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A Manuscript of Euclid's *De Speculis*: A Latin Text of MS 98.22 of the Archivo y Biblioteca Capitulares de la Catedral, Toledo

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Introduction

De speculis is a Latin translation of Euclid's *Catoptrica* which was translated into Latin probably in Sicily in the mid-twelfth century. The large number of extant manuscripts of *De speculis*, of which at least 52 MSS are known, indicates its wide circulation in the Middle Ages.[1]

I published in 1992 a book which contains three different versions of Euclid's *De speculis*: the original Latin translation made from Greek text (Text I, 18 MSS) and (2) the paraphrased Latin versions made from Text I (Text II, 5 MSS and Text III, 1 MS).[2] During my stay at University of Wisconsin-Madison in 1996–97, I have had an opportunity to examine seven additional manuscripts which were new to me and to reconsider *De speculis*' textual tradition.[3]

Of seven additional manuscripts, the Toledo manuscript which is reproduced here with English translation on facing pages deserves special attention in some important points. Before turning to the points, however, it is beneficial to make some short comments on the Greek original itself. Scholars have doubted the authenticity of the *Catoptrica*. However, recent work by Knorr, Jones, and myself verifies its authenticity.[4] The work seemed to have a lot of false assertions and deficiencies

[1] In the number of extant manuscripts, *De speculis* is only second to the most popular textbook on optics, Pecham's *Perspectiva Communis*. As for the manuscript information, see David C. Lindberg, *A Catalogue of Medieval and Renaissance Optical Manuscripts*, Toronto, 1975.

[2] Ken'ichi Takahashi, *The Medieval Latin Traditions of Euclid's Catoptrica: A Critical Edition of* De speculis *with an Introduction, English Translation and Commentary*, Kyushu University Press, 1992.

[3] I would like to express my gratitude to Prof. Lindberg, University of Wisconsin-Madison, USA for providing me with a number of microfilm copies of *De speculis*.

[4] Wilbur R. Knorr, "Archimedes and the Pseudo-Euclidean *Catoptrics*: Early Stages in the Ancient Geometric Theory of Mirrors," in *Archives internationales d'histoire des sciences*, vol. 35 (1985), pp. 28–105; Alexander Jones, "On Some Borrowed and Misunderstood Problems in Greek Catoptrics," in *Centaurus*, vol. 30 (1987), pp. 1–17; Ken'ichi Takahashi, *op. cit.*, pp. 13–33.

and lacked the mathematical rigor, unworthy of the author of the masterpiece of Greek mathematics, entitled the *Elements*. To mention just one example among many for the sake of my argument below, Prop. 7 seemed to assert or presume the same contents as Prop. 16 which states the location of images in plane mirrors. In other words, the preceding proposition seemed to be dependent on the subsequent proposition. This is an example that allegedly lacks mathematical rigor. However, closer scrutiny of the theoretical contents of these as well as others, shows that the so-called false assertions, deficiencies, and lack of mathematical rigor resulted from their misunderstanding. The rehabilitation of Euclidean authenticity itself is an interesting story, but this topic is regrettably beyond the scope of the present article. In any case, it is worth noting that the Greek original seemed to be far from Euclidean perfection as exemplified in his *Elements*. Its lack of sophistication encouraged some intelligent medieval readers to expand the text with their own comments.

Our Toledo manuscript has a text that belongs to the tradition of Latin paraphrased versions. These versions have in common the tendency to expand the original text in various fashions which reflect the "didactic concerns" of medieval scholars.[5] In other words, the versions show a typical pattern of the modification that a certain medieval text undergoes when transmitted from one generation into another. I would like to put beforehand these concerns in seven items which are as follows.

(1) Attempt to reorganize *De speculis* (Replacement of Props. 16–18 before Prop. 7)
(2) Additional comments to the postulates
(3) Rewriting of the original proofs and sometimes of the enunciations
(4) Formalization of proof steps
(5) Citations of theorems in the *Elements* and other sources
(6) Proliferation of the cases in question
(7) Criticism of the original proofs

Our manuscript shares item (1) with Text III. It brings propositions 16–18, which are so numbered in the Greek original and therefore also in the Text I, immediately before proposition 7. The former propositions teach how to determine the location of images produced by the plane, the spherical concave and spherical convex mirrors, while the latter proposition seemed to assert or presume the same contents as Prop. 16 as said before. Even though the interpretation that Prop. 7 seemed to presuppose Prop. 16 was nothing but a misunderstanding from the historical point of

[5]I have borrowed the pertinent expression "didactic concerns" from Prof. John E. Murdoch's paper, "The Medieval Euclid: Salient Aspects of the Translation of *Elements* by Adelard of Bath and Campanus of Novara," in *Revue de synthèse*, vol. 89 (1968), pp. 67–94.

view, the rearrangement of the propositions nonetheless shows some kind of logical acuteness on the part of the scholars who wrote Text III and our manuscript.

Item (2) is found in Text III, but not in Text II. In this respect, our manuscript follows the pattern of the former. Although they are common in putting comments on the postulates 1–4, their comments are different in terms of contents.

As stated before, the paraphrased Latin versions (Texts II and III) are different from Text I. The difference is found only in the proofs proper, not in the postulates and the enunciations of the propositions except for minor variants and omissions. While our manuscript, as well as Texts II and III, rewrites the original proofs, its new proofs noticeably differ from those of Texts II and III, as can be easily recognized if one compares these three texts. And there is more to it. Although our manuscript belongs to the paraphrased versions, it is a rare exception because it sometimes rewrites the enunciations of propositions into more natural Latin: noticeably in Props. III, IV, XIII, XVI, XVII, XVIII, XIX, XXII, XXIV, XXVI and XXVIII. This state of affairs is naturally expected to happen because of the fact that the original Latin enunciations in Texts I, II and III are at times clumsy due to the characteristic word-for-word translation technique of the twelfth century. The following examples of rewriting will suffice to show the fact:

Proposition XXII

[Text I] In convexis speculis a minoribus speculis minora apparent ydola.

[Our MS] A minoribus convexis speculis apparent ydola minora.

Proposition XXIV

[Text I] In concavis speculis si super centrum oculus ponatur, ipse tantum oculus apparet.

[Our MS] Si super centrum ponatur oculus in concavis speculis, nichil videtur nisi oculus.

As an illustration of items (4) and (5) let us first cite Proposition II. I add underlines to show them up with corresponding numbers.

[II] Qualicumque speculo visus incidit equales faciens angulos, is per se ipsum reflectetur.

(4) Verbi gratia: (4) si quis hoc asserat falsum esse, tunc AB posito speculo [Figure 2a], ab oculo qui est E radius EC ad speculum descendat equales faciens angulos in punctum C, non per se reflectetur ut dicit proterviens. Ergo per alium reflectetur quam sit EC. Reflectatur ergo versus G. (4) Inde sic: EC et CG sunt radii reflexionum, ergo anguli ECA et BCG sunt equales (5) per primum huius. Sed anguli ECA et ECB sunt equales (5) ex ypothesi. Ergo dempto ECA communiter accepto ECB angulus [est]

> equalis angulo BCG, equalis[*del.*] quod est [inpossibile (5) per] premissa G[eometrie].
>
> (4) Amplius in convexo: sit ABC convexum speculum [Figure 2b], et oculus E, a quo radius EB ad speculum ABC inmissus in B punctum equos angulos, scilicet ABE et CBE, efficiat. Dico ergo quod radius EB reflectitur in se. (4) Quod si falsum esse quis asserat, reflectatur ad punctum G. Intellecto igitur DBF plano speculo ad punctum B contingento ABC, (4) sic arguere: EB et BG sunt radii reflectionum, ergo ABE et CBG anguli sunt equales (5) per primum huius. Sed ABE et CBE sunt equales (5) ex ypothesy. Ergo dempto ABE communiter hinc inde accepto erit CBG equalis angulo CBE. Quod est inpossibile (5) per premissa ad Geometriam.
>
> (4) Itemque in concavo: sit concavum speculum ABG [Figure 2c], oculus qui est E, a quo ad ABG speculum in punctum B radius EB mittatur, in B faciens angulos equales ABE et GBE. Dico igitur quod radius EB reflectitur per se. (4) Quod si quis asserat esse falsum, tunc reflectatur ad punctum C per B. (4) Inde sic: EB et BC sunt radii reflexionum. Ergo anguli ABE et CBG sunt equales (5) per primum huius. Sed ABE et GBE anguli sunt equales (5) ex ypothesy. Ergo dempto ABE communiter hinc inde accepto. erit EBG equalis EBC [*lege* CBG], quod inpossibile relinquatur (5) per premissa ad Geometriam.

