SCIAMVS 7 (2006), 81–144

# A preliminary study in the critical assessment of diagrams in Greek mathematical works

Ken Saito

*Department of Human Sciences*
*Osaka Prefecture University*

## I Introduction: critical text vs. uncritical diagrams in modern editions

We read Greek mathematical works in critical editions, or in modern translations of these. The editor provides a text reconstructed from the primary manuscripts through established procedures of textual criticism, and we can examine the editor's individual decisions by consulting the critical apparatus.

The text of a mathematical work is almost always accompanied by diagrams. It is quite certain that these existed in the original, because it is often impossible to understand the text without them. However, the figures accompanying the text in the editions we read, have not been critically assessed using procedures similar to those applied to the text. Indeed, the figures in modern editions are often different from any found in the extant manuscripts, and sometimes the editors seem to have invented new figures on the basis of ideas of mathematical consistency and generality.[1]

Critical examination of the diagrams is therefore necessary. Such an undertaking requires a comparison of the diagrams in a number of manuscripts, but this is not an easy task. Some diagrams are hard to see because of the poor condition of the manuscript; lines and labels are not always readable. Hence, one should first transcribe the figures, just as one transcribes the text when producing an edition.

This article reports a preliminary study of the diagrams using a simple computer program. We discuss the general characteristics of the figures, and then make a case study of the particularly complicated situation found for proposition III.25 of the

[1]The diagrams have not been altogether ignored by historians. Ver Eecke, in his translation of the *Spherics* of Theodosius (1927), sometimes refers to the drawings in the manuscripts or in previous editions (propositions II.10, 19, 20). However, his aim was to give more general, mathematically correct figures.

Jones, in his edition of Book VII of Pappus (1986), gave an apparatus to the diagrams (2:620–627), an attempt which should be noted as a forerunner to the more recent studies of diagrams (such as [De Young 2005] and [Keller 2005]), for which the influence of [Netz 1999] has been decisive.

*Elements*. Appendix 1 describes the program we used to transcribe the figures (the program is distributed gratis), and Appendix 2 gives example transcriptions: the figures of all 48 propositions of Book I of the *Elements*, in six principal manuscripts (four in Greek, two in Latin).

# II General characteristics of the diagrams in manuscripts

## II.1 Overspecification

When we look at the diagrams in manuscripts, the first thing we realize is that they are much less general than those printed in modern editions. Where modern editors shows us parallelograms, in the manuscripts we find rectangles, even squares (e.g., Appendix 2, prop. I.35, 36, 42–45). For a proposition treating any triangle, modern editions give a scalene, general triangle, while we often encounter an isosceles or right triangle in manuscripts (e.g., Appendix 2, prop. I.4, 8, 17–21, 25, 26, 47).

Let us call this phenomena in the manuscript diagrams *overspecification*. A question that immediately comes to mind is whether the overspecification originates with the ancient authors, or is the result of simplification and modification on the part of medieval scribes. Although it is difficult to give a definitive answer to this question, we are inclined to think that overspecification had its origin in antiquity. There are two principal reasons for this position. Firstly, this phenomenon can be found in almost every proposition and it is unlikely that independent modification of the diagrams occurred on such a large scale. Secondly, as we will see below, some figures in the manuscripts are simply incorrect as an accurate metrical representation of the geometric objects, although they are nonetheless capable of representing the geometric situation at issue. These "incorrect" drawings suggest that diagrams are not meant to be a strict reproduction of the spatial relationships of geometric objects along the lines of a photograph, but are rather meant to be a schematic representation. Overspecification can also be understood as a feature of schematic representation.

## II.2 Incorrect diagrams

Sometimes the diagrams in the manuscripts are simply wrong and yet, nevertheless, serve quite well to represent the configuration of the geometric objects treated in the propositions that they accompany.

A conspicuous example was pointed out by Reviel Netz: in Archimedes' *Sphere and Cylinder*, the figure of a dodecagon inscribed in a circle appears several times. The manuscript diagrams represent the sides of the dodecagon not by straight lines but by concave curves, so that they can easily be distinguished from the circumfer-