might have been intended as a presentation or display copy rather than a student's working copy.

Cairo, Dār al-Kutub, Magāmīʿ 846/4

The *manẓūma*, occupying folios 61b–67b, is the last within a collection of four *manẓūmāt*, apparently all copied in the same elegant *naskhī* hand (King 1981, 288; 1986, 163).[27] The first three *urjūza* in the collection are:[28]

1. أرجوزة في مواليد الائمة وفياتهم ومناقبهم (Didactic verses on the births of the imams and their deaths and their virtues)
2. أرجوزة في المواريث (Didactic verses on legacies / inheritances)
3. أرجوزة في الزكاة (Didactic verses on giving alms)

The text is arranged in two neatly justified columns, fourteen lines per page, with a space separating the hemistichs of each couplet. There is some evidence to suggest that red ink was used to highlight portions of the text—for example, headings are much lighter and more difficult to read than the remainder of the text—although it is difficult to be certain because the original manuscript is not available for inspection and the library provides only scans from the microfilms. There are no illustrative figures included in the treatise.

The last folio contains an addition to the standard colophon stating that Muḥammad Riḍā ibn ʿAzīzullāh copied four *arājīz* in the year 1100 AH (1689 CE):

تمت الاراجيز الأربعة بقلم ... الأقل الخليقة بل لا شيء في الحقيقة محمد رضا ابن عزيز الله في سنة ١١٠٠ هزار وصد

The date of the completion of the *manẓūma*, given a few lines earlier, is the last day of Jumādā II, 1056 AH.

Mashhad, Murtadawī, ms. 2612/12

This treatise, the twelfth in the codex, occupies pages 252–264.[29] The text is written in a neat *naskhī* and includes most diacritic points. The *manẓūma* is arranged in two parallel columns of fifteen lines per page with a sign similar a colon (:) centered in the space separating the two hemistichs of each couplet. There are guide words at

[27] The Dār al-Kutub Library catalogs the collection under the title *Urjūza fī sīrat al-nabī*, the title used for the first *manẓūma* in the collection.

[28] There are several other examples of four *manẓūmāt* in one codex—such as Tehran, Majlis Shūrā, ms. 11 and Tehran, Millī, ms. 4818. This feature suggests the possibility that each collection may contain the four *manẓūmāt* attributed to al-Ḥurr al-ʿĀmilī here.

[29] The manuscript has been paginated rather than foliated by the modern cataloger.

the end of each folio to ease the transition to the next folio. The treatise contains no marginalia apart from the final page, which contains several extended annotations. There are no illustrative figures included in this copy of the treatise.

The copyist has left blank a space for 28 lines following the first word of couplet 25. The easiest suggestion to explain this phenomenon is that the copyist recognized that a folio was missing from the archetype being copied.[30] The first word of couplet 25 is present—presumably it was the guide word supplied to transition to the missing folio.

The manuscript shares many variants with Qom, Āyatallāh Marʿashī, ms. 2786, suggesting that the two may be genetically related.

Khvānsār, Fāḍil Khvānsārī, ms. 240/11

The treatise appears to be the final component of the codex, occupying folios 292b–298a.[31] The label or title is written in black ink and centered in the line, following the last eight lines of the preceding *manẓūma*. Both have been copied in the same hand. Thus the manuscript may contain all four of the *manẓūmāt* found in the Cairo, Dār al-Kutub, Magāmīʿ 846, although only an inspection of the complete codex can reveal the validity of this hypothesis.

The text, written in a somewhat coarse and cramped *naskhī*, is arranged in two parallel justified columns, 15 lines per page, with a blank space separating the two hemistichs of each couplet. Copying errors are usually corrected directly in the text by lining out the incorrect word or portion of the word and writing the correction above it. There are guide-words at the end of each folio to assist the reader to the new folio. There are no illustrative figures to accompany the text.

The binding is somewhat tight, so that the words at the inner margins of each leaf are sometimes nearly obscured—especially at the top. The paper is apparently rather thin and the photographers did not use a dark sheet behind the pages when photographing, so that the text is often somewhat obscured by text from the opposite side of the leaf bleeding through.

A statement following the traditional colophon gives the name of the copyist as Ibrahīm ibn Ḥājjī Muḥammad ʿAlī al-ʿĀmilī and date of copying as 16 Ṣafar 1111 AH (12 August 1699 CE). The form of the copyist's name suggests a possible familial relation to al-Ḥurr al-ʿĀmilī. I have not been able to confirm this hypothesis.

The colophon reads:

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[30] The copyist could quickly learn that a folio was missing because the guide word at the end of the folio did not match the first word of the next folio in the codex. The fact that 28 lines are left blank suggests that the manuscript from which he was copying had fourteen lines per page.

[31] There is no folio 297 because the modern cataloger has mistakenly omitted that number from the foliation.

وكتبه بيده الفانيه تراب اقدام المؤمنين
المحتاج إلى عفو الله ورحمته الراجي
شفاعة نبية وايمته البالي
ابراهيم ابن حاجي محمد علي العاملي
عامل الله بغفرانه وحشره ووالدي في بضاته
وذلك في ستة عشر من شهر سفر ستة ألف ومایة وإحدى عشر

VI Edition of the *Manẓūma*

In this section, I present an edition of the treatise. My goal in the edition has been to produce a robust text which seems to be typical of what a late medieval scholar might have encountered. In the editing process, I have used Tehran, Majlis Shūrā 11/4 as my base text. This manuscript contains the entire text with no lacunae and with few difficult readings or variants. I have collated the remaining copies with this manuscript. In most cases it is difficult to ascertain with certainty any genetic connections among the examined manuscripts.

Since the major differences between manuscripts lie in the title, preamble, and colophon, I decided to edit each of these sections independently. Each is followed by its paraphrase/translation.

Most copies of the *manẓūma* place the two hemistichs in parallel columns, separated by a space. A few indicate the separation using a small sign. I have followed the latter convention, using an asterisk (⁎) to signal the break between the two hemistichs.

Manuscript sigla:

Tehran, Majlis Shūrā, ms. 11/4 : طا
Tehran, Majlis Shūrā, ms. 4816/1 : طب
Tehran, Majlis Shūrā, ms. 10253/2 : طج
Tehran, Dāʾirat al-Maʿārif, ms. 833/3 : طد
Tehran, Millī, ms. 4818 : طم
Qom, Masjid Aʿẓam, ms. 764/3 : قا
Qom, Āyatallāh Marʿashī Najafī, ms. 2786 : قب
Qom, Islamic Heritage Revival Center, Urmawī collection, ms. 4384/9 : قج
Cairo, Dār al-Kutub, Magāmīʿ, ms. 846/4 : قد
Mashhad, Murtadawī, ms. 2612/12 : مم
Khvānsār, Fāḍil Khvānsārī, ms. 240/11 : خف

VI.1 Title

The manuscripts use several different titles and terms to describe the didactic verses. Some manuscript titles use the term *urjūza* while others use the term *manẓūma* or another form derived from the same triliteral root (*n-ẓ-m*). The latter formulation is, as can be seen from the following list of titles, the more common among the manuscripts I have examined.

When considering the titles, I have divided the manuscripts into two broad groups. The first group of manuscripts are those in which the *manẓūma* is clearly part of a collection of rhymed treatises by al-ʿĀmilī and copied by the same scribe. The key features that identify these collections are (a) the title line immediately follows the end of another *manẓūma* (on the subject of almsgiving) without a break and without a traditional *basmala*, and (b) both didactic verses are copied in the same hand. The second group of manuscripts are those in which the *manẓūma* on geometry appears as an independent treatise, not associated with another *manẓūma* by al-ʿĀmilī. In this group of manuscripts, the *manẓūma* begins with a title at the head of a folio (sometimes there is even an independent title page) and contains a traditional *basmala*. In each of these two groups, I list those titles formulated with the term *manẓūma* first, followed by titles formulated using the term *urjūza*.

VI.1.1 Collections of Manẓūmāt *by al-ʿĀmilī*

1. Tehran, Majlis Shūrā, ms. 11/4. There is no independent title page or *basmala*. The treatise opens with a one-line centered title statement, written in red (p. 142):

   منظومة في أشكال التأسيس في علم الهندسة نظم محمد الحر

   "Didactic verses (*manẓūma*) on *Ashkāl al-taʾsīs* in geometrical science, a versification (*naẓm*) of Muḥammad al-Ḥurr"

2. Tehran, Majlis Shūrā, ms. 10253/2. There is no independent title page or *basmala*. The treatise opens with a two-line centered title statement, written in black (f. 7b):

   وقال ناظما لأشكال
   التأسيس في علم الهندسة

   "He said: a rhyming (*nāẓiman*) for *Ashkāl al-taʾsīs* in geometrical science"

3. Qom, Islamic Heritage Revival Center, Urmawī Collection, ms. 4384/9. There is no independent title page or *basmala*. The treatise opens with a centered one-line title statement written in red (f. 103b):

وقال ناظما لأشكال التأسيس في علم الهندسة

"He said: A rhyming (*nāẓiman*) for *Ashkāl al-ta'sīs* in geometrical science"

4. Tehran, Millī, ms. 4818. There is no independent title page or *basmala*. The treatise opens with a one-line title statement written in red (f. 34a):

ارجوزة في أشكال التأسيس في علم الهندسة

"Didactic verses (*urjūza*) on *Ashkāl al-ta'sīs* in geometrical science"

5. Qom, Āyatallāh Fāḍl Khvānsārī, ms. 240/11. There is no independent title page. The treatise opens with a one-line title statement written in black (f. 292b):

ارجوزة في أشكال التأسيس في علم الهندسة

"Didactic verses (*urjūza*) on *Ashkāl al-ta'sīs* in geometrical sciences"

6. Cairo, Dār al-Kutub, Magāmīʿ 846/4. There is no independent title page or *basmala*. The treatise opens with a centered one-line title statement, probably written in red ink because it is faint and difficult to read in the scans provided by the library (f. 61b):[32]

[ارجوزة] في أشكال التأسيس في علم الهندسة

"Didactic verses (*urjūza* on *Ashkāl al-ta'sīs* in geometrical science"

VI.1.2 Mixed Collections or Independent Treatises

7. Mashhad, Murtadawī. ms. 2612/12. The title, apparently in another hand, has been placed in the upper right margin of the first page (p. 252):

هذه منظومة في علم الهندسة للشيخ المحدث الحر العاملي لحمد الله

"This versification (*manẓūma*) on geometrical science is by the sheikh [and] transmitter of *ḥadīth* al-Ḥurr al-ʿĀmilī with praise to Allāh"

The *manẓūma* opens with a standard *basmala* centered above the two columns.