The author of our manuscript always begins proofs with "*verbi gratia.*" Then he uses "*amplius,*" "*itemque,*" and "*item*" as an indicator of a transition to the next step of a proof. He frequently uses "*inde sic*" in almost every proposition in order to introduce a proof of a particular assertion made immediately before. Of the same kind are "*sic arguere,*" "*quod sic probatur* [or *probetur*]" (Props. I, VI, XVIII, XXII, XXVI, XVIII and XXIX), "*quod sic arguas*" (Prop. V), "*sic astruere* [or *astrue* or *astrues*]" (Props. XII and XIX), "*probantur ... hoc modo*" (Prop. XVII), "*sic*" (Prop. XVIII), "*sic procede*" (Prop. XX), and "*astruendum sic*" (Prop. XXXI). An alternative proof is indicated by "*vel sic levius*" (Prop. V). The parts of proofs are often indicated in the text by the following phrases: "*Propositi autem hec est pars prima*" (Prop. XIV), "*hec est prima* [or *secunda*] *pars propositi*" (Props. XV and XIX), "*Restat igitur secundam partem astuere*" (Prop. XVIII), "*que prima pars est propositi*" (Prop. XXIX), "*Et hoc est propositorum secundum*" (Prop. XXIX), and "*Tertium vero sic constabit*" (Prop. XXIX). In some propositions, the typical Euclidean QEDs are added in the following manners: "*hoc erat propositum*" (Prop. VI), "*Et hoc erat propositum ostendere*" (Props. XI and XXI), and "*Et sic habetur propositum*" (Prop. XXV).

In Prop. II cited above, he uses the phrase "*si quis hoc asserat falsum esse*" and the like to introduce an indirect proof [*reductio ad absurdum*]. For the same purpose he also uses such phrases as "*eius contrarium si quis asserat*" (Prop. III), "*indirecte* *sic astruas*" (Prop. XX), "*dicet* [or *proponet* or *ponet*] *falsigrafus*" (Prop. XXII), "*Ad eius rei inprobationem*" (Prop. XXII) and "*et per inpossibile*" (Prop. XXIV). Indirect proofs end with the following phrases: "*quod obviat ypothesi*" (Prop. III), "*quod* [or *hoc*] *est inpossibile*" (Props. II, V, XXII and XXVII), "*sequitur inconveniens*" (Prop. VIII), "*sic contrarium propositi obtineres*" (Prop. XXII). In passing the term, "*falsigrafus,*" deserves our special notice, because it also appears in some Latin manuscripts of Archimedes's *De quadratura circuli* and *De curvis superficiebus* whose author was John of Tynemouth according to W. R. Knorr.[6]

As for item (5), the citation above shows the internal referencing [*per primum huius*], Euclidean references [*per premissa ad Geometriam*] and logical elucidation [*per hypothesy*]. In other cases, the internal references are introduced by such phrases as "*per precedens huius*" (Prop. VI), "*ex suppositione VII^{eme} propositionis huius*" (Prop. XI), "*per penultimam huius*" (Prop. XXI), "*de quo dictum est in penultima propositione*" (Prop. XXVII). Euclidean citations are simply given either by the book-number of the *Elements* such as "*per primum librum G[eometrie]*", "*ex primo G[eometrie],*" and so forth (*passim*) or by a technical term such as "*per contrapositionem*" (Props. VII and XXII). The other source materials are occasionally mentioned: Euclid's *Optics* [Props. XIV and XV], an anonymous [*Mathematical* ?] *Collections* [Prop. XVIII] and an anonymous *Book on chords and arcs* [Prop. XXVI].

Item (6), the proliferation of the cases in question, is found in propositions IV and XIX. In these propositions the different configurations not found in Greek original are considered, which are preceded by the phrases, "*Item alia dispositio ad idem*" and "*Item in alia dispositione*" in proposition IV and "*Item alia dispositio*" in proposition XIX. The appearance of this item exemplifies the logical exhaustiveness on the part of a medieval scholar.

Item (7) evidently shows a critical temper of our author. He criticizes Euclid's proof in proposition XIX as follows: "*Sed quod in puncto ab E[u]clide dicto sectionis, ..., nec intellexi nec intelligi.*" And he offers a new configuration to support his claim. A mention to a false argument added in proposition XVIII (*Item hoc argumentum falsum est*) is also a reflection of his critical temper. Of this kind, even though to a lesser degree, are the critical remarks preceded by such phrases as "*notandum tamen quod ...*" (Props. XII and XXVIII), "*nota quod ...*" (Props. XIV and XVI) and "*hoc intelligendum ...*" (Prop. XV).

Finally mention should be made to our author's distinctive expressions. First, his stock phrase for proportionality is "*que est proportio A ad B ea est C ad D*" (Props. XXIX and XXX), which is found in Text III (Comments to Postulate 3) though the word, "*proportio,*" is often omitted. On the other hand, we find in Text II such phrases as "*que est proportio A ad B eadem est C ad D*" (Props. XXIX and

[6] Wilbur R. Knorr, "Paraphrase Editions of Latin Mathematical Texts: *De figuris ysoperimetris,*" in *Medieval Studies*, vol. 52 (1990), pp. 132–189.

XXX) and "*proportionaliter erit sicut A ad B ita C ad D*" (Prop. XXVII), whereas we find in Text I such phrases as "*sicut A ad B ita C ad D*" (Postulate 3) and "*proportionaliter erit sicut A ad B ita C ad D*" (Prop. XXVII) as a result of a faithful translation from Greek.

Second, we find a phrase "*esche bien seche* (a very dry food)" in Proposition XXXII. At least to my knowledge, the appearance of French words in the midst of the Latin text is quite exceptional. This may tell us something of the provenance of our author. However, the paucity of evidence does not permit us to say something definite. I report only this fact for the future study of the identification of our author. Third and lastly, he alludes to his own experience in the same proposition: "*Huius autem consimile probavi per lapidem qui dicitur ...*" This may also be an illustration of his effort to show his originality.

In sum, our Toledo manuscript is a uniquely paraphrased Latin version. In a sense it can be characterized as a hybrid version of Texts I and III. This is the reason why it deserves a separate publication here.

Editorial Procedures

In establishing the text, I followed the same procedure as done in my book.

The capitalization, punctuation and paragraphing are all mine. Capitalization is also employed to represent geometrical magnitudes used in both the figures and the text, though our manuscript has them in small letters.

Square brackets [ ] have been employed in the text and the translation to enclose my editorial insertions. The difference of proposition number between our text and the Heiberg's Greek text (= Text I) is indicated, for instance, as [VII(H16)] which means that Proposition VII of our text is the same as Proposition 16 of Heiberg's edition. A question mark [?] draws attention to a doubtful reading, and an exclamation mark [!] has been used as equivalent for *sic.* In the translation, Euclidean citations are simply put, for example, as *Elm*-I which means Book One of the *Elements.* Illegible parts of the manuscript are indicated by either [*illeg.*] or [...].

The following abbreviations and Latin terms have been employed.

add. (*addidit*)	has added
addidi	I have added
del. (*delevit*)	has deleted
hab. (*habet*)	has
iter. (*iteravit*)	has repeated
lac. (*lacuna*)	blank
lege	read

Text and Translation of

De Speculis

Based on

Toledo, Archivo y Biblioteca Capitulares de la Catedral MS 98.22,

fols. 87v–89v,

13th–14th century

De sepeculis

[PETITIONES]

[1] Visum rectum esse cuius media terminos recte continuant, id est, cuius partes intermedie directe terminos continuant. Recte dicit ad differentiam radii reflexi.

[2] Visu omnia recta videri. Id est recte et hoc quantum ad apparentiam non quantum ad rei veritatem.

[3] Speculo posito in plano visaque altitudine aliqua, que perpendicularis sit illi plano, fient proportionaliter, sicut que inter speculum videntem recta ad speculi et altitudinis intermediam rectam ita videntis altitudo ad perpendicularem plano altitudinem. Evidens est hec petitio et est practica ad quamlibet altitudinem mensurandum.

