III Appendices

III.1 Rules and Examples of the *Bījagaṇita*

Here I briefly describe the mathematical rules and examples given in the verses of the BG and the solutions of the examples given in the prose parts. I express them in modern algebraic notation for easy reference and apprehension.

III.1.1 Chapter 1: Six kinds of operations on positive and negative numbers

1: Salutation.

Bhāskara salutes to (1) avyakta (the invisible), which is the sole primary seed of all the visible material world conceived by the Sāṃkhya philosophy, (2) īśa (the Lord), who is the unique supreme seed of all that is visible, and (3) avyakta-gaṇita (the invisible mathematics or the mathematics with unknown numbers), which is the seed of the vyakta-gaṇita (the visible mathematics or the mathematics with known numbers).

2: Introduction.

'Since the visible ⟨mathematics⟩ (i.e., mathematics with known numbers) told before ⟨by me in the *Līlāvatī*⟩ has the invisible ⟨mathematics⟩ as its seed, and since, without the reasoning of the invisible ⟨mathematics⟩, problems can hardly be understood (i.e., solved) ⟨even by intelligent persons and⟩ not at all by less-intelligent persons, I speak about the operations with seeds.'

Cf. Q2, 73, 94, and Q15

3ab: Addition.

Let a and b be positive numbers. Then

$$a + b = a + b, \quad \dot{a} + \dot{b} = \overset{\bullet}{\widehat{a+b}},$$

$$a + \dot{b} = \begin{cases} a - b & (b < a) \\ \overset{\bullet}{\widehat{b-a}} & (a < b) \end{cases}, \quad \dot{a} + b = \begin{cases} \overset{\bullet}{\widehat{a-b}} & (b < a) \\ b - a & (a < b). \end{cases}$$

E1:

1. $\dot{3} + \dot{4} = \dot{7}$. 2. $3 + 4 = 7$. 3. $3 + \dot{4} = \dot{1}$. 4. $\dot{3} + 4 = 1$.

3cd: Subtraction.

Let a and b be positive numbers. Then

$$a - b = a + \dot{b}, \quad \dot{a} - \dot{b} = \dot{a} + b, \quad a - \dot{b} = a + b, \quad \dot{a} - b = \dot{a} + \dot{b}.$$

To these the rule for the sum (3ab) is applied.

E2ab:
1. $3 - 2 = 1$. 2. $\dot{3} - \dot{2} = \dot{1}$. 3. $3 - \dot{2} = 5$. 4. $\dot{3} - 2 = \dot{5}$.

4a: Multiplication.
Let a and b be positive numbers. Then

$$a \times b = ab, \quad \dot{a} \times \dot{b} = ab, \quad a \times \dot{b} = \dot{\widehat{ab}}, \quad \dot{a} \times b = \dot{\widehat{ab}}.$$

E2cd:
1. $2 \times 3 = 6$. 2. $\dot{2} \times \dot{3} = 6$. 3. $2 \times \dot{3} = \dot{6}$. 4. $\dot{2} \times 3 = \dot{6}$.

4b: Division.
Let a and b be positive numbers. Then

$$a \div b = a/b, \quad \dot{a} \div \dot{b} = a/b, \quad a \div \dot{b} = \dot{\widehat{a/b}}, \quad \dot{a} \div b = \dot{\widehat{a/b}}.$$

E3:
1. $8 \div 4 = 2$. 2. $\dot{8} \div \dot{4} = 2$. 3. $\dot{8} \div 4 = \dot{2}$. 4. $8 \div \dot{4} = \dot{2}$.

4cd: Square and square-root.
Let a be a positive number. Then

$$a^2 = a^2, \qquad (\dot{a})^2 = a^2,$$

$$\text{square-roots of } a = \sqrt{a} \text{ and } \dot{\sqrt{a}},$$

$$\dot{a} \text{ does not have a square-root.}$$

E4ab: Square.
1. $3^2 = 9$. 2. $\left(\dot{3}\right)^2 = 9$.

E4cd: Square-root.
1. Square-roots of $9 = 3$ and $\dot{3}$. 2. $\dot{9}$ does not have a square-root.

III.1.2 Chapter 2: Six kinds of operations on zero

5ab: Addition and subtraction.
Let a be a positive number. Then

$$a + 0 = 0 + a = a, \quad \dot{a} + 0 = 0 + \dot{a} = \dot{a}, \quad a - 0 = a, \quad \dot{a} - 0 = \dot{a},$$

$$0 - a = \dot{a}, \quad 0 - \dot{a} = a.$$

E5ab:

1. $3 + 0 = 0 + 3 = 3$. 2. $\dot{3} + 0 = 0 + \dot{3} = \dot{3}$. 3. $0 + 0 = 0$. 4. $0 - 3 = \dot{3}$. 5. $0 - \dot{3} = 3$. 6. $0 - 0 = 0$.