[32] From the scans provided by the library, which were made from the microfilm, it is impossible to read the first word. But the colophon states: *tammat al-arājīz al-arbaʿa*, suggesting that the title should begin with *urjūza*.

8. Tehran, Dāʾiratu-l-Maʿārif, ms. 833/3. The treatise opens with a *basmala* (apparently added later in the upper margin but in the hand of the copyist) and one-line title statement written in black (p. 70):

نظما للشيخ محمد الحسين لأشكال التأسيس في علم الهندسة

"A rhyming (*naẓman*) of Sheikh Muḥammad al-Ḥussayn for *Ashkāl al-taʾsīs* in geometrical science"

9. Tehran, Majlis Shūrā, ms. 4816/1. The manuscript places the title on a separate title page (f. 3a):

منظومة أشكال التأسيس بخط محترم ناظم آن وهو الشيخ المحدث الفقيه والمجتهد النسبية محمد الحسن الحر العاملي مؤلف وسائل الشيعة

"A versification (*manẓūma*) of *Ashkāl al-taʾsīs* written by the outstanding versifier (*nāẓm*), namely the Sheikh, the transmitter of *ḥadīth*, the legal scholar [and] the interpreter of legal principles, Muḥammad ibn al-Ḥassan al-Ḥurr ah-ʿĀmilī, author of *Wasāʾil al-Shīʿa*"

10. Qom, Masjid Aʿẓam, ms. 764/3. An author-title statement, apparently in a hand other than that of the copyist, the last third of which is inverted, is placed in the upper margin, preceding the *basmala*, which is centered on the first line of the page (f. 138b):

نظم أشكال التأسيس للشيخ الأصل الأكمل الأوصد محمد بن الحسن بن علي الحر العاملي طاب قواه العاملي نسبة إلى عامله قريه من قرى الشام

"A versification (*naẓm*) of *Ashkāl al-taʾsīs* of the Sheikh, [who is] noble [and] outstanding [and] unique, Muḥammad ibn al-Ḥassan ibn ʿAlī al-Ḥurr al-ʿĀmilī—may his strength be excellent. [The apellation] al-ʿĀmilī is in reference to his working in one of the villages of [greater] Syria."

11. Qom, Āyyatallāh Marʿashī Najafī, ms. 2768. The treatise opens with a author-title panel, written in black ink, surrounded by a double scallop border. Below, in a smaller second panel is a standard *basmala*, also in black ink. Half the author-title panel is now missing due to manuscript damage. This makes it impossible to know whether the original title used the term *manẓūma* or *urjūza*. The modern cataloger has imposed the title:

أرجوزة في الهندسة

"Didactic verses (*urjūza*) on geometry."

VI.2 The Preamble

Most medieval treatises begin with a preamble or preface which names the author and explains a bit about the contents of the treatise and the author's motivation for writing. This *manẓūma* also begins with a preamble, which may be divided into three parts. The first section names the author and includes several typical pious invocations and honorific epithets. We find two somewhat different versions of this first section of the preamble. The shorter version is found in the majority of manuscripts. A variant of this shorter version omits the first line, which names the author of the treatise. This alternate version of the shorter preamble occurs only in Majlis Shūrā 10253/2. The reason for this omission is unclear. Perhaps it was done because this *manẓūma* is copied as part of a larger collection of didactic verse (often four treatises—see the description of the manuscripts in Section V) and the information may have been provided earlier in the collection and hence was considered redundant here.

يقول راجي عفو رب عادل * محمد الحر الفقير العاملي
حمداً لمن ابدع شكل ما ابتدع * مؤسّسا على اقتدار ما صنع
وأظهرت طبائع الأشياء * حكمته لنا بالا خفاء
وصلواته على النبي * محمد ذي الشرف الجلي
عالم ما لم يحظ أهل العلم * به كجذر العدد الأصم
ونسبة القطر ومنتهي العدد * صلى عليه الواحد العذل الصمّد
وآله الائية الهداة * وصحبه الأكارم الثقات

Muḥammad al-Ḥurr al-ʿĀmilī, who is needy [and] who is hoping for the pardon of the Lord who is just, says: Thanks [be] to the one who originated the perceptual form (*shakl*) of whatever he contrived, the one who founded whatever he has made according to [his] potency, the one who makes visible the constitution of things, whose wisdom is for us without doubt. And may his blessing be upon the prophet Muḥammad the one possessing conspicuous honor, the one who is knowledgeable of that which the practitioners of the science do not describe, such as the root of the irrational number and the ratio of the

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1 يقول ... العاملي] (−) **طج.** 2 مؤسّسا] مؤسا: **قا، طد، طج** = **مو: خف.** 3 الأشياء] للأشياء: **طج.** خفاء] ~~خلاف~~ *above*: **طج.** 4 النبي] (+) النبي: **قب.** 5 عالم] **لم: خف.** 6 عليه] (−) **طج.** الواحد] لواحد: **قب.** العذل] الفرد: **طد، ڤج.** 7 وآله] ما له: **قب.** الائية] الائقة: **طا** = ~~الأظهار~~ ثمة: **طد.** الأكارم] الاركام: **ڤج.** الثقات] النفات: **طا** = الثقاة: **طد** = الثقال: **خف.**

diameter [to the circumference] and the extreme limit of numbers.[33] May the blessing of the One, the Impartial, the Everlasting be upon him, and [upon] his family [and] the rightly guided imams and [upon] his companions [and] the most noble authorities.[34]

The longer version of this first part of the preamble is found in only two of the manuscripts examined in this study (Majlis Shūrā 4816/1 and Qom, Masjid Aʿẓam, 764/3). The foundation text is Tehran, Majlis Shūrā 4816/1. It reads:

قال الفقير المرتجي ذا المنن * عبيده محمد بن الحسن
عاملي مولاه بالإحسان * والعفو والرحمة والغفران
حمداً لمن ابدع شكل ما ابتدع * مؤسساً على اقتدار ما صنع
واظهرت طبائع الأشياء * حكمته لنا بلا خفاء
عالم ما لم يخط أهل العلم * به كجذر العدد الأصم
ونسبة القطر ومنتهي العدد * سبحانه من أحد فرد صمد
وصلواته على النبي * محمد ذي الشرف الجلي
وآله الائمة الهداة * وصحبه الأكرام الثقات

The needy one, the one full of the hope of the one possessing blessings [for] his little servant, Muḥammad ibn al-Ḥassan Al-ʿĀmilī, who is his servant with respect to good deeds, forgiveness, grace, and pardon, said: Thanks [be] to the one who originates the perceptual form (*shakl*), the one who founded whatever he has made according to [his] potency, the one who makes visible the constitution of things, whose wisdom is for us without a doubt, the one who is knowledgeable of that which the practitioners of the science do not describe, such as the root of the irrational number and the ratio of the

[33] The author mentions here three classic unsolvable problems that would have been known to any student of mathematics: finding the root of an irrational number, finding the value of π (the ratio of the diameter to the circumference of a circle) and finding the greatest number. The implication seems to be that even though these matters are unknown to or insoluble by mathematicians, they are known to Allah.

[34] The copyist of Majlis Shūrā 11 seems to have misread his exemplar. He has written *nifāt* rather than *thiqāt*. The two words are almost identical in orthography, differing only in the number of diacritic points used.

6 أحد فرد] فرخ احد: قا. 8 الأكرام] الاكارم: قا.

diameter [to the circumference] and the extreme limit of numbers. May praise be to the one who alone is everlasting.

May God, [who is] one, incomparable, eternal be magnified! And may his blessing rest upon the prophet Muḥammad, who possesses conspicuous honor and [upon] his family [and] the rightly guided imams and his companions [and] the most noble authorities.

The longer form of the preamble opens with the same verb (to say) as the shorter version, but in the longer version, the verb is placed in the past tense rather than the present tense. The significance of this tense change could be debated, but I am personally inclined to regard it simply as a feature of the rajaz meter used in these didactic verses. The longer version of the preamble also provides a fuller statement of the author's name, as well as additional pious epithets and conventional honorific titles. This longer preamble, because it shares most of its content and approximately half its phrasing with the shorter preamble, appears to me to be a modification of the shorter in several points by another editor who was also accomplished as a composer of didactic verse. For this reason, I suspect that it is a later version of the preamble that was composed by al-ʿĀmilī himself.

The second part of the preamble introduces the subject matter of the treatise. Unlike the first part of the preamble, we find only a few copyist variants but no major variants in substance. Perhaps we find multiple formulations of the first part of the preamble because it contains more flowery forms of description and so offers more oppotunities for exercising the scribe's compositional abilities. Since the second part of the preamble is more prosaic and matter-of-fact, there are fewer tempations to "improve" the formulation. This part of the preamble is introduced by the stereotypical term *wa-baʿd* and states the importance of the subject and also provides a kind of rationale for the composition of the didactic verses which follow.

The foundation text is Tehran, Majlis Shūrā 11/4.

وبعد فأعلم أنّ علم الهندسة * منه البراهين غدت مقتبسة
لعملي مساحة وجبر * وهيئة وغير ذاك فأدر
وهو مؤسس بلا شك على * أشكال تأسيس فكن محصّلا
وقد أردت نظمها ليسهلا * بذاك حفظها فخذها جملا 4

---

[1] غدت] عدت: **قد، طم** = عذب: **طج.** [2] لعملي] لعلمي: **قب، خف.** فأدر] فأذر: **طا.** [3] وهو . . . محصّلا] (–) **طد.** مؤسس] قيوس: **قد.** شك] (+) ~~ولا~~: **جق.** فكن] لكن: **قب.** [4] ليسهلا] كي يسهلا: **قا، جق، طب، طج، طد.** بذاك] بذلك: **طج.** حفظها] حفظهما: **طا، طج.**

أذكر قبل ذكرها مقدمة * فيها المبادئ وهي المقدمة

> Now then, you should know that the demonstrations from the science of geometry are adopted for the practice of *misāḥa* (surveying) and algebra and *hay'a* (configuration of the heavens) and such-like disciplines. So you should ponder [it carefully]. And it (the science of geometry) is without doubt the foundation of the *Ashkāl al-ta'sīs* (by al-Samarqandī). So be [a person] who acquires [knowledge of it].
>
> And indeed I desired their versification in order to ease, by that [method], their memorization. So accept it as a summary. I discuss, before discussing them (the demonstrations), [some] preliminaries in which are the fundamental concepts.[35] This is the introduction.