[4] In planis speculis loco occupato, in quem cathetus cadit a conspecto, non res conspecta iam videtur. Et e contrario loco non occupato in quem cathetus cadit etc. Id est si cadat radius visualis in punctum in quem cadit linea perpendicularis a re conspecta, eadem non videbitur, quod ex prima propositione liquefiet. Item si radius perpendicularis a re conspecta cadat in speculum et iterum ab oculo, quia in infinitum protracti non concurrerunt, res conspecta non videbitur.

[5] In convexis speculis comprehenso loco per quem a conspecto in centrum spere ducitur cathetus, non iam videbitur quod conspectum est. Idem accidit in concavis.

[6] Si in ad vas mitatur quid sumaturque distantia ut non iam videatur, eodem spatio existente si aqua infundatur, videbitur quod inmissum est.

[PROPOSITIONES]

[I] In planis convexisque et concavis speculis, visus in angulis equalibus revertetur.

Verbi gratia: sit speculum AB planum [Figure 1a], altitudo videntis AE, res visa D sitque tam AE quam BD perpendicularis ad AB speculum vel sperem consuperficialem AB. Inde sic: que est proportio AG ad GB eadem est AE ad BD per tertiam huius petitionem et angulus EAG equus angulo GBD per petitionem

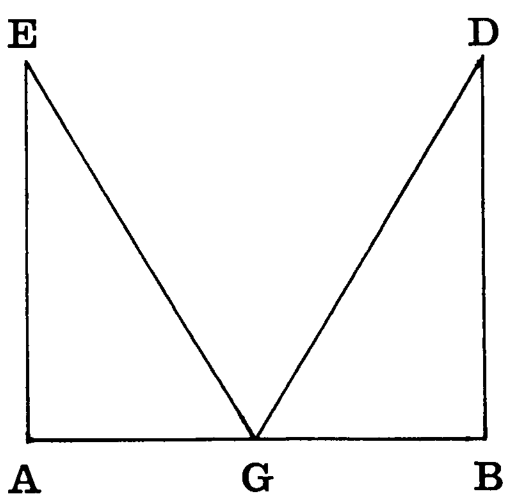

Figure 1a

ON MIRRORS

[Postulates]

[1] A visual ray is straight, whose intermediaries are on a line straight with the extremities, that is, whose intermediary parts are in a straight line continuous with the extremities. He [Euclid] says "*recte* (on a line straight with)" in distinction from the reflected visual ray.

[2] By vision all objects of sight are seen direct[ly]. That is "*recte* (directly, or rectilinearly)," and this is true as long as appearance is concerned, but not as long as the actual truth is concerned.

[3] When a mirror is placed on a plane and some height is seen which is perpendicular to the plane, there arises a proportion such that as the line between the mirror [and] an observer is to the intermediary line between the mirror and the height, so is the height of the observer to the height perpendicular to the plane. This postulate is evident and is useful for measuring heights whatever.

[4] In plane mirrors, if the place is occupied on which the perpendicular falls from an object of sight, the object of sight is no longer seen. And on the contrary, if the place is not occupied on which the perpendicular falls *etc.* That is, if a visual ray falls on the point on which a perpendicular line falls from an object of sight, the object will not be seen, which is manifest from the first proposition. Moreover, if perpendicular rays fall on the mirror from an object of sight and also from an eye, the object of sight will not be seen, because they do not converge even if extended *in infinitum.*

[5] In convex mirrors, if the place is covered through which a perpendicular is drawn from an object to the center of the sphere, the object that was being seen will no longer be seen. The same thing happens in concave mirrors.

[6] If something is placed into a vessel and a distance is so taken that it may no longer be seen, with the distance held constant if water is poured, the thing that has been placed will be seen [again].

[Propositions]

[I] IN PLANE, CONVEX AND CONCAVE MIRRORS, A VISUAL RAY WILL BE REFLECTED AT EQUAL ANGLES.

For example: Let AB be a plane mirror [Figure 1a], AE be a height of an observer, and an object of sight be D and let both AE and BD be perpendicular to the mirror AB or to the sphere whose surface is common with AB. Then as follows: AG : GB = AE : BD by the third postulate of this book and $\angle$EAG = $\angle$GBD by

V G[eometrie]. Ergo trianguli EAG et GBD sunt similes per VI G[eometrie]. Anguli respicientes proportionalia latera sunt equales ex eodem. Ergo anguli EGA et DGB adinvicem equantur. Sed illi idem sunt anguli reflexionum. Ergo anguli reflexionum sunt equales in plano speculo.

Amplius in convexis: sit convexum speculum ABC [Figure 1b], oculus F, visum E. Dico ergo quod angulus EAB et angulus CAF, anguli scilicet reflexionum, sunt equales ad rei evidentiam. Intelligatur planum speculum GD scilicet contingens CAB convexum speculum ubi incidit radius visualis ab F ad GD. Verum sic probatur: angulus GAF est equalis angulo EAD ex premisso. Sed et angulus CAG equus est angulo BAD ex tertio G[eometrie]. Et in primo ergo contractim per V[*sic*] G[eometrie] angulus GAF et CAG sunt equales angulis EAD et BAD. Sed anguli GAF et CAG constituunt angulum CAF. Item anguli EAD et BAD constituunt angulum EAB. Ergo anguli CAF et EAB sunt equales. Et hoc est propositum.

Item in concavis: sit concavum ABC [Figure 1c], oculus G, res visa E. Dico igitur quod anguli reflexionum scilicet [*lege* sunt] ABE et CBG. Quod sic probatur. Posito speculo plano DBF simliter, anguli EBD et GBF equales sunt ex precedenti huius, CBF vero et ABD equales ex tertio G[eometrie], ergo ABE et CBG sunt equales per premissa G[eometrie].

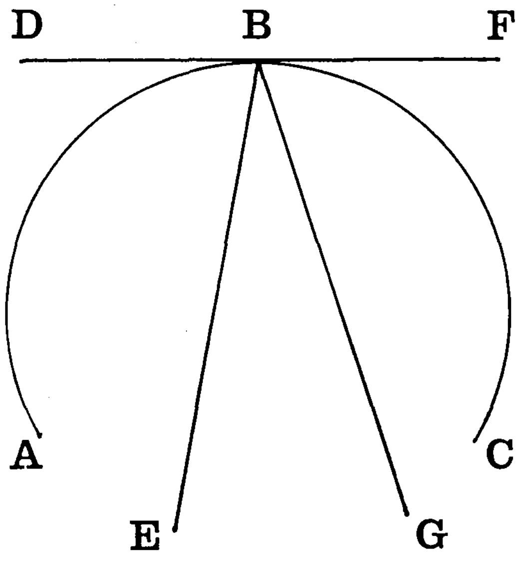

Figure 1c

[II] Qualicumque speculo visus incidit equales faciens angulos, is per se ipsum reflectetur.

Verbi gratia: si quis hoc asserat falsum esse, tunc AB posito speculo [Figure 2a], ab oculo qui est E radius EC ad speculum descendat equales faciens angulos in punctum C, non per se reflectetur ut dicit proterviens. Ergo per alium reflectetur quam sit EC. Reflectatur ergo versus G. Inde sic: EC et CG sunt radii reflexionum, *Elm*-V-postulate. Therefore △EAG ∼ GBD by *Elm*-VI. The angles subtending proportional sides are equal by the same. Therefore ∠EGA = ∠DGB. And they are the very angles of reflection. Therefore the angles of reflection are equal in a plane mirror.

Furthermore in the case of convex [mirrors]: let a convex mirror be ABC [Figure 1b], an eye be F, and an object of sight be E. Then I say that ∠EAB and ∠CAF, *scilicet* the angles of reflection, are evidently equal. Let us think of a plane mirror GD which is tangent to the convex mirror CAB where a visual ray is incident on GD from F. The truth is proved as follows: ∠GAF = ∠EAD by the preceding [argument]. But ∠CAG = ∠BAD by *Elm*-III-[16]. Therefore first by addition of *Elm*-V [*sic*], ∠CAF+∠CAG = ∠EAD+∠BAD. But ∠GAF and ∠CAG constitute ∠CAF, while ∠EAD and ∠BAD constitute ∠EAB. Therefore ∠CAF = ∠EAB. And this is what was proposed.

