S C I A M V S

SOURCES AND COMMENTARIES IN EXACT SCIENCES

Volume 3, August 2002

Chief editor: Michio Yano
E-mail: yanom@cc.kyoto-su.ac.jp

Managing editor: Ken Saito
E-mail: ksaito@hs.cias.osakafu-u.ac.jp

CONTENTS

SCIAMVS
(ISSN 1345–4617)

Published by SCIAMVS, Kyoto, Japan.
Printed by Hokuto Printing Company

Editorial

This third issue of SCIAMVS includes five articles by scholars of six nationalities, and concerning five different languages. Once again the importance of a close reading of primary texts is emphasised. Four of the articles include new editions of previously unknown or imperfectly known documents, with English translations and commentaries; the fifth (Hayashi's), a close comparison between two recensions of a text. The documents may be written on clay (as in the first article), or may be tables rather than written texts (as in the third). An accurate knowledge of the primary materials is a prerequisite for any statements on the history of mathematics, and, in the case of tables and diagrams, the layout of the text is also important. Through its large format SCIAMVS can accommodate large-scale reproductions of the primary documents, and all the requisite scripts can be printed elegantly. The presentation of original texts provides the primary documentation that any scholar can refer to, if he or she has any doubts about a particular interpretation. The texts themselves are usually accompanied by literal translations, which preserve the rhetorical, syntactical and notational styles of the original texts. In mathematics, unlike other subjects, two levels of 'translation' are required: the literal translation, and the 'translation' into modern notation. A typical example is that of algebra, in which a literal translation of an Arabic or Latin text may read 'a substance (square) and twenty-one dirhams (units) are equal to ten unknowns', but this may be 'translated' into: $x^2 + 21 = 10x$. If the first level is omitted, there is a real danger of misrepresenting the original author, or at least losing many of the nuances of his mathematical culture. The provision of these two levels is also important to ensure that the history of pre-modern mathematics does not remain the prerogative of mathematicians, but can also be appreciated by students of the relevant cultures, and integrated into a larger picture of the society in which the texts were written.

Following previous editorials I would like to continue to place the research presented in these volumes of SCIAMVS into a scholarly tradition, but one stretching back beyond Otto Neugebauer and Kiyosi Yabuuti, to the turn of the nineteenth to twentieth centuries. Two articles in this volume (the second and the fifth) depend on editions of Greek mathematical works made by Johan Ludvig Heiberg; a third (my own), on those of Latin mathematical texts made by Nikolai Bubnov. For over a hundred years the editions of these towering scholars have served as the basic texts for the history of ancient and medieval Western mathematics. Heiberg (1854–1928) was professor of classical philology at Copenhagen University from 1896 until 1924. He specialised in editing texts on Greek mathematics, but also dealt with philosophy

and medicine. He edited the works of Archimedes (twice), Euclid (together with H. Menge), Apollonius of Perga, Hero of Alexander, Serenus Antinoensis, Ptolemy (the *Almagest* and minor astronomical works), and Theodosius (*Sphaerica*), as well as Simplicius' commentary on Aristotle's *De caelo*, Paulus Aegineta's medical encyclopedia, and several texts by the father of Greek medicine, Hippocrates. He supplied all his editions of works of mathematics and astronomy with Latin translations, and his texts of Euclid and Apollonius in turn provided Thomas Heath with the material for making his well-known English translations of the two authors. Heiberg's bibliography consists of over 200 items, and most of his editions remain the texts still in use today. Nikolai Bubnov, on the other hand, who was born in 1858, and was a professor at the University of Kiev from 1891 until the Revolution, after which he held a chair in Ljubljana, is known for only one work: *Gerberti postea Silvestri II papae Opera Mathematica* (972–1003), published in Berlin in 1899 (another text, *Arithmetische Selbstständigkeit der europäischen Kultur, Ein Beitrag zur Kulturgeschichte*, Berlin, 1914, is not so readily available). In this work, however, he did much more than edit the mathematical works more or less securely attributed to Gerbert d'Aurillac (who was Pope Silvester II from 999–1003). He surveyed a large number of manuscripts of mathematical texts written in this period and immediately afterwards, and established the mutual relationships both between these texts and between the manuscripts that contain them. Thus he identified the Latin writings on the astrolabe that were translated directly from Arabic, and showed the order of dependence of later texts by means of a system of symbols that scholars still use. He did a similar thing with early texts on the abacus and the early medieval texts on land-surveying and geometry. Altogether over fifty texts are included in his book, and any scholar working on Latin texts on the abacus, the astrolabe or geometry before ca. 1100 must begin with the editions made by Bubnov.