From this second part of the preamble, we learn three things. First, the author proposes a way to situate the science of geometry within the spectrum of the mathematical sciences—it provides, he says, the formal underpinning for such sciences as surveying and configuration [of the heavens] (or mathematical cosmography). The reference to algebra may at first reading seem more puzzling. It may refer, however, to the fact that *Ashkāl al-ta'sīs* includes several propositions drawn from *Elements*, book II. This portion of the *Elements* has been subjected to a wide range of interpretations over the past 150 years among historians of mathematics, several of whom have seen book II as somehow linking algrebra and geometry or even as a form of crypto-algebra.[36]

The second important point we learn from the author is that the science of geometry is the foundation (*mu'assas*) of the *Ashkāl al-ta'sīs*. Thus it is clear that this treatise in didactic verse is regarded as part of geometrical science. The reader (a student?) is urged to acquire knowledge of this science.

And finally, the author tells us that he undertook the construction of these didactic verses for the purpose of making it easier to memorize the treatise. This statement implies that the goal of education, in the eyes of the author at least, was to memorize prescribed texts. But in light of the contents of the *manẓūma*, and especially the omission of most of the proof sketches from these didactic verse, the

[35] By these fudamental introductory concepts he has in mind the definitions, axioms, postulate, and common notions that Euclid placed at the beginning of his geometry.

[36] For a historical introduction to these debates, readers may refer to Herz-Fischler (1987, 37–50) or Høyrup (2016).

[1] فيها المبادئ] فيها الميادي: **قد** = فها المبادي: **خف** = تبدئ المبادي: **طب، طج، طد، قا.**

memorization may have been intended only to provide a general scaffolding of the overall structure of the *Ashkāl al-ta'sīs* which the student would be able to fill in from his study of the original treatise.

VI.3 Versification of *Ashlāl al-ta'sīs*

In order to make it easier to locate each proposition within the versification, I have introduced overlining of the proposition number. This procedure is not typical of the manuscripts that have been studied, but is not unknown. For example, such overlining can also be found in Tehran, Majlis Shūrā 4816/1. In most cases, the number appears at the beginning of the proposition, but in a few cases, it is mentioned only at the end of the proposition.

النقطة التي غدت كما علم * شياً له وضع وليس ينقسم
والخط ما كان له طول بلا * عرض ولا عمق وحيث حصلا
طول وعرض فهو سطح ومتى * كان الجميع فهو جسم ثبتا
والجسم قد ذكرته استطرادا * ولم يكن بذاته مرادا
ونقطة نهاية الخط كما * خط نهاية لسطح فأعلما
وآخر الجسم هو السطح وإن * يلتق خطان قويمان ومن
شرطهما أن لا يكون فيهما * تحدّب ولا إتحاد فأفهما
فههنا ناحية منسطحة * تعرف بالزاوية المسطحة
وإن يقم خط له استقامة * على نظيره في الاستقامة
فإن تجد زاويتين كانتا * عن جانبي ذاك وقد تساوتا
فهو عمود وهما فأعرفهما * قائمتان في اصطلاح العلما

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[1] ينقسم] تنقسم: **قد، طم.** [2] عرض ولا عمق] عمق ولا عرض: **طم.** وحيث] فحيث: **قا، طب.** [3] طول] ~~حصلا~~ طول: **خف.** ومتى] ومتر: **قب.** جسم ثبتا] تسما جسم يا فتى: **قد** = ~~تسما~~ جسم يا فتى: **طم** = جسم يا فتي: **قا.** [4] استطرادا] استردا: **قا.** [6] الجسم هو السطح] السطح هو الجسم: **قا.** [7] فأفهما] فأفتهما: **طم** = ~~فأعلما~~ *above* : **مم.** [8] فههنا] فههنا ~~زاوية~~ : **مم.** منسطحة] مسخطة: **قا، طد، طم.** [9] وإن] فإن: **طب.** خط] (–): **مم.** له] ~~على~~: *above* **طب.** [10] زاويتين] زوايتين: **طم.** جانبي] جانبتي: **قج.** [11] عمود وهما] عمودهما: **قج.** وهما] فهما: **قا.** فأعرفهما] فأفهما: **قا.**

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[4] مرادا] ناقص من نسخة قب أربع وثالثون أبيات التي ما يلي هذا البيت.

ثم التي أصغر منها حادة * وذات الانفراج تلك الزائدة
والشكل هيئة إذا احاط حد * بالشئ أو أكثر من حد تعد
ثم المربع الذي الأضلاع * منه تساوت وكذا الأرباع
أعني الزوايا فأفهم الخفايا * والمستطيل القائم الزوايا
مختلف الأضلاع والمعيّن * أضلاعه سوًا وذاك بيّن
لكنه خال من القوائم * فأفهم ودع ملامة اللوائم
ثم الشبيه بالمعيّن إنتفى * عنه التساوي والقيام فأعرفا
لكن يساوي ضلعه المقابل * للآخر الذي له يقابل
وهكذا وما عدا ما قد وصف * يعرف في اصطلاحهم بالمنحرف
والمتوازيان في الخطوط * لا يتلاقيان بالتخطيط
لو اخرجا بالوهم لا إلى مدى * فأسمع هداك الله منهج الهدى
وكل مقدارين يضرب الأقل * في أكثر أو عكسه فأعلم حصل
سطح توازت منه أضلاع وقد * احاط خطان به فليعتمد
أي مستقيمان وكل قائمة * ترى المساواة لديهم لازمة
ما بينها وبين كل زاوية * قائمة وهي لها مساوية
ولا يحيط مستقيمان كما * قد اجمعوا بالسطح أي وحدهما
لم يتصل خط له استقامة * بمستقيمين على استقامة

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[1] ثم التي أصغر] *damaged, illegible* **: طد.** الانفراج] الافراد **: قا.** [2] تعد] بعد **: طا.** الأرباع] الابادع **: جق.** [5] وذاك] وذك **: قا.** [6] القوائم] القدائم **: طا.** فأفهم ودع ملامة اللوائم] وذاك باد عند كل عالم **: طب، قا، قب، مم.** [8] له] (−) **: طد.** [9] في اصطلاحهم] باصطلاحهم **: قا.** [10] في الخطوط] فالخطوط **: خف.** يتلاقيان] متلاقيان **: طا.** [11] مدى] المدى **: جق.** فأسمع] فأفهم **: طب، قا، قب، مم.** الله] للام **: طج.** منهج] منهاج **: طج، طد، جق.** [12] فأعلم] أيضا **: طب، طد، قا، قب ، مم** = (−) **: طج.** [13] توازت] تساوت **: طا، طج، طد، طم، جق، قد، خف.** وقد * احاط] * وقد احاط **: طب.** [14] مستقيمان] مستقيما **: قا.** ترى] نرى **: طا.** المساواة] المساوات **: جق.** [15] وبين] وبنى **: طج.** [16] أي] *above* **طد.** [17] على] *above* **قا.**

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[3] ثم] قد ناقص من نسخة مم أربعة وعشرين أبيات التي يلي هذه الكلمة.

فصاعداً قط وهذا واضح * لكل واضح جلي فاضح
هذا الذي اوردت في المقدمة * فقد مضت احكامها مقدمة
وبعدها اشرع في الأشكال * لينجلي بها صدى الأشكال
وهي ثلثون وخمس تكفل * بيان ما مضى وهذا الأول
إن قام خط ذو استقامة على * آخر أيضاً مستقيم حصلا
قائمتان أو شبيهتان * في الوضع قدرانهما والثاني
منها كما اثبت بالبرهان * تقريره إذا التقى خطان
في نقطة طرف خط أخرا * أي مستقيم أوّلا وآخرا
فإن يكن يحدث عن جنبيهما * قائمتان أو مساو فهما
أي ذانك الخطان خط متحد * وثالث الأشكال فأفهم واعتمد
خط بخطين قويمين التقى * فإن بدأ في جانب واتفقا
أعظم من قائمتين قدّرا * يحصل أقل منهما في آخرا
وذانك الخطان لا بد وإن * يلتقيا إن اخرجا فليعلمن
وذا ضروري كما ادعاه * أقليدس الرئيس في أولاه
واعترض القوم عليه الجوهري * وعمر الخيّام ثم الأبهري
وبعده الطوسي وإبن الهيثم * ومعهم قاضي حما فليعلم
قالوا المقادير بغير غاية * إذ تتجزئ لا إلى نهاية
فقد يجوز فيهما التقارب * بلا تلاق وهو قول كاذب

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[1] واضح] أوضح: **طا، طد، طم.** لكل واضح جلي فاضح] فلا يزيده وضوحا شائح: **طد** = فلا يزيده وضوحا حا شائح: **جڤ** = فلا يزيده وضوحا شارح: **طج، طد، طم** = فلا يزيده وضوحا شادح: **طا، خف.** [3] لينجلي بها صدى] لكي تزيد ظلمة: **طم، خف** = لكي تزيل طلمة: **طا** = لكي يزيل ظلمة: **جڤ، قد.** [5] قام] كان: **طد.** ذو استقامة] فاستقامة: **طا.** مستقيم] مستقيما: **طم.** [6] قدرانهما] قدرا بهما: **طج، طم، جڤ، قد، خف** = قدراتهما: **طا.** [7] بالبرهان] بالبراهان: **طد.** تقريره ... خطان] *damaged, illegible* : **طد.** تقريره] تقديره: **طا.** التقى] انتفى: **طا.** [8] مستقيم] مستقسما: **طد.** [9] جنبيهما] جنبهما: **طج.** [10] ذانك] ذاتك: **طج.** متحد] متحمد: **طج.** وثالث] فا وثالث: **طب.** [11] بخطين] بخين: **خف.** [13] وذانك] وذلك: **طا، طد، قا.** يلتقيا إن] يلتقيان: **طا، طد، جڤ.** [14] في] *above* **قا، قد.** أولاه] أولاده: **قد.** [16] قاضي] قامي: **طا.** حما] (–): **خف.** [17] قالوا] فابوا: **قا.** بغير] ~~لذي الدر~~: *margin* **طب** = لغير: **قب، خف.** إذ] ~~قد~~: *above* **طب** = إن: **قا.** نهاية] النهاية: **قا.** [18] تلاق] تلاف: **طا، قب.**