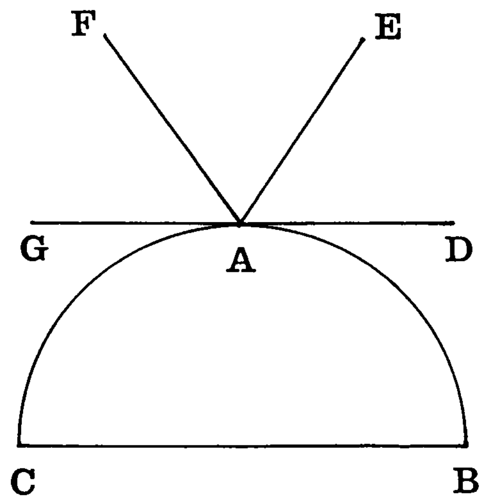

Figure 1b

Moreover in the case of concave [mirrors]: let a concave mirror be ABC [Figure 1c], an eye be G, and an object of sight be E. Then I say that the angles of reflection are ∠ABE and ∠CBG, which is proved as follows: Let a plane mirror be placed similarly [as before], [then] ∠EBD = ∠GBF by the preceding of this book; but ∠CBF = ∠ABD by *Elm*-III; therefore ∠ABE = ∠CBG by *Elm*-[I]-assumptions.

[II] IN ANY [TYPE OF] MIRRORS, WHEN A VISUAL RAY FALLS MAKING EQUAL ANGLES [*i.e.* PERPENDICULARLY], IT WILL BE REFLECTED ON ITSELF.

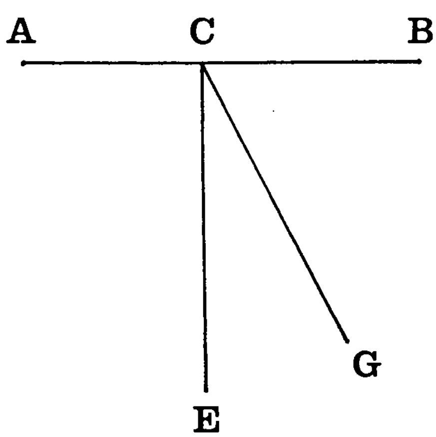

Figure 2a

For example: If someone asserts that this is false, then let AB a mirror [Figure 2a], and from the eye E let a ray EC descend to the mirror, making right angles at point C, [then] it would not be reflected on itself as the wanton person says. Therefore it would be reflected along a line other than EC. Therefore it would be reflected toward G. Then as follows: Because EC and CG are the rays of reflection, ergo anguli ECA et BCG sunt equales per primum huius. Sed anguli ECA et ECB sunt equales ex ypothesi. Ergo dempto ECA communiter accepto ECB angulus [est] equalis angulo BCG, equalis[*del.*] quod est [inpossibile per] premissa G[eometrie].

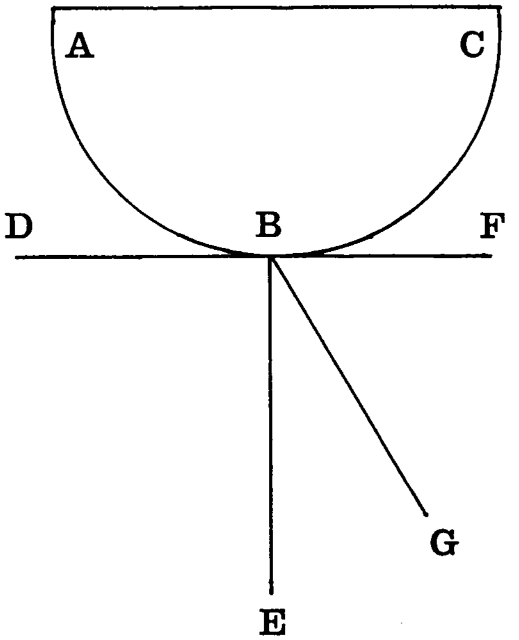

Figure 2b

Amplius in convexo: sit ABC convexum speculum [Figure 2b], et oculus E, a quo radius EB ad speculum ABC inmissus in B punctum equos angulos, scilicet ABE et CBE, efficiat. Dico ergo quod radius EB reflectitur in se. Quod si falsum esse quis asserat, reflectatur ad punctum G. Intellecto igitur DBF plano speculo ad punctum B contingento ABC, sic arguere: EB et BG sunt radii reflectionum, ergo ABE et CBG anguli sunt equales per primum huius. Sed ABE et CBE sunt equales ex ypothesy. Ergo dempto ABE communiter hinc inde accepto erit CBG equalis angulo CBE. Quod est inpossibile per premissa ad Geometriam.

Itemque in concavo: sit concavum speculum ABG [Figure 2c], oculus qui est E, a quo ad ABG speculum in punctum B radius EB mittatur, in B faciens angulos equales ABE et GBE. Dico igitur quod radius EB reflectitur per se. Quod si quis asserat esse falsum, tunc reflectatur ad punctum C per B. Inde sic: EB et BC sunt radii reflexionum. Ergo anguli ABE et CBG sunt equales per primum huius. Sed ABE et GBE anguli sunt equales ex ypothesy. Ergo dempto ABE communiter hinc inde accepto, erit EBG equalis EBC [*lege* CBG], quod inpossibile relinquatur per premissa ad Geometriam.

[III] Qualicumque incidens visus in speculum inequales faciens angulos is neque per se ipsum neque per minorem eorum reflectitur.

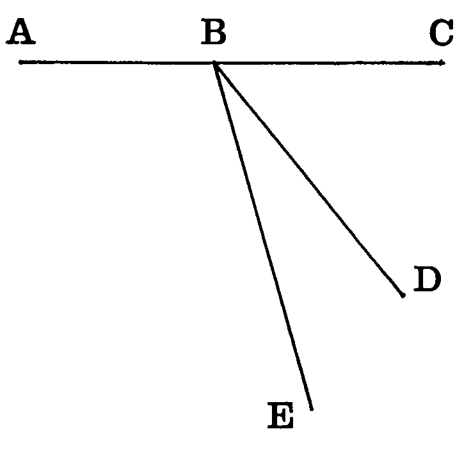

Figure 3a

Verbi gratia: sit planum speculum ABC [Figure 3a], oculus E, radius incidens ad ipsum in puncto B sit EB, et incidat ad inequales angulos scilicet ABE et CBE sitque CBE minor. Dico igitur quod EB neque per se ipsum reflectetur neque per CBE. Eius contrarium si quis asserat, reflectatur primo per se. Sed reflectitur ad equales angulos per primam huius. Ergo

∠ECA = ∠BCG by the first [proposition] of this book. But by hypothesis, ∠ECA = ∠ECB. Therefore by taking away the common angle ∠ECA, ∠ECB = ∠BCG, which is [impossible by] *Elm*-assumptions.

Furthermore in the case of a convex [mirror]: let ABC be a convex mirror [Figure 2b], an eye be E from which the ray EB, emitted to the mirror ABC on point B, forms equal angles, *scilicet* ∠ABE and ∠CBE. Then I say that the ray EB will be reflected on itself. Now if someone asserts this to be false, then let it be reflected to point G. Now let us think of a plane mirror DBF tangent to ABC at point B, [then] argue as follows: because EB and BG are angles of reflection, by the first [proposition] of this book ∠ABE = ∠CBG; but ∠ABE = ∠CBE by hypothesis; therefore by taking away the common ∠ABE at both equations, it would be that ∠CBG = ∠CBE. This is impossible by *Elm*-assumptions.

Moreover in the case of a concave [mirror]: let a concave mirror be ABG [Figure 2c], an eye be E from which let a ray EB be emitted to point B on the mirror ABG, making equal angles at B, ∠ABE and ∠GBE. Then I say that a ray EB will be reflected on itself. Now if someone asserts this to be false, then let it be reflected to C through B. Then as follows: because EB and BC are the rays of reflection, ∠ABE = ∠CBG by the first [proposition] of this book; but ∠ABE = ∠GBE by hypothesis; therefore by taking away the common ∠ABE at both equations, it would be that ∠EBG = ∠[CBG], which would be left impossible by *Elm*-assumptions.