In spite of the greater facilities for travel, for reproducing material, and for communicating information available to us, and our advanced computer technology, it is difficult to match the achievement of these two scholars. But our aims are the same: to produce texts that are faithful to the evidence provided by the manuscripts (or clay-tablets), but that are also informed by mathematical competence. Both Heiberg and Bubnov presumed that their readers were competent in Latin (if not also in Greek). Alas, this is no longer the case, and English translations of texts have to be provided rather than Latin ones. But by providing English translations, and the second level of 'translation' into modern mathematical terms, we are going further than our nineteenth- and early twentieth-century predecessors. We are trying to make mathematics of early periods and cultures understandable to as wide an audience as possible.

July 7, 2002
Charles Burnett

SCIAMVS 3 (2002), 3–90

TU 11
A Collection of Rules for the Prediction of Lunar Phases and of Month Lengths

Lis Brack-Bernsen
Wissenschaftsgeschichte
Universität Regensburg

Hermann Hunger
Institut für Orientalistik
Universität Wien

SCIAMVS 3 (2002), 91–108

The Abacus at Echternach in ca. 1000 A.D.

Charles Burnett

Warburg Institute
University of London

The single sheet of manuscript from the Benedictine monastery of Echternach, catalogued as no. 770 in the Bibliothèque nationale de Luxembourg, is probably the nearest thing we have to the abacus board devised by the famous mathematician and educator of the late tenth century, Gerbert d'Aurillac (d. 1003). This article describes the board in the light of what we know of Gerbert's own instrument and also introduces another smaller example of exactly the same board. Both examples have hitherto been unnoticed by scholars.[1]

* * * *

1 The Gerbertian Abacus

From the late tenth century until at least the mid twelfth century the principal method of studying practical arithmetic in the schools of Western Europe was that of the abacus with marked counters. This was a a board or sheet of parchment ('abacus board') on which parallel lines were drawn to provide columns for powers of ten. Counters marked with each of the numerals were placed in the columns and successively replaced in the course of an arithmetical calculation. Rules were written to describe the sequences of procedures, especially for division and multiplication.

[1]I am very grateful to Luc Deitz, the curator of the manuscripts and rare books collection (Réserve précieuse) of the Bibliothèque nationale de Luxembourg for introducing me to the manuscript-sheet, and inviting me to give a talk at the library on its relation to the early history of the introduction of Arabic numerals into Europe, on 19 October 2001, as well as for showing me a preliminary study of his on the logical schemata contained on the sheet and correcting an early draft of this paper. Throughout this article I am indebted to the research of Menso Folkerts, whose article, 'Frühe Darstellungen des Gerbertschen Abakus' (in *Itinera mathematica: Studi in onore di Gino Arrighi per il suo 90a compleanno*, eds R. Franci, P. Pagli and L. Toti Rigatelli, Siena 1996, pp. 23–43) constitutes the starting point for the study of early depictions of the abacus, and who has generously shared his knowledge and research materials with me. I have also benefited from the advice of David Juste, Daibhi O Croinin, Alison Peden and Thomas Falmagne, and from the services of Reiner Nolden of the Stadtarchiv of Trier. The contents of the manuscript sheet as a whole will be the subject of a monograph written jointly by Luc Deitz, Thomas Flamagne and the present author.

SCIAMVS 3 (2002), 109–125

A New Reading of *Method* Proposition 14 : Preliminary Evidence from the Archimedes Palimpsest (Part 2)

Reviel Netz
Stanford University

Ken Saito
Osaka Prefecture University

Natalie Tchernetska
Trinity College, Cambridge

I Introduction[1]

In the first part of this article (*SCIAMVS* 2(2001): 9–29) we published an edition of The Archimedes Palimpsest, coll. 110v–105r, containing about half of the text of what Heiberg called '*Method* proposition 14'. The text was especially interesting in that it included a previously unsuspected application of Lemma 11 of the *Method* (= *Conoids and Spheroids*, prop. 1) with a proportion involving infinitely many objects. We briefly touched on the implications of this new reading for the history of Greek ideas concerning infinity, and for Archimedes' position in the history of the pre-calculus.

The full text of proposition 14 begins before, and ends after, the text published in the first part. The columns 110v–105r constitute together one side (the second one) of a single Archimedes folio, 105–110. The proposition apparently begins at the very beginning of that folio, and ends on another folio (158–159). A year ago, when the first part was published, we had available to us digital images for 110v–105r alone. Now that we possess the digital images for the entire proposition we complete and revise the edition accordingly. In terms of columns of text as defined by Heiberg, we now publish five new columns, which, together with the four previously published, cover the entire proposition. The new columns are printed as bold in the sequence:

110r. col. 1 – 105v. col. 1 – 110r. col. 2 – 105v. col. 2 – 110v. col. 1 – 105r. col. 1 – 110v. col. 2 – 105r. col. 2 – **158r. col. 1**

[1]The same words of thanks from the first part of the article hold here as well. This study would have been impossible without the crucial contribution of many people: William Noel, curator of manuscripts, Cathleen Fleck, assistant curator, Erin Loftus, Conservation technician, and Abigail Quandt, senior conservator of manuscripts, all of the Walters Art Museum; Roger Easton and Keith Knox of the Rochester Institute of Technology; William Christens-Berry of Johns Hopkins University; Michael Toth of R.B. Toth Associates; and the owner of the Archimedes Palimpsest.