وكل عقل بسواه شاهد * وكل ما كان كذاك فاسد
رابعها أن هنا مثلثا * شابه ضلعاه وما قد حدثا
بينهما أعني بذاك الزاوية * ضلعين من مثلث وزاوية
بينهما فاستوت الجميع * أي الزوايا الست والضلوع
ويستوي المثلثان ثم إن * إحديهما كانت أقل يا فطن
فخطها الذي غدا لها وتر * قطعاً له عن وتر الأخرى قصر
وعكسه حق وهذا الخامس * من غير إشكال به والسادس
متى استوى خطان من مثلث * فطرفا قاعدة المثلث
زاويتان وهما سواء وإن * اخرج ضلعاه إذن يحدث من
تحتهما زاويتان وهما * أيضاً سواء بالدليل فأعلما
هذا الذي لشغف المأمون * به غدا يعرف بالمأموني
سابعها متى استوى زاويتا * مثلث يستو أيضاً يا فتى
ضلعاه أعني الموترين لهما * ثامنها مثلثان فيهما
ستة أضلاع سوا النظير * يشبهه في الآخر النظير
فتستوي كذا الزوايا فيهما * كل لمثله كما قد علما
ويستوي المثلثان فأعلم * وتاسع الأشكال فأسمع وأفهم

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[2] هنا] ههنا: **طد.** حدثا] حدنا: **طا.** [4] أي] إلى: **قا.** [6] الذي غدا] الذي غد: **طج** = التي غدا: **قب، مم** = ذاك الذي: **طب، قا.** غدا لها] غداها: **طد** = لهما: **طب.** وتر] وتري: **طا.** [7] وعكسه] وعكسها: **قا.** حق] حقا: **خف.** به] *above* طد. [8] استوى] استوت: **طا، قا، قب، مم.** خطان] ساقان: **طب، قا.** مثلث] ~~مسدس~~: *below* **قب.** [9] اخرج] اخرجي: **طج.** ضلعاه إذن] ساقاه به: **طا، طب، طم، قا** = ضلعاه به: **طد.** [10] سواء] سواك: **طج.** بالدليل] فبذاك: **طب، قا** = (–): **طج.** [11] هذا] هو: **قد** = هذي: **قا.** يعرف] يغرف: **طم.** بالمأموني] بالمأمون: **طد، قا، قب، قد.** [12] ~~سابعها~~ . . . يستو] *damaged,* *illegible*: **طد.** استوى] استوت: **طا، قب، مم.** زاويتا] زاوية: **قب.** [13] الموترين] الوترين: **ڃ** = الموتران: **خف.** ~~ثامنها~~] ثالثها: **طد.** فيهما] بينهما: **قب** = (–): **طد.** [14] النظير] النضير: **طد.** [15] فتستوي] ويستوي: **مم** = وتستوي: **قد.** كذا] أيضا: **طج، طد، طم، قا.** الزوايا] لزوايا: **طم.** فيهما] فهما: **طب، قا** = منهما: **طا، طج** = فمنهما: **ڃ.** [16] فأسمع وأفهم] فأصغ وأفهم: **طب، قا، قد** = فأفهم وأعلم: **قب.**

أنا نريد نخرج العمودا * ولا يكون خطه محدودا
من نقطة في الخط فلنخط إلى * بعدين عنها بالسواء لنجعلا
ربعين من دائرة تقاطعا * ونصل النقطة والتقاطعا
فيحصل العمود والعاشر أن * نخرجه من نقطة له بأن
نجعل تلك مركز الدائرة * تقطّع ذاك الخط وهي دائرة
ثم ننصف الذي داخلها * بنقطة ونخرج الخط لها
وإن ترم معرفة الحادي عشر * ذاك الذي شاع لديهم واشتهر
خطان قد تقاطعا فحدثت * ثم زوايا أربع تقابلت
ثنتان منها اشبهت ثنتين * قابلتاهما بغير مين
كل لما قابله يساوي * نزهك الله عن المساوي
واستمع التقرير الثاني عشر * يا مبرزاً في الفهم عن كل البشر
كل مثلث إذا اخرجنا * ضلعاً له خارجة وجدنا
زاوية خارجة أعظم من * داخلتيه بانفراد ثم إن
ضلع من الثلاث طال رسما * فهو يقينا موتر للعظمى
وذلك الثالث عشر ثبتا * وبعده الرابع عشر قد أتى
عظمى الزوايا أطول الأضلاع * يوترها قطعاً بلا نزاع

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1 أنا] إذا: **قب.** نريد] (−): **طج.** خطه] خطوط: **قا** = *above*: **طد.** 2 فلنخط] فليخط: **طد** = فلتخط: **طا، طج، قب، جق، مم.** بعدين عنها بالسواء] بعد(+)ين *above* ~~معين به~~: *margin* **طب.** بعدين] تعدين: **طم.** لنجعلا] ليجعلا: **طا، قب، مم.** 3 تقاطعا] يقاطعا: **طد** = لقاطعا: **قب** = فقاطعا: **خف.** ونصل] :ولنصل **قا** = وفصل: **طا، قب، مم.** 4 نخرجه] يخرجه: **مم، خف** = تخرجه: **طم، جق.** 5 نجعل] فعل: **طم.** 6 ننصف] تنصف: **طا، طد، طم، قب، جق، مم.** الذي] التي: **طد، قب، مم.** 7 الحادي عشر] الحاد عشر: **قب.** واشتهر] وشتهر: **طج.** 8 فحدثت] (−): **طج.** ثم] ثم ارجع: **طم، قد.** 9 اشبهت] اشتبهت: **قب، مم، خف.** 10 قابله] قاله: **قا.** يساوي] مساوي: **طب، طج، طد، قا، جق.** 11 واستمع التقرير] واضع إلى تقريري: **طب** = واضع للتقرير في: **قا** = خارجه التقرير: **طج.** 12 اخرجنا] (+) اخرجنا: **مم.** خارجة] (−): **طج** = (+) وضعنا: **قب.** 13 داخلتيه] داخليه: **قب.** 14 يقينا] لقسنا: **قب.** 15 الثالث عشر] الثالث ~~قد~~: **طب.** وبعده] وبعد: **طد.** الرابع عشر] الرابع: **مم.** 16 عظمى] عظم: **قا.** أطول] أطوال: **قب.** يوترها] لوترها: **قب.** بلا] *above*: **طد.**

نريد أن خامس عشرها نضع * مثلثاً لكن بشرط أن يقع
فيه استواء كل ضلع مع خط * يفرض لكن الخطوط يشترط
أن لا يكون اثنان منها إلا * أطول من ثالثها قطعا لا
أنقص أو مساويين إذ لا * يمكن في مثلث ذا أصلا
وذاك بالبرهان قطعاً يعمل * كذاك بالفرجار وهو أسهل
سادس عشر ينبغي أن تعملا * زاوية تكون فاعرفها على
أية نقطة من الخط بها * زاوية مفروضة قد اشبها
سابع عشر أن تساوي الضلع مع * زاويتين من مثلث يقع
زاويتين مع ضلع ساويا * من آخر المثلثان استويا
أعني تساوت منهما الأضلاع مع * كل الزوايا ثم إن خط وقع
على قويمين استوت مبادلة * في جانب الخط مع المبادلة
أو اشبهت خارجة لداخلة * أو كانت الواقعتان داخلة
في جهة قائمتين أو كما * ساواهما كان التساوي دائما
ما بين ذينك القويمين أتى * وذلك الثامن عشر ثبتا
تاسع عشر عكس ما تقدمه * ومكمل العشرين أيضاً تممه
كل مثلث إذا اخرجنا * ضلعاً له خارجة وجدنا
زاوية خارجة تشبه ما * قابل من داخلتيه ثم ما

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[1] نريد أن خامس عشرها] خامس عشر معاها نريدا: **قب.** بشرط] يشرط: **طج.** [2] يفرض] يفرط *above*: **قا.** يشترط] تشترط: **قا.** [3] إلا] *damaged, illegible*: **طد.** أطول ... لا] *dam-aged, illegible*: **طد** = (−): **قا.** أطول] أزيد: **طب، قا.** من] (+) ثلثها: **طب.** [4] أنقص ... لا] (−): **قا** أنقص] زيد: *above* **طب.** مثلث] مثلثين: **طا، طم، قب، قد، مم، خف.** ذا] (−): **قب، مم.** أصلا] صلا: **طا.** [5] بالبرهان] بالبرها: **خف.** يعمل] بعمل: **طج.** [6] ينبغي] نبتغي: **طب، طد، قد** = يتبغي: **طم.** تعملا] نعملا: **جق** = يعملا: **طم.** [7] اشبها] انتها: **مم.** [8] تساوي] يساوي: **طد، جق.** يقع] وقع: **طد.** [9–10] زاويتين ... مع] (−): **جق.** [9] المثلثان] المثلث أن: **قب، مم.** [10] منهما] فيهما: **قب.** خط] خطه: **قب** = خصا: **خف.** وقع] يقع: **طد.** [12] أو اشبهت] لو شبهت: **خف.** [13] كما] ما: **طب، قا.** التساوي دائما] التساوي أيضا: **قد** = التوازي ذو ما: **طب، قا.** [14] ثبتا] اثبتا: **قا.** [15] ومكمل العشرين] وبعده العشرون: **طب، قا.** تممه] تمه: **مم.** [16] وجدنا] علمنا: **طد.** [17] داخلتيه] داخليه: **قب** = داخلته: **طا.**

يحصل من ثلاثة أعني التي * داخلة قائمتين ساوت
وبعد فالحادي والعشرين إذا * اردت تقريراً له فهكذا
إذا الخطوط المتوازيات مع * شرط التساوي في المقادير وقع
لها خطوط وصلت أطرافها * فإنها قد جمعت أوصافها
وهي التساوي والموازاة أتت * وبعده الثاني وعشرون ثبت
إذا تقابلت من السطوح مع * شرط التوازى في الخطوط فيقع
ما بينها وهي لها أضلاع * إن تتساوى فلتضح اسماع
كذا الزوايا المتقابلات * من غير شك متساويات
نعم وأقطار السطوح السالفة * منصّفات فدع المخالفة
وثالث العشرين في الأوضاع * سطحان متوازيا الأضلاع
كانا على قاعدة بينهما * ما بين متوازيين فهما
قطعا سواء إن وجدا في جهة * واحدة فأعلم بغير شبهة
ثم إذا كان لكل قاعدة * مع التساوي فهما كواحدة
هذا هو الرابع والعشرونا * وبعده الخامس والعشرونا
تقريره كل مثلثين * في جهة بين موازيين
كانا على قاعدة فاستويا * وهكذا قاعدتان استويا