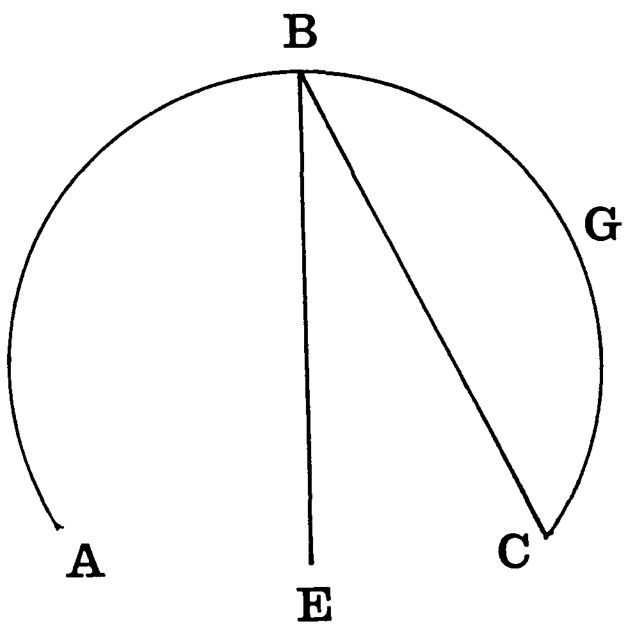

Figure 2c

[III] IN ANY [TYPE OF] MIRRORS, WHEN AN INCIDENT VISUAL RAY MAKES UNEQUAL ANGLES [*i.e.* FALLS OBLIQUELY], IT IS REFLECTED NEITHER ON ITSELF NOR ON [THE SIDE OF] THE SMALLER [ANGLE].

For example: Let a plane mirror be ABC [Figure 3a], an eye be E, and let EB be a ray incident on the mirror at point B, falling at unequal angles, *scilicet* ∠ABE and ∠CBE, ∠CBE being smaller. Then I say that EB will be reflected neither on itself nor on CBE. If someone asserts its contrary, first let it be reflected on itself. But it is reflected at equal angles by the first [proposition]of this book. Therefore it angulus ABE est equus angulo CBE, quod obviat ypothesy.

Sit item quod reflectatur per CBE minorem angulum scilicet ad punctum D, sic: EB et ED [*lege* BD] sunt radii reflexionum. Ergo anguli ABD [*lege* ABE] et EBD [*lege* CBD] sunt equales. Sed angulus CBE maior est angulo CBD per premissa ad G[eometriam]. Ergo CBE angulus maior est angulo ABE, quod obviat ypothesy.

Amplius in convexis: sit ABC convexum speculum [Figure 3b], oculus E a quo ad B inmittatur radius faciens angulos inequales scilicet ABE et CBE. Dico igitur quod EB radius neque per se neque per CBE reflectitur. Eius contrarium si quis asserat, reflectatur primo per se. Inde sic: EB reflectitur per se. Sed reflectitur et in equales angulos per primum huius. Ergo angulus ABE equus est angulo CBE, quod obviat ypothesy.

Sit item quod reflectatur per minorem ad punctum D, sic: EB et BD sunt radii reflexionum. Ergo anguli ABD [*lege* ABE] et CBD sunt equales per primum huius. Sed CBE maior est angulo CBD per premissa ad G[eometriam]. Ergo CBE maior est angulo ABE, quod obviat ypothesy.

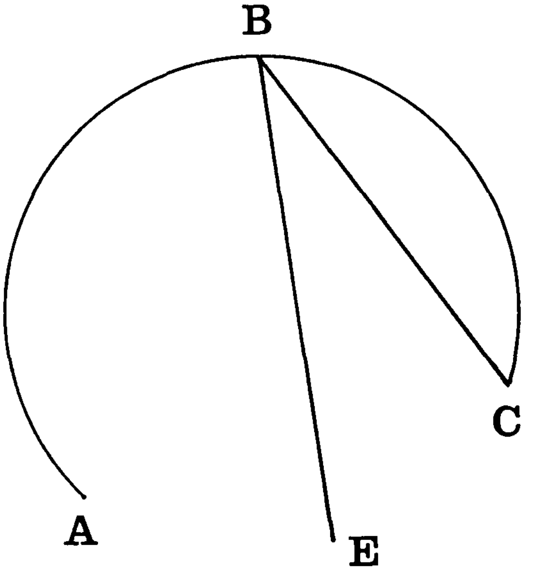

Figure 3c

Idem et eodem modo sequatur inconveniens in ABC concavo [Figure 3c], scilicet quod si EB reflectitur per se, angulus ABE angulo CBE, maior scilicet minori, contra ypothesim adequatur. Si vero per CB angulum ad C partem reflectatur, tunc CBE contra ypothesim maior fiet angulo ABE, quod item obviat ypothesy.

[IV] Visus a planis speculis et convexis repercussi non concidunt nec erunt paralelli.

Verbi gratia: sit planum speculum ABC [Figure 4a], oculus E, unus radius incidens EB, alius vero ED. Reflexi vero sint BF et DG. Dico igitur quod BF et DG would be that ∠ABE = ∠CBE, which is contrary to the hypothesis.

Similarly let it be that it would be reflected to the side of smaller angle, ∠CBE, *scilicet* to point D. [Argue] as follows: because EB and [BD] are the rays of reflection, ∠[ABE] = ∠[CBD]; but ∠CBE > ∠CBD by *Elm*-assumptions; therefore ∠CBE > ∠ABE, which is contrary to the hypothesis.

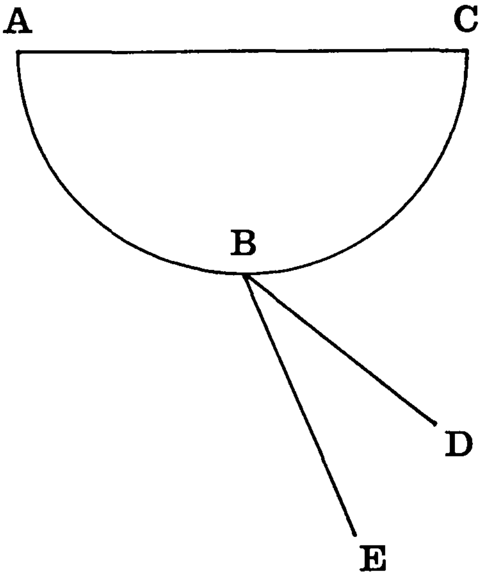

Figure 3b

Furthermore in the case of convex [mirrors]: Let ABC be a convex mirror [Figure 3b], and an eye be E from which let a ray be emitted to B, making unequal angles, ∠ABE and ∠CBE. Then I say that the ray EB is reflected neither on itself nor on CBE. If someone asserts its contrary, first let it be reflected on itself. Then [argue] as follows: EB is reflected on itself; but it is also reflected at equal angles by the first [proposition] of this book; therefore it would be that ∠ABE = ∠CBE, which is contrary to the hypothesis.

Similarly let it be that it would be reflected along the side of smaller angle to point D. [Argue] as follows: because EB and BD are the rays of reflection, ∠[ABE] = ∠CBD by the first [proposition] of this book; but ∠CBE > ∠CBD by *Elm*-assumptions; therefore ∠CBE > ∠ABE, which is contrary to the hypothesis.

In the case of a concave [mirror] ABC [Figure 3c], contradiction would follow in just the same manner, *scilicet* that if EB is reflected on itself, it would be that ∠ABE = ∠CBE, *i.e.* the larger would be equal to the smaller, contrary to the hypothesis. Moreover if it is reflected to the side of C along the angle CB[C], then contrary to the hypothesis it would be that ∠CBE > ∠ABE, which is again contrary to the hypothesis.

[IV] VISUAL RAYS REFLECTED BY PLANE AND CONVEX MIRRORS WILL BE NEITHER CONVERGENT NOR PARALLEL.

For example: Let there be a plane mirror ABC [Figure 4a], an eye E, and let an incident ray be EB and another be ED while let reflected [rays] be BF and DG. Then I say that BF and DG are nei-

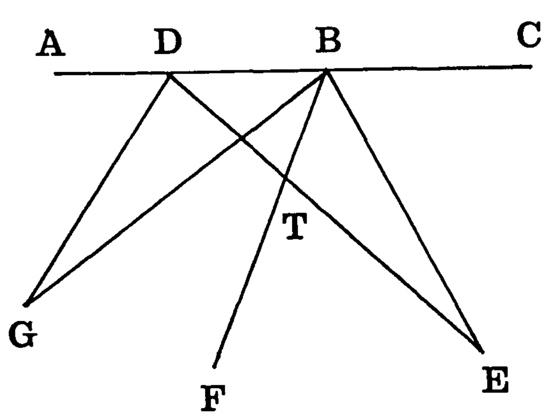

Figure 4a

neque concidunt neque sunt paralelli. Eius contrarium si quis asserat, sit primo quod concidant ad punctum G, inde sic: angulus ADG est maior angulo DBG per primum librum G[eometrie]; Sed angulus DBG est equus angulo CBE per primum huius. Ergo angulus ADG est maior CBE per V librum G[eometrie]. Sed angulus ADG est equus angulo EDB per primum huius. Ergo per quintum librum G[eometrie] EDB angulus est maior CBE, intrinsecus scilicet extrinseco maior, quod est inpossibile per primum librum G[eometrie].