5cd: Multiplication, division, square, square-root.

Let a be a positive or a negative number. Then

$$0 \times a = a \times 0 = 0,$$
$$0 \div a = 0, \quad a \div 0 = \frac{a}{0} \text{ (zero-divisor)},$$
$$0^2 = 0, \quad \sqrt{0} = 0.$$

E5cd:

1. $0 \times 2 = 0$. 2. $0 \div 3 = 0$. 3. $3 \div 0 = \frac{3}{0}$ (zero-divisor). 4. $0^2 = 0$. 5. $\sqrt{0} = 0$.

6: Zero-divisor and God: a simile.

Bhāskara compares the zero-divisor to God (Viṣṇu) for its infinite (ananta) and permanent (acyuta) nature: $\frac{a}{0} \pm b = \frac{a}{0}$. For the story on which this simile is based, see *Mahābhārata*, Poona ed., 6.30.17–19 and 6.31.7–8.

III.1.3 Chapter 3: Six kinds of operations on unknown numbers

Section 3.1: Six kinds of operations on an unknown number

7: Names of the values of unknown numbers.

The relative adverb yāvattāvat and the color names kālaka, nīlaka, pīta, lohita, etc. are employed for designating unknown numbers.

In BG 68p1, Bhāskara refers to additional color names and the consonants, ka etc., to be used for the same purpose. In BG E44p, he says that intelligent persons (matimat) may use the initial letters of the names of the things whose quantities are to be known.

In this appendix, I use x, y, z, etc. for the original unknown numbers of a particular problem and s_1, s_2, s_3, etc. for yāvattāvat, kālaka, nīlaka, etc. If the problem involves only one unknown number, then I use x for the original unknown number and s for yāvattāvat.

8ab: Addition and subtraction.

Of two names (letters) of the same kind (samāna-jāti), the sum or the difference is taken. Two names of different kinds (vibhinna-jāti) stand separately.

For the sake of convenience, I hereafter write '$-a$' in place of $\dot{a}$: for example, '$(x + 1) + (2x - 8) = 3s - 7$' in place of '$(x + 1) + (2x + \dot{8}) = 3s + \dot{7}$' (E6.1), and '$2x - (-6x + 8) = 8s - 8$' in place of '$2x - (\dot{6}x + 8) = 8s + \dot{8}$' (E7cd).

E6: Addition.

1. $(x+1)+(2x-8) = 3s-7$. 2. $(-x-1)+(2x-8) = s-9$. 3. $(x+1)+(-2x+8) = -s+9$. 4. $(-x-1)+(-2x+8) = -3s+7$.

E7ab: Addition.

$(3x^2+3)+(-2x) = 3s^2-2s+3$.

E7cd: Subtraction.

$2x-(-6x+8) = 8s-8$.

8cd–9: Multiplication, division, square, square-root.

A color × rūpas = the color; product of the same color = power (square, cube, etc.) of the color; product of different colors = bhāvita ('what is produced'); apply the rules given in the L to the rest (division, square, square-root).

10: Multiplication by parts (khaṇḍa-guṇanā).

The same algorithm that is given in L 14cd. This algorithm can be used also for the square of unknown numbers and for the multiplication and the square of karaṇīs.

E8: Multiplication.

1. $(5s-1)\times(3s+2) = 15s^2+7s-2$. 2. $(-5s+1)\times(3s+2) = -15s^2-7s+2$. 3. $(5s-1)\times(-3s-2) = -15s^2-7s+2$. 4. $(-5s+1)\times(-3s-2) = 15s^2+7s-2$.

11: Division.

The same algorithm that is given in L 18ab but worded for the division involving colors (unknown numbers).

Examples in 11p1 (the reverse of E8): 1. $(15s^2+7s-2)\div(3s+2) = (5s-1)$. 2. $(-15s^2-7s+2)\div(3s+2) = (-5s+1)$. 3. $(-15s^2-7s+2)\div(-3s-2) = (5s-1)$. 4. $(15s^2+7s-2)\div(-3s-2) = (-5s+1)$.

E8ef: Square.

$(4s-6)^2 = 16s^2-48s+36$.

12: Square-root.

Take the square-roots of the squares of colors contained in the given quantity and subtract twice the product of every twos of those roots from the rest. If the given quantity contains a rūpa, take its square-root first.

Example in 12p1 (the reverse of E8ef): the square-root of $(16s^2-48s+36) = 4s-6$.

Kṛṣṇa refers to the other root, yā $\dot{4}$ rū 6 or $(-4s+6)$, and Colebrooke includes it in his translation but AMG do not mention it. Of course, Bhāskara knew the existence of two roots of a positive number (cf. BG 4c, 21 and E14cd) but his intention here is to obtain the very root from which the square is obtained in the previous problem (E8ef). See the first sentence of 12p1. For a similar case see E10p5. He attaches