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[1] من] (+) ونساو: **طب.** [2] وبعد فالحادي والعشرين] وبعده الحادي وعشرون: **طب، طج، قا.** [3] إذا] إذ: **قب، مم.** المتوازيات] المتوازيان: **طم** = متوازيات: **طا.** وقع] مع: **طا.** [4] أطرافها] بأطرافها: **قب.** أوصافها] بأوصافها: **قب.** [5] وهي] أعني: **طب، طج، طد، قا.** ثبت] أتت: **طم، قد** = ثلث: **طج.** [6] تقابلت] (+) مع: **طج.** التوازى] التساوي: **طا، طم، قب، قد، مم.** في] وفي: **قد.** الخطوط] الخط: **طج.** فيقع] فتقع: **طا، طد، طم، جق، قد، مم.** [7] بينها] بينهما: **طج.** [8] الزوايا] زوايا: **جق.** [9] نعم] هكذا نعم: **قب.** فدع] فأطرح: **طب، قا.** [10] وثالث العشرين] وثالث العشرون: **قا.** سطحان] ~~سطحطا~~ سطحان: **قا.** متوازيا] متساويا: **مم** = متواضيا زيا: **طم.** [12] وجدا] وجدنا: **طا، قب، مم.** [13] ثم . . . قاعدة] *damaged, illegible*: **طد.** [14] وبعده] فبعده: **قب.** [15–197.2] تقريره كل مثلثين . . . على] كل مثلث وسطح حصلا * يتبعه السابع والعشرونا: **طم.** [15] مثلثين] (+) مثلثين: **مم.** موازيين] متوازيين: **طد، قا، جق.** [16] فاستويا] فاستوتا: **جق، قد.** استويا] استوتا: **طد، جق، قد.**

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[4] أوصافها] نقص من نسخة خف ثلثين أبيات التي يلي هذا البيت. [8] متساويات] نجد بقية هذه المتظومة في الهامش في نسخة طد.

وذلك السادس والعشرونا * يتبعه السابع والعشرونا
كل مثلث وسطح حصلا * مع التوازى في خطوطه على
قاعدة واحدة تجويهما * خطان متوازيان فهما
في جهة واحدة فالسطح * ضعف مثلث وذا يصحّ
وأستمع الثامن والعشرينا * فإنّني بيّنته تبيينا
سطحان أضلاعهما كل لكل * وازى وساوى الارتفاعان يدل
على بيان النسبتين القاعدة * فإن تكن ناقصة أو زائدة
يتبعها السطحان في المقدار * كذا المثلثان باستظهار
وتاسع العشرين حسبما ورد * سطحان متوازيا الأضلاع قد
حلّا بسطح تتوازى أضلاعه * وكل سطح منهما اجتماعه
بالآخر الموصوف فوق القطر عن * جنبيه عند نقطة فليعلمن
في عرفهم هما المتممان * وبالدليل متساويان
ثم الثلثون بلا إشكال * شكل العروس أحسن الأشكال
كل مثلث تكون قائمة * إحدى زواياه فتلك القائمة
وترها مربعاً يساوي * مربع الضلعين والتساوي
قد صحّ بالبرهان والحادي إذا * شكل الثلثين مضى فهكذا
إذا ضربت الشيء لاستعلامه * في الشيء ساوي الضرب في أقسامه
والثان من بعد الثلثين جعل * كما نقول وكما أيضاً نقل

2–1 وذلك السادس والعشرونا * ... على] (−): **طده** 2 وسطح] ومسطح: **قب، مم.** على] علا: **قا.** 3 تجويهما] (−): **طد.** فهما] وهما: **طج، جق، قد.** 5 وأستمع] واصغ إلى: **طب، قا.** فإنّني] فأتي: **طج، طم.** تبيينا] تبيننا: **طب، قا** = تبينا: **طا، طج، مم.** 6 الارتفاعان] الارتفاعات: **قا، مم.** 7 النسبتين] التبيين: **طج.** فإن] قال: **طج.** 8 يتبعها] تبعها: **طا، طم، قب، جق، قد، مم.** المقدار] المقدر: **طج.** المثلثان] المربعان: **طا، طج، طد، طم، قب، جق، قد، مم، خف.** 9 حسبما] حسبها: **قا** = جسما: **طج.** الأضلاع] للأضلاع: **طج.** 10 بسطح] تسطح: **طج.** سطح] (+) مثة: **قب.** منهما] منها: **قا.** 12 المتممان] المتممات: **طا.** 14 تكون] يكون: **طد.** 15 وترها] أوترها: **قب، مم.** 16 والحادي] (−): **طج.** فهكذا] وهكذا: **طا، طج، طد، قب، مم.** 17 لاستعلامه] لاستغلامه: **طج.** 18 والثان] والثاني: **طا، طد، قا، فب.** نقول] تقول: **طب، قا، قد، خف.**

ضرب سطوح الخط في اجزاه لا * يزيد عن مربع له ولا
ينقص والثالث أي من بعد أن * تمضى الثلثون تماماً فأعلمن
مربع الخط يساوي فأعرف * مربعي قسميه مع مضعّف
مضروب سطح القسم في القسم ومن * بعد فرابع الثلثين فإن
نصّف خط وقسمنا بعد ذا * مجموعه مختلفين فإذا
جمعت سطحي أحد القسمين * في الآخر احفظه وبعد ذين
أضف مربع الزيادة التي * في النصف من قسم له واستثبت
تجده في الحالين ساوي قدراً * مربع النصف وقس لكي ترى
وخامس بعد الثلثين لمن * يطلب تقريرا له فليعلمن
فقد بدأ بيانه بلا خفاء * وذاك أن كل خط نصّفا
وبعده زيد عليه آخر * على استقامة وذاك ظاهر
مجموع سطح الخط والزيادة * معاً إذا ضربت في الزيادة
أضف إلى مربع النصف فما * قام يساوي بعد ذاك فأعلما
مربع النصف مع الزيادة * وعند ذاك تمت الأفادة

VI.4 Paraphrase of the versification of *Ashkāl al-taʾsīs*

Because this treatise is versified, it is impossible to translate literally. I have tried to give a reasonably close paraphrase of the *manẓūma* without attempting to reproduce either the rhyme or meter of the original Arabic. When the author uses terminology

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[1] سطوح] السطوح: **قب.** الخط] *above*: **طد.** ولا] فلا: **خف.** [2] تمضى] يمضى: **قا.** [3] مربعي] (+) ~~ضلعية~~: **طب، قد.** قسميه] قسمته: **طج** = (+) ~~مته~~: **مم.** مضعّف] منصّف: **طا، قب، مم.**
[5] وقسمنا] أو قسمنا: **قب، مم.** فإذا] فإنها: **قب، خف.** [6] أحد] إحدى: **قب.** احفظه] احفظ: **طج.**
وبعد ذين] بغير مين: **طا، طج، طد، طم، قب، ڃ، قد، مم، خف.** [7] من] عن: **طب، طج، طد، قا، ڃ.**
واستثبت] فاستثبت: **مم.** [8] تجده] تحده: **قد.** الحالين] الحاكين: **طج.** مربع] من نصع: **طج.** لكي]
لكن: **طا، قب، مم.** [9] تقريرا له] علم حكمة: **طب، قا.** [10] بيانه] تقريره: **طب، قا.** خفاء] حقا: **طم.**
كل] كلي: **قا.** نصّفا] نصفاه: **قب.** [12] مجموع] (+) ~~ذا~~: **طب.** [13] إلى] لهما: **طد** = لها: **طب، قا.**

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[10] نصّفا] قد نقص من نسخة طج أربعة عشر أبيات التي بعد هذا البيت.

different from that commonly encountered in the Euclidean tradition and in the *Ashkāl al-ta'sīs*, I have indicated the actual reading in transliteration.

I have introduced standard punctuation marks into the translation. There is no punctuation in the original Arabic of the *manẓūma*.

Words enclosed in square brackets [ ] are not found in the original. They are added in order to complete the meaning in English. Words placed in parentheses ( ) are intended to explain the statements in the text which might not be clear from a simple word-by-word translation.

Following the request of one of the referees, I have introduced boldface type to indicate proposition numbers. A similar practice has been followed in a minority of the Arabic manuscripts examined. I have also introduced numeration of the definitions and postulates. I have introduced also references correlating these definitions and postulates with those of Euclid, following the numeration scheme of Heath's English translation (Heath 1908). These references are enclosed in angled brackets (< >). The Roman numeral represents the number of the book, and the Arabic numeral represents the number of the definition.

The translations of the enunciations as found in *Ashkāl al-ta'sīs* are my own. I based them on the edition of the commentary by Qāḍīzāde al-Rūmī (1984), although in a few doubtful cases I also consulted Tehran, Majlis Shūrā, ms. 6878.

[Definitions]

[1] The point is that which is considered, as is known, something having position and not divisible <I, 1>.

[2] The line is that which has length but no width and no height <I, 2>.[37]

[3] Now, where length and width occur, there exists a surface <I, 5>.

[4] And when they all (that is, length, width, height) occur, a solid [is] established <XI, 1>.[38] And indeed I have mentioned the solid [as] a digression (*istiṭrād*); it is not [something] intended (*murād*) on account of its essence.[39]

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[37] The author has added the phrase "and no height" which is not found in *Ashkāl al-ta'sīs*. The phrase is, however, found in the commentary of Qāḍīzāde al-Rūmī (1984, 39).

[38] The Arabic term *jism* is often translated as either solid or body. In the context of Euclidean discourse, the term refers to an object that has extension in each of the three dimensions—length, width, height.

[39] This statement was apparently added by al-'Āmilī. I interpret the statement to mean that the definition of the solid was added for the sake of an intellectual completion, not because it was required in a later proposition in the *Ashkāl al-ta'sīs*. A similar statement is found in the commentary of Qāḍīzāde al-Rūmī (1984, 40), who also labels the statement *istiṭrādan*, "since there is no need for it in this treatise."

[5] And a point is the terminus (*nihāya*) of the line <I, 3> just as a line is the terminus of the surface <I, 6>, as you should know.

[6] And the end (*ākhar*) of the solid is the surface <XI, 2>.

[7] And if two straight lines meet one another, with the condition that there be no curvature in the two of them and no merging together—understand—the region of the surface here is known as the planar angle <I, 8>.