Sit item quod sint paralleli et reflectantur ad G vel F per T [Figure 4a]. Inde sic: BF et DG equidistant, ergo CBF, CDG per primum G[eometrie]. Cum linea AC super ipsas incidat. ergo angulus DBF equus est angulo ADG [propter ?] premissa ad G[eometriam] et primum eiusdem. Sed ADG et DBF sunt equales CDE et CBE per primum huius. Ergo illi quatuor invicem sunt equales, scilicet ADG et DBF et CBE et CDE, per premissa ad G[eometriam]. Ergo angulus GDE et angulus FBE sunt equales per primum G[eometrie] et premissa ad G[eometriam]. Sed angulus FTE est maior angulo FBE per primum G[eometrie]. Ergo idem FTE est maior angulo GDE per primum V G[eometrie], intrinsecus scilicet equales extrinseco, quod inpossibile relinquetur in equedistantibus lineis per primum G[eometrie].

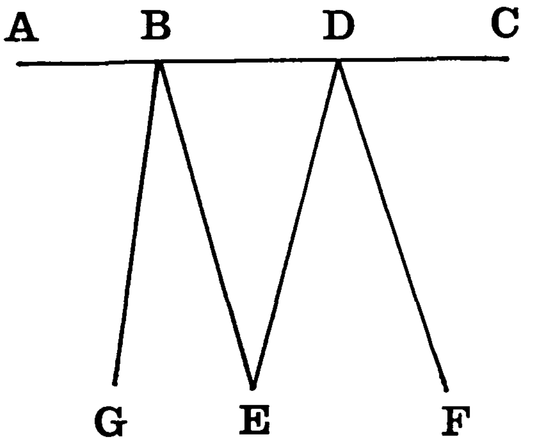

Figure 4b

Item alia dispositio ad idem: sit ABCD [Figure 4b] speculum planum ad quod ab E oculo ad B et D incidant radii reflectantur et eidem radii ad F et G. Dico ergo quod neque concidunt neque sunt paralleli. Quod si quis asserat eos concidere, tunc ergo anguli DBG et BDF minores sunt duobus rectis per primum G[eometrie]. Ergo ABG et CDF sunt maiores duobus rectis et maiores quam illis per premissa ad G[eometriam] et primum. Sed ABG et CDF sunt equales angulis DBE et BDE. Ergo DBE et BDE sunt maiores DBG et BDF per quintum G[eometrie]. Hoc autem inpossibile relinquitur per premissa ad G[eometriam].

Ponantur item BG et DF radii reflexi equidistare. Ergo anguli DBG et BDF sunt equales duobus rectis per primum G[eometrie]. Ergo anguli ABG et CDF sunt equales duobus rectis ex eodem. Sed DBE et BDE anguli sunt equi angulis ABG et CDF ex primo huius. Ergo idem DBE et BDE anguli sunt equales duobus rectis. Ergo triangulus BDE habet duos angulos rectos et insuper acutum, quod inpossibile est per primum G[eometrie]. Relinquitur ergo quod concidant vel sint paralleli esse inpossibile.

Amplius in convexis: sit convexum speculum ABCD [Figure 4c], oculus E, inmittanturque radii ad speculi puncta B et C. Dico igitur quod neque conther convergent nor parallel. If someone asserts its contrary, first let it be that they would converge to point G. Then [argue] as follows: By *Elm*-I, $\angle$ADG > $\angle$DBG; but by the first [proposition] of this book, $\angle$DBG = $\angle$CBE; therefore $\angle$ADG > $\angle$CBE by *Elm*-V; but by the first [proposition] of this book, $\angle$ADG = $\angle$EDB; therefore by *Elm*-V, $\angle$EDB > $\angle$CBE, that is, the interior [angle] would be larger than the exterior. This is impossible by *Elm*-I.

Further let it be that they be parallel and be reflected to G or F through T [Figure 4a]. Then [argue] as follows: because BF and DG are parallel, CBF and CDG [are parallel]by *Elm*-I; since line AC falls on them, $\angle$DBF = $\angle$ADG by *Elm*-assumptions and *Elm*-I; but $\angle$ADG and $\angle$DBF are equal [respectively] to $\angle$CDE and $\angle$CBE by the first [proposition]of this book; therefore these four, that is, $\angle$ADG, $\angle$DBF, $\angle$CBE, and $\angle$CDE, are equal by *Elm*-assumptions; therefore $\angle$GDE = $\angle$FBE by *Elm*-I and *Elm*-assumptions; but $\angle$FTE > $\angle$FBE by *Elm*-I; therefore by *Elm*-V-1 the same $\angle$FTE is larger than $\angle$GDE, *i.e.* the interior [angle] would be equal to the exterior, which is left impossible in the case of parallel lines by *Elm*-I.

Moreover another layout for the same: Let there be a plane mirror ABCD [Figure 4b], toward which let rays be fallen from the eye E to B and D, and let them be reflected toward F and G. Then I say that they are neither convergent nor parallel. Now if someone asserts that they be convergent, then $\angle$DBG + $\angle$BDF < 2$\angle$R by *Elm*-I; therefore $\angle$ABG + $\angle$CDF > 2$\angle$R and $\angle$ABG + $\angle$CDF > $\angle$DBG + $\angle$BDF by *Elm*-assumptions and -I; but $\angle$ABG and $\angle$CDF are [respectively] equal to $\angle$DBE and $\angle$BDE; therefore $\angle$DBE + $\angle$BDE > $\angle$DBG + $\angle$BDF by *Elm*-V. However, this is left impossible by *Elm*-assumptions.

Further let us suppose that reflected rays, BG and DF, are parallel. Then $\angle$DBG + $\angle$BDF = 2$\angle$R by *Elm*-I; therefore $\angle$ABG + $\angle$CDF = 2$\angle$R by the same; but $\angle$DBE and $\angle$BDE are [respectively] equal to $\angle$ABG and $\angle$CDF by the first [proposition] of this book; therefore $\angle$DBE + $\angle$BDE = 2$\angle$R; therefore triangle BDE would have two right angles and an acute [angle] as well, which is impossible by *Elm*-I. Wherefore there remains that it is impossible for them to be convergent or parallel.

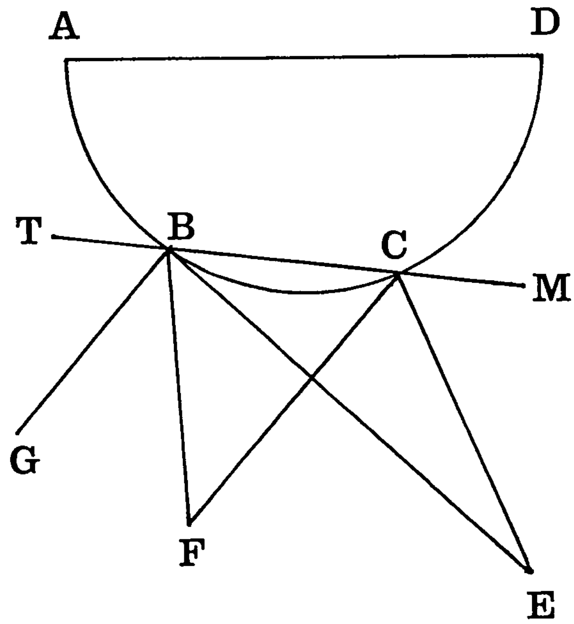

Figure 4c

Furthermore in the case of convex [mirrors]: Let there be a convex mirror ABCD [Figure 4c], an eye E, and let rays be emitted to points B and C on the mirror. Then I say that they be neither cidunt nec erunt paralleli. Quod si quis eos asserat concidere, tunc ad punctum F concidant. Intelligatur igitur planum speculum TBCM. Inde sic: angulus TBE [*lege* TBF] maior est angulo MCE per primum G[eometrie]. Sed angulus TBE [*lege* TBF] equus est angulo MBE per primum huius. Ergo MBE maior est angulo MCE etiam per primum G[eometrie]. Relinquitur inpossibile.

Si vero dicatur quod BG et CF radii reflexi equidistant, habebitur quod triangulus BCF habet duos angulos rectos per primum huius et primum G[eometrie]. Hoc autem inpossibile relinquatur ex primo G[eometrie].

