

Book XVI: A Mediaeval Arabic Addendum To Euclid's *Elements*

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I Introduction

Euclid's *Elements* has been transmitted into many languages since it was first composed in Greek and has provoked extensive discussions in many branches of this transmission. The Arabic transmission was no exception. Once it became available in Arabic during the eighth and ninth centuries, Euclid's mathematical classic was "corrected", summarized, given addenda, reduced to extracts, commented upon in whole and in part, given alternate demonstrations, paraphrased, and much more. These discussions were not limited to any specific time period, but continued to appear regularly until the nineteenth and twentieth centuries.

This paper presents a hitherto unstudied addendum to an edited version of a popular Euclidean treatise. The addendum, entitled Book XVI, comprises nineteen propositions describing techniques for constructing polyhedra within other polyhedra or within spheres. Thus it develops themes related to those of book XV, itself a late Greek addendum to the genuine Euclidean treatise. The contents are, however, in the tradition of Archimedes rather than Euclid. The first part of Book XVI describes construction of semiregular truncated polyhedra inscribed within regular polyhedra (Platonic solids) and within spheres. The addendum ends with what may be the earliest discussion of the construction of a representative example from each of the classes of semiregular polyhedra known today as prisms and antiprisms. Thus, although the editor or copyist who attached this book XVI to the manuscript clearly intended this material as a kind of continuation of the *Elements*, the intention of the original author is less clearly Euclidean.

II Manuscript

Book XVI is appended to a medieval Arabic manuscript the bulk of which is an edited version of the well-known and influential redaction *Tahrīr Kitāb Uqlīdis* by Naṣīr al-Dīn al-Ṭūsī (597 AH/AD 1201 - 672 AH/AD 1274). This Book XVI is at present known only in this unique copy whose colophon gives the date of copying as