[8] And if a line possessing straightness stands upright upon [a line] corresponding to it in straightness, such that the two angles that are found on the two sides of that [line] are equal to one another, it is a perpendicular (*ʿamūd*). And the two of them (the two angles), one should know, are two right angles (*qā'imatān*) <I, 10>, according to the convention of the learned.

[9] Then that [angle] which is smaller than it (the right angle) is acute <I, 12> and [that angle] possessing obtuseness exceeds that (the right angle) <I, 11>.

[10] And the figure (*shakl*) is a configuration (*hay'a*) [resulting] if a boundary or more than one boundary surrounds something <I, 14>.

[11] Then the square [is] that whose sides are equal and likewise the four (*arbāʿ*) [parts] <I, 22>—I mean, the angles—so understand the secrets.

[12] And the rectangle (*mustaṭīl*) [has] right angles [and] different (or unequal) sides <I, 22>.[40]

[13] And the rhombus [has] its sides equal, and that is evident, but [is] devoid of right angles <I, 22>—therefore understand and cease finding fault.

[14] Then the rhomboid lacks [both] equality and perpendicularity <I, 22>—let it be known—but its opposite sides are equal [one] to the other which is opposing to it.

[15] And so on. Whatever does not fall among what has been described is known in their technical terminology as a trapezia <I, 22>.

[16] And two lines parallel to one another do not meet one another on being drawn even if extended in imagination to any distance <I, 23>. Listen. May Allah guide you in the path of guidance.

[17] Any two magnitudes, [when] the smaller is multiplied (*ḍaraba*) into the larger or the reverse—understand—produce a surface the sides of which are parallel.[41] It is [said to be] bounded by two [contiguous] lines from it. Any two straight

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[40] The term *mustaṭīl* is not the most typical formulation of this definition. In the Arabic translation of the *Elements* the term is rendered as *al-mukhtalif al-ṭūlayn*. But it was the term used by al-Samarqandī in the *Ashkaāl al-ta'sīs*. It was also the term preferred by Naṣīr al-Dīn al-Ṭūsī in his *Taḥrīr Uqlīdis*.

[41] Only one of our manuscripts (Tehran, Majlis Shūrā 4816/1) has the term "parallel" at this point; all others have "equal." The reading found in the *Ashkāl al-ta'sīs* is "parallel" (De Young 2001, 80). For that reason, I have retained the reading despite the majority of the testimonia.

[lines] be may be [so] employed.[42]

[Postulates]

[1] And you should observe [that] equality exists of necessity between any right angle and every [other] right angle <I, Postulate 4>.

[2] And two straight lines are not able to surround by themselves any surface, however they may be joined together.[43]

[3] A straight line is not continued rectilinearly by two straight lines such that the two form a line—it can never be.[44] And this is clear, for anything clear is obvious.

Now, I have clarified what I wanted in the introduction, and so the introduction has brought to an end the provisions (*aḥkām*) of the introduction.

And after it (the introduction) he set his sights on the propositions (*ashkāl*)—[which] are thirty five [in number]—in order that there may be dispelled through them [any] reverberation (*ṣadan*) of ambiguity (*ishkāl*).[45] [And] he took upon himself the demonstration of what follows.

Now this is the **first**: If a line possessing straightness stands upright upon another [line], also straight, then the two of them produce two right [angles] or [two angles] resembling them (that is, the two right angles) with regard to their quantities.[46]

And the **second** of them is just as it may be established through the demonstration of its stipulation: when two lines meet one another in a point [which is] the endpoint of another line, that is, the first [line] and other [line, each being] straight, such that there is formed on the sides of the two of them two right [angles] or the equivalent, the two of them, that is, these two lines, are a united (*muttaḥid*) line.[47]

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[42] The definition is found in the Arabic transmission at the beginning of book II, although it is not found in the Greek transmission.

[43] This postulate is not found in the Greek primary transmission, although it was already mentioned as an alternative postulate by the Greek commentator Proclus (Heath 1908, I, 232). The postulate is, however, commonly found in the Arabic transmission.

[44] This is another alternate postulate that is sometimes found in the Arabic transmission of the *Elements*.

[45] A minority of manuscripts replace this clause with the statement "in order to dissipate the obscurities of the propositions."

[46] *Ashkā al-ta'sīs*: If a straight line stands upright upon another straight line, the angles formed on the two sides of it are either two right angles or are equal to two right angles.

[47] *Ashkāl al-ta'sīs*: If two straight lines are joined at a point which is the terminus of another line, then, if there are formed on the two sides of it (the third line) two right angles, the two lines are a

And the **third** of the propositions—let it be understood and relied upon—[is]: should a line meet two straight lines, then if one starts on one side and the two [angles] that occur are greater than two right angles in measure, there are produced [angles] smaller than two [right angles] on the other [side], [then] these two lines of necessity meet one another if extended—you can be sure of that—and it has a necessity just as Euclid, the Master, maintained in his preliminaries (*awwalāhu*). But al-Jawharī and Omar al-Khayyām, then al-Abharī and after him al-Ṭūsī and ibn al-Haytham and together with them Qāḍī Ḥamā[48] have objected to it—let it be known. They said [that] magnitudes exist without end (*ghāya*) since they may be divided without end. Thus it is permitted that the two of them get closer and closer without meeting. But it is a false statement and every intelligent [person] without exception bears witness [to that]; everything like that is incorrect (*fāsid*).[49]

And the **fourth** of them is that there being a triangle whose two sides and what the two produce between the two of them—I mean by that the angle—are similar (*shābih*) to two sides from [another] triangle and the angle between them, then the

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straight line.

[48] I have not been able to identify this person.

[49] *Ashkāl al-ta'sīs*: If a straight line falls on two straight lines then if the sum of the two interior angles which are on one side of that line is less that two right angles, the sum of the two interior [angles] on the other side is greater than two right angles because the two sums are like (or equal) to four right angles, just as occurred in the first [proposition]. Thus what is between the two lines on that side is narrower than on the other. So one of them is is inclined to the other of necessity. The two of them, on being extended on that side, are getting closer to one another of necessity. Now, the conclusion of getting closer to one another is necessarily meeting one another… Euclid did not demonstrate this proposition. But a group of outstanding practitioners of geometry made a demonstration and raised objections to it. They said that [since] there is established in scholarship the division of continuous magnitudes is without ending. And this continual division of the approaching, is correlated with not ending in meeting one another. They composed on the demonstration of this proposition treatises containing propositions and arguments (*maqā'il*), such as the treatises ascribed to scholars in geometry, like Ibn al-Haytham and Omar al-Khayyām and al-Jawharī and Naṣīr al-Dīn al-Ṭūsī and Athīr al-Dīn al-Abharī and Qāḍī Ḥamāh. And there is no doubt that what they mentioned concerning the continual approaching without meeting is a matter the falsity of which is attested by clear intellect. And if someone were to permit that [view], then approaching would be impossible also as well as the inconceivability of extending a line from a point to another. Everything they mentioned in their treatises is false because they are based upon the extension of lines. Consequently each of these treatises is not devoid of many falsehoods in the definitions or mistakes or use of non-geometrical premises, as some of them made clear in falsifying the statement of the other, as well as all [of them] sharing in being more obscure than that preliminary (or definition).

whole—that is the six angles and the sides (*ḍulū'*)—are equal to one another and the two triangles are equal to one another.[50]

Next, if one of the two of them is smaller (*aqall*), oh clever one, then the line that becomes (*ghadā*) its chord falls short of the chord of the other [angle]—it is a section (*qaṭ'*) of it. And the converse is [also] true. This is the **fifth** [proposition] in which there are no obscurities (*ishkāl*).[51]

The **sixth**: when two sides (*sāqān*) from a triangle are equal to one another, then at the two ends (*ṭaraf*) of the base of the triangle there are two angles and the two of them are equal to one another; and if its two sides are extended, there are produced beneath the two of them (that is, the two equal angles) two [other] angles and they are also equal to one another according to the demonstration (*bi-l-dalīl*).[52] Now you should be aware [that] this [proposition], on account of al-Ma'mūn's fascination (*shaghf*) with it, is known as "al-Ma'mūnī."[53]

The **seventh** of them: When two angles of a triangle are equal to one another, then also, Oh youth, its two sides—I mean, the chords of the two of them—are equal.[54]

The **eighth** of them: Two triangles in which there are six sides, each one [being] equal respectively to the corresponding [side which] is similar (*yushbihu*) in the other [triangle], similarly the angles in the two of them are equal to one another, each to its correlate (*mithlihi*), just as was made known; and the two triangles are equal to one another. So be informed [about it].[55]

And the **ninth** of the propositions—listen and understand—is that we want to extend a perpendicular from a point on a line and the line being unbounded. So let us draw at two equal distances from it (the given point) so that we make two quarter

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[50] *Ashkāl al-ta'sīs*: If two sides and the angle between them from [one] triangle are equal to two sides and the angle between them from another triangle, the remaining pair of sides and the two triangles are equal to one another.

[51] *Ashkāl al-ta'sīs*: If one of the two angles is smaller than the other in the two mentioned triangles, its chord is smaller than the chord of the other.

[52] *Ashkāl al-ta'sīs*: The two angles that are upon the base of an isosceles triangle are equal to one another; likewise, the two which are produced under the base if the two sides are extended.

[53] Both the *Ashkāl al-ta'sīs* and the commentary of Qāḍīzāde al-Rūmī (1984, 74) provide only the statement that "this proposition is nicknamed al-Ma'mūnī." The origin of the author's more extended explanation is obscure (Kunitsch 1993, 206–208).

[54] *Ashkāl al-ta'sīs*: If two angles of a triangle are equal to one another, the sides which are chords of the two of them are equal to one another.