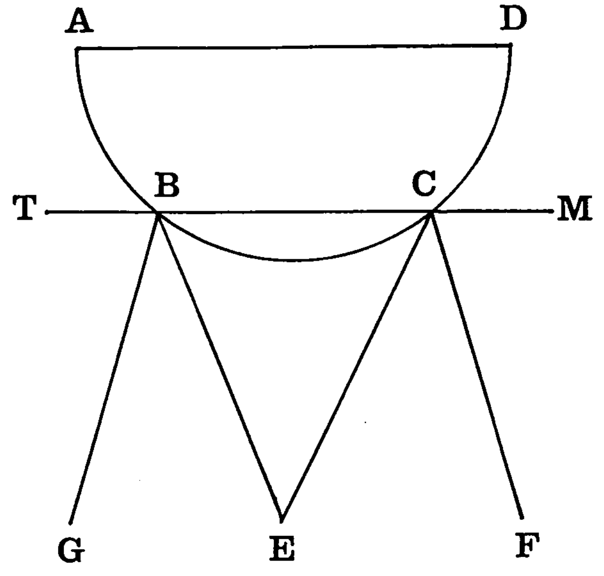

Figure 4d

Item in alia dispositione: sit ABCD speculum, E vero sit oculus [Figure 4d]. Dico radii emissi ab E ad speculi puncta que sunt B et C neque concidunt neque erunt paralleli. Quos [*lege* Quod] si quis asserat concidere, intellecto speculo plano TBCM, ergo non reflectentur per se quia triangulus EBC habet duos angulos rectos per secundum huius et primum G[eometrie], quod est inpossibile. Reflectantur igitur ad F et G et [ipsis ?]. Ergo anguli CBG et BCF sunt minores duobus rectis. Ergo TBG et MCF sunt maiores duobus rectis per primum G[eometrie]. Sed idem sunt equales CBE et BCE per primum huius. Ergo CBE et BCE sunt maiores duobus rectis per V G[eometrie]. Ergo CBE triangulus habet duos angulos maiores duobus rectis, quod est inpossibile per primum G[eometrie].

Item sint BG et CF radii reflexi equidistantes. Quod si esset ergo CBG et BCF anguli sunt equales duobus rectis. Ergo TBG et MCF sunt equales duobus rectis. Sed idem sunt equales CBE et BCE per primum huius. Ergo CBE et BCE sunt equales duobus rectis per primum G[eometrie], [quod est inpossibile] per [primum] G[eometrie].

[V] In cavis speculis si vel super [*illeg*: centrum vel super periferiam] vel extra ponatur oculus, visus repercussi concidunt.

Verbi gratia: sit speculum concavum ABCD [Figure 5a]. Sit oculus E a quo inmittantur radii EB [EC ?] EM. Et a quibuslibet igitur illorum reflectentur per se, quia omnes radii secundum [equales angulos ?] reflectabuntur per se ex primo huius. Aliter enim non reflectantur ad equos angulos. [Sed si sic reflectuntur ?] concidunt in oculum. Ergo si oculus sit in centro, radii reflexi concidunt.

convergent nor parallel. Now if someone asserts that they be convergent, then let them be convergent to point F. Then let us think of a plane mirror TBCM. Then [argue] as follows: by *Elm*-I, ∠[TBF] > ∠MCE ; but by *Elm*-I, ∠[TBF] = ∠MBE; therefore, ∠MBE > ∠MCE also by *Elm*-I. This is left impossible.

Now if it is shown that the reflected rays, BG and CF, are parallel, it will be shown that triangle BCF would have two right angles by the first [proposition] of this book and *Elm*-I. However, this is left impossible by *Elm*-I.

Similarly in another layout: Let ABCD be a mirror, and E be an eye [Figure 4d]. I say that the rays, emitted from E to points B and C on the mirror, are neither convergent nor parallel. Now if someone asserts [them to be] convergent, by introducing a plane mirror TBCM, then they would not be reflected on themselves, because triangle EBC would have two right angles by the second [proposition] of this book and *Elm*-I. This is impossible. Therefore they are [respectively] reflected to F and G. Therefore ∠CBG + ∠BCF < 2∠R. Then by *Elm*-I, ∠TBG + ∠MCF > 2∠R. But they are [respectively] equal to ∠CBEand∠BCE by the first [proposition] of this book. Therefore by *Elm*-V, ∠CBE + ∠BCE > 2∠R. Wherefore triangle CBE would have two angles larger than two right angles, which is impossible by *Elm*-I.

Similarly let BF and CF be the reflected parallel rays. Now if so, then ∠CBG + ∠BCF = 2∠R. Therefore ∠TBG+ ∠MCF = 2∠R. But they are [respectively] equal to ∠CBE and ∠BCE. Therefore ∠CBE+ ∠BCE = 2∠R by *Elm*-I, [which is impossible] by *Elm*-[I].

[V] IN CONCAVE MIRRORS, IF AN EYE IS PLACED EITHER ON [THE CENTER OR ON THE CIRCUMFERENCE], OR OUTSIDE IT, REFLECTED VISUAL RAYS CONVERGE.

For example: Let a concave mirror be ABCD [Figure 5a]. Let an eye be E from which are emitted the rays, EB, [EC], and EM, and then they will be reflected on themselves from any of those points, because all rays will be reflected at [equal angles] on themselves by the first [proposition] of this book. For otherwise, they are not reflected at equal angles. [But if they are reflected that way,] they converge to the center. Therefore if the eye is on the center, the reflected rays converge.

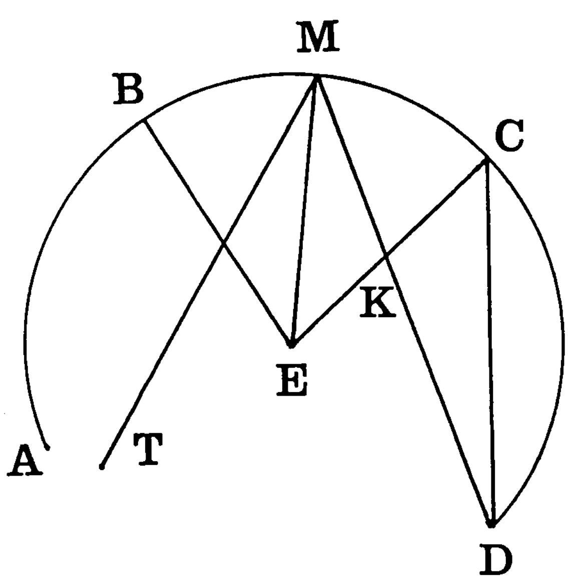

Figure 5a

Ponatur autem oculus in periferia et sit D a quo exmittantur ad C et M puncta, reflectantur vero ad T et E [Figure 5a]. Dico quod CE et MT reflexi scilicet radii concidunt. Quod sic arguas: arcus CD est minor arcu MD. Ergo corda CD est minor corda MD. Ergo angulus DCD est minor angulo DMD per III^{um} G[eometrie]. Sed MCE angulus equus est angulo DCD et item angulus BMT equus est angulo DMD. Ergo anguli DCD et MCE sunt minores angulis DMD et BMT, ita quod [...] per V G[eometrie]. Ergo angulus DCE maior est angulo DMT per primum huius et primum G[eometrie]. Sed angulus DKE est maior angulo DCE per primum G[eometrie]. Ergo angulus DKE est maior DMT per V^{tum} G[eometrie]. Sed angulus DKE et MKE sunt equales duobus rectis. Ergo MKE et KMT sunt minores duobus rectis per primum G[eometrie] et ypothesym. Ergo MT et CE concurrent per primum G[eometrie]. Vel sic levius: MT et CE equidistant. Ex ypothesy ergo anguli KMT et MKE sunt equales duobus rectis per primum G[eometrie]. Sed angulus DKE est maior angulo KMT. Ergo anguli DKE et MKE sunt maiores duobus rectis, quod est inpossibile per primum G[eometrie].