SCIAMVS 3 (2002), 127–192

A Paraphrased Latin Version of Euclid's *Optica* A Text of *De visu* in MS Add.17368, British library, London

Ken'ichi Takahashi
Graduate School of Social and Cultural Studies Kyushu University, Japan

Takako Mori
Faculty of Education Ehime University Japan

Youhei Kikuchihara
Graduate School of Social and Cultural Studies Kyushu University, Japan

I Introduction

The present article provides for the first time a paraphrased version of Euclid's *De visu* which is a Latin translation of his *Optica*. It has already been acknowledged that there are many versions of it. Out of thirty-three MSS now known to be extant, Lindberg[1975] distinguishes seven versions according to the difference of *incipits* for definition 1 and proposition 1. Moreover, he notices that some manuscripts have alternative enunciations for one or more of their propositions, which are drawn from *De radiis visualibus*, the most popular Arabo-Latin translation of Euclid's work. Our manuscript of *De visu*, which is contained in MS Add.17368, British library, London, belongs to this category of manuscripts. In order to investigate the historical significance of our manuscript, let us begin by picking out those manuscripts of this type from Lindberg[1975], which may be arranged in chronological sequence.

MSS of *De visu* with alternative enunciations drawn from *De radiis visualibus*

- **Version 1 (6 MSS out of 21)**

  #1. London, British Museum, MS Add. 17368, fols.60r-69r. 12th c.

  #2. Oxford, Bodleian Lib., MS Auct. F. 5.28, fols. 17(57)r-24(64)r. 13th c.

  #3. Oxford, Bodleian Lib., MS Corpus Christi Coll. 251, fols. 1r-7v. 13th c.

  #4. Venice, Bibl. Naz. Marciana, MS Zanetti Lat. 332 (Valentinelli XI.6), fols.242r-251v. 13th c.

  #5. Erfurt, Wissensch. Bibl., MS Ampl. Q.385, fol. 210r. 14th c. Fragment

  #6. Leeuwarden, Provinciale Bibl., MS B.A.Fr.57, fols.59r-68r. 15th c.

SCIAMVS 3 (2002), 193–230

Notes on the Differences between the Two Recensions of the *Līlāvatī* of Bhāskara II

Takao Hayashi

Science and Engineering Research Institute
Doshisha University
Kyoto Japan.

Introduction

The *Līlāvatī* (abbr. L) is a Sanskrit work on arithmetic and mensuration (computational geometry) composed by Bhāskara II in or a little before A. D. 1150. The number of the manuscripts (more than six hundred) of the work listed in the CESS,[1] as well as the number of the commentaries on it (more than thirty),[2] proves that it has been the most popular mathematical textbook in India used by a number of people of the entire subcontinent. It is regrettable that virtually no study based on those manuscripts has been done yet on the history of the transmission of such an influential work. So far, we only know that there are some differences between the northern and the southern recensions of the work,[3] represented respectively by the Ānandāśrama edition (I call it ASS)[4] and by the Hośiarapura edition (VIS).[5]

The ASS is accompanied by two commentaries, one by Gaṇeśa and the other by Mahīdhara. Gaṇeśa, son of Lakṣmī and Keśava of the Kauśikagotra, composed

---

[1]CESS: David Pingree, *Census of the Exact Sciences in Sanskrit*, Ser. A, Vols. 1–5, Philadelphia: American Philosophical Society, 1970–1994. For the manuscripts of the *Līlāvatī*, see A4, pp. 300–308, and A5, pp. 254–257. The number of manuscripts may be multiplied if we take into account those which are left and forgotten at corners of a number of family libraries.

[2]CESS, A4, pp. 299–300.

[3]See R. C. Gupta's article cited in III.15. See also fn. 5 below.

[4]ASS: The *Līlāvatī* edited with Gaṇeśa's *Buddhivilāsinī* and Mahīdhara's *Līlāvatīvivaraṇa* by Dattātreya Āpaṭe, *et al.*, 2 parts, Anandashrama Sanskrit Series 107, Poona: Anandashrama Press, 1937.

[5]VIS: The *Līlāvatī* edited with Śaṅkara and Nārāyaṇa's *Kriyākramakarī* by K. Venkateswara Sarma, Vishveshvaranand Indological Series 66, Hoshiarpur: Vishveshvaranand Vedic Research Institute, 1975. Yano and I have pointed out some of the variant readings found in the VIS in our Japanese translation of the ASS edition, which has been included in: *Collection of Astronomical and Mathematical Works in India*, edited by Michio Yano, Kagaku no Meicho Series 1, Tokyo: Asahi Press, 1980, pp. 139–372.