[55] *Ashkāl al-ta'sīs*: If each one of the sides of a triangle are equal to each one of the sides of another triangle, their angles are equal, each to its corresponding [angle]; and the two triangles are equal to one another.

circles intersecting one another. We connect the [given] point and the intersection of the two [quarter circles]. Thus there is produced the perpendicular.[56]

The **tenth** is that we [should] extend it (the perpendicular) from a point (not on the given line) to it (the line). We make that [point] the center of a circle such that [it], namely the circle, intersects that line. Then we bisect that [part of the line] which is inside it (the circle) at a point and we extend the line [from the given point] to it (the bisecting point). Indeed, you should crave understanding (*ma'rifa*).[57]

The **eleventh** is that which has been made known among them and it is famous: [When] two lines intersect one another, there are produced four angles lying opposite one another, a pair of them resembling (*ashbahat*) the pair lying opposite them, without prevarication, each one is equal to that which is opposite it—may Allah keep you blameless with regard to vile deeds—and you must listen closely to the stipulation.[58]

The **twelfth**, oh you who are superior in understanding among all mankind, [is]: [For] any triangle, if we extend a side of it externally, we find [that] the exterior angle is greater than [either of] its two [opposite] interior [angles] in isolation.[59]

Then, if a side from the three [sides of a triangle is] drawn longest (*ṭāl*), then it is indisputably (*yaqīnā*) a chord of the largest [angle]. That is [what] the **thirteenth** [proposition] established (*thabbatā*).[60]

After it the **fourteenth** sets out [that] the chord of the largest angle is the longest side, [which is] absolutely (*qaṭ'an*) without controversy.[61]

The **fifteenth** of them: We want to erect a triangle but with the condition that there occurs in it the equality of each side with a specified line but the lines have the stipulation that no two of them can be other than exceeding the third, [they can] by no means [be] less than or equal to the third of them since it is not at all possible

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[56] *Ashkāl al-ta'sīs*: We want to extend from a point on a line a perpendicular to it.

The stipulation that the line should be unbounded is found in the commentary of Qāḍīzāde al-Rūmī (1984, 87).

[57] *Ashkāl al-ta'sīs*: We want to extend from a point to a line a perpendicular.

[58] That is, when a straight line intersects another straight line, it forms two angles on one side of the line and two angles on the other side of it. Both pairs of angles are equal to two right angles, as shown in the first proposition. Each of these pairs of angles is equal to the angle lying opposite it. This statement is more difficult to understand than the statement of the enunciation in the *Ashkāl al-ta'sīs*: The pair of opposite angles formed when any two lines intersect one another are equal to one another.

[59] *Ashkāl al-ta'sīs*: [If in] any triangle one of its sides is extended, the exterior angle is greater than any one of its two opposite interior angles.

[60] *Ashkāl al-ta'sīs*: The longest side of a triangle is the chord of the greatest angle.

[61] *Ashkāl al-ta'sīs*: The greatest angle of a triangle has as its chord the longest side.

in a triangle according to principle, and that [is] according to the demonstration. Likewise it (the triangle) may be constructed using the compass and it is easier.[62]

The **sixteenth**: It is required that we construct an angle at some point of the line—Be cognizant of it—such that it looks exactly like (*ashbaha*) a specified angle.[63]

The **seventeenth**: if a side together with two angles from a triangle be equal to the side together with two angles from another, the two triangles are equal to one another, I mean that the sides of the two of them together with all the angles are equal to one another.[64]

Next, if a line fall on two straight (*qawīm*) [lines], [such that] the alternate [angle] on one side of the line is equal to the alternate [angle] or the exterior [angle] is exactly like (or equal to) the interior [angle] or the two [angles] falling interiorly on one side of it are two right angles or are like the two of them (that is, the two angles are equivalent to two right angles), it comes about in every case that there is an equality of what is between those two straight lines.[65] That is [what] the **eighteenth** [proposition] establishes.[66]

The **nineteenth** is the converse of what preceded it and the complement (*mukmil*) [of it].[67]

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[62] *Ashkāl al-ta'sīs*: We want to construct a triangle each of whose sides is equal to one of three given lines, on the condition that each pair of them together is longer than the third.

Qāḍīzāde al-Rūmī (1984, 107) explains the visual demonstration using three compasses but does not explicitly say that it is "easier."

[63] *Ashkāl al-ta'sīs*: We want to construct at a given point from a straight line an angle like (*mithl*) a given angle.

In the Arabic branch of the Euclidean transmission, *mithl* is often used in the sense of being equal. The triliteral root *sh-b-h*, from which *ashbaha* is derived, is usually used in the sense of being similar, although in this proposition it is used in the sense of being equivalent.

[64] *Ashkāl al-ta'sīs*: If two angles and a side from a triangle are equal to two angles and a side from another triangle, each to its respective [component], the other pair of angles and the remaining sides are equal to one another respectively; and the triangle [is equal] to the triangle.

[65] In other words, the two lines are always equidistant, which is another way of stating that they are parallel to one another. According to proposition three, the two lines will intersect if they are approaching on one side of the incident line. Only when the two lines are always equidistant will they never approach one another and meet.

[66] *Ashkā al-ta'sīs*: Any two straight lines upon which a straight line falls, and if the two alternate angles be equal to one another, are parallel to one another; likewise if the exterior angles be as the interior angles; and likewise if the two angles that are on one side [of the incident line] are equal to two right angles.

[67] The author, perhaps for brevity, does not even state the enunciation.

Ashkāl al-ta'sīs: If a straight line falls upon two parallel lines, the two opposite [angles] are equal

And the **twentieth** also completes it: [For] any triangle, if we extend a side of it exteriorly, we find [that] the exterior angle is the same as (*tashbaha*) the two opposite interior [angles]; moreover what results from the three [angles]—I mean those which are interior—is equal to two right angles.[68]

And after [it is] the **twenty-first**: If you wish an account of it, [it is] thus: if [there be] parallel lines, with the condition of mutual equality in respect to magnitudes, [and] there occur for them lines connecting their endpoints, [the lines] are united in their (that is, the given lines) characteristics, namely mutual equality and parallelism. [The proposition] is completed.

And after it, the **twenty-second** is established: When [sides from] surfaces [which] lie opposite one another, on condition of mutual parallelism among the lines, there occurs [to] whatever is between them, namely their sides, that they are equal to one another; then let it be clear—listen—[that] the opposite angles likewise are without doubt equal to one another. Yes, and the diagonals of the preceding areas are bisectors—now cease the contradicting.[69]

And the **twenty-third** [is] concerning positions: Two surfaces of parallel sides that are on a [single] base [and] between two [lines] parallel to one another are certainly (*qaṭ'an*) equal to one another if the two occur on the same side [of the base]—Understand it without doubting.

Next, if there be for each [surface of parallel sides in the previous proposition] a base [of its own], together with equality [of the bases] to one another, the two of them (the parallelograms) are as one (that is, the surfaces are equal to one another). This is the **twenty-fourth** [proposition].[70]

And after it [is] the **twenty-fifth**, whose statement [is]: Any two triangles, [both] being on [one] side [of a line], between two parallel lines, [and] on [one and the same] base, are equal to one another.[71]

And in like manner, [if they be upon] two [separate but equal] bases they are equal to one another. That is the **twenty-sixth** [proposition].[72]

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to one another and the exterior [angle] is as the interior [angle].

[68] Every triangle of straight sides, one of whose sides is extended, its exterior angle is equal to its two opposite interior [angles] and the three angles are equal to two right angles.

[69] *Ashkāl al-ta'sīs*: The opposite sides of surfaces having parrallel sides are equal to one another.

[70] *Ashkāl al-ta'sīs*: Any two surfaces having sides parallel to one another [which] are on the same side of two bases equal to one another and between two lines parallel to one another are equal to one another.

[71] *Ashkāl al-ta'sīs*: Two triangles [which] are on one side of a single base [and] between two lines parallel to one another are equal to one another.

[72] *Ashkāl al-ta'sīs*: Any two triangles [which] are on the same side of two equal bases [and] betwee two lines parallel to one another are equal to one another.

And there follows it the **twenty-seventh**: Any triangle and a surface [endowed] with parallelism in respect to its lines, the two [figures] occurring on a single base [and] two parallel lines encompassing the two of them, the two of them [being] on the same side [of the base], then the [parallelogrammic] surface is double the triangle. And this is shown to be correct.[73]

Listen carefully to the **twenty-eighth** of which I am now giving an explanation: Two surfaces, the sides of which [being] parallel each to the other (*kull li-kull*) and the two altitudes [being] equal to one another, are shown [to be] according to the ratios of the base—whenever the base [becomes] less or becomes greater, [the ratio] of the two surfaces follows it in magnitude. And likewise [for the case of] two triangles through [the same] demonstration (*istiẓhār*).[74]

And the **twenty-ninth**, according to what has been mentioned: Two parallelograms that occur (*ḥallā*) within a parallelogram, and each of the two parallelograms being described on either side of the diagonal [and] joined to the other at a point—you should know [that] in their convention the two of them are the complements. And according to the proof (*dalīl*), they are equal to one another.[75]

Then the **thirtieth** [which is] without obscurity—[called] the "Bridegroom Proposition," is the most excellent of the propositions: [For] any triangle, one of whose angles is right, the square on its (the right angle's) chord is equal to the square of the two sides [of that right angle] and the mutual equality is proved true by the demonstration.[76]

And the **first**, since the thirtieth proposition is past, [is] thus: If something be multiplied—seek information concerning it (*li-sti'lāmihi*)—into something, it is the

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[73] *Ashkāl al-ta'sīs*: Any surface of parallel sides and triangle [which] are on the same side on a single base [and] between two lines paralle to one another, the surface is double the triangle [in area].

[74] *Ashkāl al-ta'sīs*: Any two surfaces [having] sides parallel to one another [and having] the altitudes equal to one another, the ratio of one of them to the other is as the ratio of its base to its base. And likewise it may be judged concerning two triangles.

Only two manuscripts refer to triangles, although triangles is the reading in the *Ashkāl al-ta'sīs*. The remaining manuscripts refer to two "squares" (*murabb'ān*).

[75] *Ashkāl al-ta'sīs*: The two complements (*mutammamān*), namely any two surfaces [having] sides parallel to one another [which] occur in a surface like the two of them (that is, having sides parallel to one another) on two sides of its diagonal [and] meeting at a single point from the diagonal [and] sharing with that surface two angles, are equal to one another.

[76] *Ashkāl al-ta'sīs*: For any right-angled triangle, the square [on] the chord of the right angle is equal to the squares [on] the two sides of it (the right angle).

The origins for the name of this proposition are obscure, but it may have come from the late antique Greek transmission (Kunitzsch 1993, 208–209).

equivalent to its multiplication into its parts.[77]

And the **second** after the thirtieth is carried out just as we stated [in the previous proposition] and just as also it was transmitted: [For] surfaces [resulting from] the multiplication of the line into [each of] its parts neither exceeds nor is deficient from its square.[78]

And the **third**, that is, after the thirtieth has passed completely: you should understand [that] the square of the line is equal—let it be known—to the squares of its two parts together with double the surface [from] the multiplication of the [one] part into the [other] part.[79]

And afterward the **thirty-fourth**: If a line is bisected and we afterward divide its entirety [into] two different (or unequal) [segments], then if there are combined two surfaces of one of the two parts into the other—take note of it—and after that there is added the square of the excess of the half in relation to one of its parts, one may establish [that] there exists in the two situations (*ḥālayn*) an equality in respect to value value in relation to the square of half [the line]—compare [the two] so that you may demonstrate [the principle].[80]

And the **fifth** after the thirtieth is for whoever seeks an account of it, so one should let it be known. Now its demonstration begins without secrecy, and that is that: Any line being bisected and afterward there be added to it another [line] rectilinearly—and that is clear—the sum of the surface of the line and the addition together, if multiplied into the addition, is joined to the square on the half, then what results (*qama*) is equal after that—understand—to the square of the half together with the addition.[81]

And with that he ends the communication (*afādah*).

VI.5 Colophon

Unlike in the case of the preamble, there is essentially one version of the colophon. I again use Tehran, Majlis Shūrā 11 as the base text for the edition.

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[77] *Ashkāl al-ta'sīs*: The product (*ḥāṣil*) of the multiplication (*ḍarb*) of something into something is the equivalent of the product of its multiplication into its parts.

[78] *Ashkāl al-ta'sīs*: The sum of the surfaces of the line into its parts is equivalent to its square.

[79] *Ashkāl al-ta'sīs*: The square on a line is equal to the sum of the squares of its parts and double the surface of one of them (the parts) into the other.

[80] *Ashkāl al-ta'sīs*: Any line [being] bisected and then divided into parts [that are] different, the sum of the surface of one of the two parts into the other and the square on the excess between the half and the part is equal to the square on the half.

[81] *Ashkāl al-ta'sīs*: Any line [being] bisected and [there being] added to it another line rectilinearly, the sum of the area of the line plus the addition into the addition and the square on the half is equal to the square on the half plus the addition.

فهاك نظماً جمع الاشكالا * واطرّح الابهام والاشكالا
وقد تركت مع الاقتدار * نظم البراهين للاختصار
فارجع إليها في كلام القوم * إن رمتها ودع حبيبي لومى
وكان نظمه بمجلسين * ولم يكونا منطاولين
فأول من أول النظم إلى * ثالث عشر قد نظمت عجلا
والمجلس الأخير وهو الثاني * آخر يوم من جمادى الثاني
من عام ست بعد خمسين مضت * من بعد الف سنة قد انتقضت
من هجرة الرسول صلى الله ما * ناح حمام ألائك أو تربما

عليه والآل الكرام القادة * والصحب أصحاب الكرام السادة
والحمد لله على ما سهلا * من العسير وازاح العللا والحمد لله على ما * من العسير وازاح العللا

Now this is a versification (*naẓman*) of the totality of the *Ashkāl* and it is a stripping away of ambiguity and obscurity. I have omitted a rhymed version of the demonstrations, despite [having] capability, for the sake of abridgment. So return to them in the words of the people, since I wished to skip them on account of [their] triviality, my dear one. And its versification (*naẓm*) was [made] in two sessions (*majlisayn*) and the two of them were not prolonged. The first is from the beginning of the poem until the thirteenth [which] were rhymed quickly. The final session, namely the second, was the last day

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3–2 وقد ... لومى] (−) **قا، طب.** 3 رمتها] رومتها: **طا.** 5 فأول ... عجلا] *margin* : **طب.** فأول] فأولا: **طب، طد، طم، قا، قد** = فأولى: **جڤ.** عجلا] *damaged, illegible* **طد.** 6 الثاني] (+) ~~ثالث عشر~~ : **جڤ.** آخر يوم ... الثاني] *damaged, illegible* : **طد.** جمادى] ربيع: **طب، قا** = جميدى: **مم.** 7 من عام] سنة: **طب، قا.** بعد الف ... انتقضت] *damaged, illegible* **طد.** سنة] حجة: **طب، قا.** انتقضت] انقضت: **طب، قا، جڤ، مم.** 8 ألائك أو تربما] ألائك أو ترنما: **قب، خف** = *damaged, illegible* : **طد.**
9 والآل] وآل: **قب.** القادة] العادة: **قد.** الكرام] الكمال: **طب، طد، قا، جڤ، قد.** 10 والحمد ... العللا] (−): **قب.** سهلا] يسهل مهما: **خف.** العللا] (+) تمت كتبها ناظمها أول جمادى الأول في التاريخ: **طب** = (+) تمت والحمد لله نظم الشيخ المرحوم الشيخ محمد الحر: **قا.** 10 العسير] الغير: **خف.** وازاح] وأراح: **خف.**

11–10 والحمد ... العللا] وجدنا هذا البيت في الهامش في نسخة قب.

> of Jumādā II of the year six after the fiftieth [year] had passed after the thousandth [year] since the hijrah of the Prophet, may Allah—who did not bewail that one (*ulā'ik*) or make a trilling [over him] (*tarannamā*)—have mercy upon him and [upon] the noble family (*al-āl*), [upon] the commanders-in-chief (*al-qādah*) and the congregation of the noble saintly (*al-sādah*) companions.
>
> And thanks be to Allah for he has made easy [what is] difficult (*al-ʿasīr*) and has removed (*azāḥa*) the irregularities (*ʿilal*) [of the meter?].

We learn several things from the colophon, in addition to the date when the treatise was completed. Perhaps the most remarkable is the claim that the versification took place in just two sessions, the first going from the beginning of the treatise through proposition thirteen and the second dealing with the remainder of the treatise. Another surprising statement is that the author did not recast the demonstrations in didactic verse because they are too "trivial." It is not completely clear what the author intends by this term—perhaps they are considered obvious once the statement of the enunciation is given? Or perhaps they are too repetitive to be easily made into didactic verse?

VII Concluding Observations

My study of this *manẓūma* has utilized eleven manuscripts. Since the contents of many Iranian libraries are not well-publicized, there may yet be other manuscripts of this *manẓūma* extant. The fact that ten of the eleven manuscript known at present have come from libraries in Iran suggests that the work was most popular in that region.[82]

It has often been suggested that the use of didactic verse for instruction was intended to help students to memorize texts required in their *madrasa* education, and al-ʿĀmilī explicitly cites this idea as one of his leading motivations in preparing these verses.[83] But at the same time, the requirements of the meter and the concise formulation of the contents could sometimes produce verses that are less easily understandable than the treatise they propose to summarize—unless one already had considerable familiarity with the subject matter. Thus the use of didactic verse

[82] Five are found in libraries in Tehran, five in libraries in the region of Qom and Mashhad, the very region where al-ʿĀmilī taught. Only one copy has so far been identified outside Iran—Cairo, Dār al-Kutub, Magāmīʿ 846/4.

[83] The use of rote memorization is mentioned (often pejoritively) in connection with *madrasa* education, but the actual practices in late medieval education are difficult to document. Nevertheless, one could easily understand why an expert in *ḥadīth* (transmission of the sayings of the Prophet Muḥammad) might emphasize memorization in his own instruction style.

could also impede the pedagogical process—especially if the author's desire was to assist the beginner in the field (van Gelder 1995, 108). For this reason, many *arājīz* spawned prose commentaries. For example, Ibn al-Hā'im's commentary on Ibn al-Yāsamīn's *al-Urjūza fī-l-jabr wa-l-muqābala*, recently studied by Mahdi Abdeljaouad (Ibn al-Hā'im, 2004), and Ibn Ghāzī's *Bughyat al-ṭullāb fī sharḥ Muniyat al-ḥisāb*, a prose commentary explaining the author's earlier *manẓūma* on rules of calculating, which was edited by Mohammed Souissi (Ibn Ghāzī 1983). But so far, no prose commentary explaining this *manẓūma* has been found—perhaps because the clarity of its contents did not require additional explication, or perhaps the popular prose commentary by Qāḍīzāde al-Rūmī on the *Ashkāl al-ta'sīs* filled that role.

Sobieroj (2016, 8) has suggested that most didactic verses such as those of our text were copied in the context of a school (*madrasa*) or mosque, rather than in a princely scriptorium. Although this suggestion seems logical, it is somewhat surprising that there are almost no annotations of any kind in these manuscripts. One might have expected that a manuscript copied for use in education might exhibit more evidence, in the form of marginalia, of reader engagement with the text.[84] Certainly the apparently deliberate omission of the demonstrations for most propositions and the decidedly awkward phrasing of some enunciations must raise some questions about the usefulness of the treatise without some sort of magisterial assistance being provided by the teacher.

Although most of the known manuscripts do not include a date of copying in their colophon, those that can be dated suggest that the interest in and use of these didactic verses, although not perhaps a short-lived phenomenon, may not have played a significant role long after the death of al-ʿĀmilī. The latest copy known from the colophons is dated 16 Ṣafar 1111 / 12 August 1699, less than a decade after the death of the author. And if this interpretation is correct, we may wonder whether use of the *manẓūma* was limited to al-ʿĀmilī and his immediate circle.

This treatise in didactic verse offers us a small window into the practice of mathematics education in the region of Mashhad during the last half of the seventeenth century—the period when al-Ḥurr al-ʿĀmilī was teaching at Mashhad. Of course, there is still much that is unknown. The organization and day-to-day functioning of the *madrasa* differed from one time to another and from one locale to another. The content of the curriculum changed and developed over time and the place of the mathematical sciences within the curriculum also changed again and again. Moreover, the place of the *madrasa* as social institution also continued to develop and change over time. To date, there is no adequate history of the institution of the *madrasa* available in English or European languages. Even attempts to sketch the history of *madrasas* in a particular locale require analysis of a wide range of sources,

[84] On the importance of marginalia, see Jacquart and Burnett (2005) and Murdoch (2003).

from biographies and biographical dictionaries to legal records of pius foundations (*waqf*) that provided financial underpinning to the *madrasas*, to institutional histories and short biographical or autobiographical vignettes in historical records, to mention some of the more widely used sources.[85] Only a few examples of didactic verse were devoted to geometry, so this *manẓūma* represents a "road less traveled" in mathematics education. Whatever our judgement about its ultimate success, the attempt to distill geometry into verse deserves our attention as a testimony to the effort expended to express the essence of the subject.

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[85] Moazzen (2011, 11–19) discusses the various sources of evidence that she used in order to produce a complete and nuanced understanding of education in a particular *madrasa* in Safavid Iran. But it is not clear to what extent her description of a single *madrasa* can be assumed to be normative for other *madrasa* even in the same region.

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