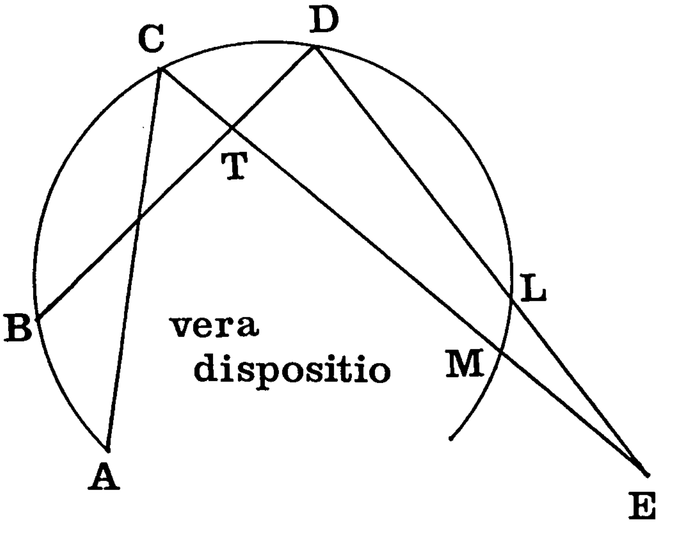

Figure 5b

Sit iterum oculus extra peryferiam, positus E scilicet [Figure 5b], inmittaturque radii ad B[*lege* D] et C. Sic: CM est maior quam DL quia est propinquior centro. Ergo angulus DCM est maior angulo LDL per tertium G[eometrie]. Ergo angulus BDE maior angulo ACE per primum G[eometrie] et primum huius. Sed angulus BTM est maior angulo BDE. Ergo idem BDM [*lege* BTM] maior est angulo ACE per V G[eometrie]. Sed angulus BTE est maior angulo BDE. Ergo angulus BTE maior est angulo ACE. Sed BTE et CTE [*lege* CTB] sunt equales duobus rectis, ergo CTE[*lege* CTB] et ACE sunt minores duobus rectis per primum G[eometrie]. Ergo DTB et CA concurrunt.

Ad parallelum est solutio [Figure 5c]. Quod levium est in termino.

Let an eye be on the circumference, and let it be D from which [rays] are emitted to points C and M and are reflected to T and E [Figure 5a]. I say that the reflected rays, CE and MT, converge. Now you should argue as follows: because arc CD is smaller than arc MD, chord CD is smaller than chord MD; therefore by *Elm*-III, $\angle DCD < \angle DMD$; but $\angle MCE = \angle DCD$ and also $\angle BMT = \angle DMD$; $\angle DCD + \angle MCE < \angle DMD + \angle BMT$, thus [...] by *Elm*-V; therefore $\angle DCE > \angle DMT$ by the first [proposition] of this book and *Elm*-I; but by *Elm*-I, $\angle DKE > \angle DCE$; therefore *Elm*-V, $\angle DKE > \angle DMT$; but $\angle DKE + \angle MKE = 2\angle R$; therefore by *Elm*-I and the hypothesis, $\angle MKE + \angle KMT < 2\angle R$; therefore by *Elm*-I, MT and CE converge. Or much easier as follows: [Assume] that MT and CE be parallel; therefore by the hypothesis and *Elm*-I, $\angle KMT + \angle MKE = 2\angle R$; but $\angle DKE > \angle KMT$; therefore $\angle DKE + \angle MKE > 2\angle R$, which is impossible by *Elm*-I.

Furthermore let an eye be placed at E outside the circumference [Figure 5b], and let rays be emitted to [D] and C. [Argue] as follows: CM is longer than DL, because it is nearer to the center; therefore by *Elm*-III, $\angle DCM > \angle LDL$; therefore $\angle BDE > \angle ACE$ by *Elm*-I and the first [proposition] of this book; but $\angle BTM > \angle BDE$; therefore the same $\angle$[BTM] is larger than $\angle ACE$ by *Elm*-V; but $\angle BTE > \angle BDE$; therefore $\angle BTE > \angle ACE$; but because $\angle BTE + \angle$[CTB] $= 2\angle R$, $\angle$[CTB] $+ \angle ACE < 2\angle R$ by *Elm*-I; therefore DTB and CA converge.

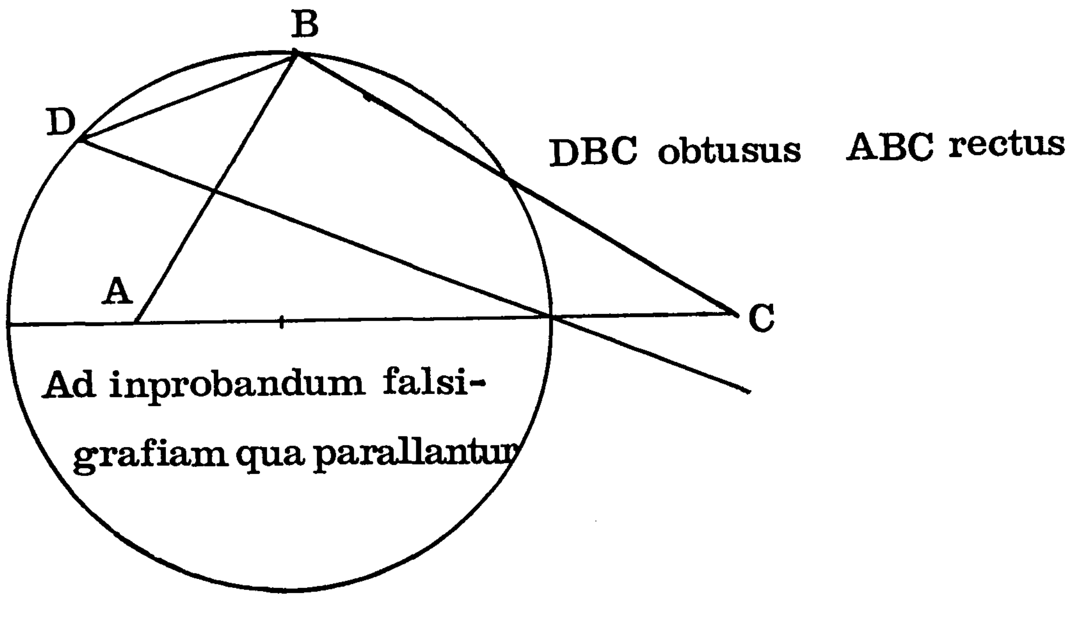

Figure 5c

The solution will be possible in the case of [their being] parallel [Figure 5c], which is easy in the end.

[VI] In concavis speculis si in medio centri et periferie ponatur oculus, aliquotiens quidem concidunt radii repercussi, aliquotiens vero non concidunt.

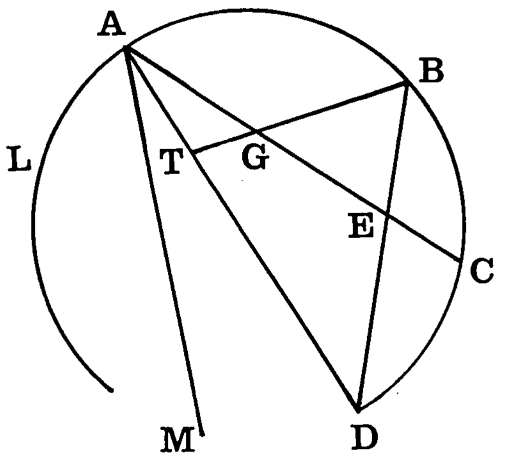

Figure 6

Verbi gratia: sit speculum ABCD [Figure 6], oculus E, a quo ad B et A puncta speculi inmitantur radii EA et EB reflectanturque ad T et M. Dico quod si intelligatur AE usque ad speculi peryferyam C scilicet, sed et EB usque ad D, si AC sit minor [*lege* maior] vel equalis BD, tunc BT et AM concidunt; si vero maior [*lege* minor] sit, tunc quandoque concidunt quandoque non.

Quod sic probatur: posito quod AC sit equalis BD primo, inde sic: AC est equalis BD. Ergo angulus BAC est equus angulo CBD. Sed LAM et ABT anguli adequantur per tertium G[eometrie] utrisque. Ergo angulus DBT est equus angulus[*sic*] angulo EAM per primum G[eometrie]. Sed angulus EGT est maior DBT angulo. Ergo idem EGT est maior EAM. Sed EGT et AGT sunt duobus rectis equales ex primo G[eometrie]. Ergo AGT et GAM sunt minores duobus rectis per primum G[eometrie]. Ergo BGT et AM concurrunt per primum G[eometrie].

Similiter si corda AC est maior corda BD, angulus EBT est maior angulo EAM per precedens huius. Ergo BGT et AM concurrunt per precedens huius.

Sit item quod AC sit minor BD. Ergo angulus BAC et LAM sunt minores angulis CBD et ABT per tertium G[eometrie]. Ergo angulus CAM maior est angulo TBD per primum et tertium G[eometrie]. Sed angulus TGE maior est angulo EBT per primum G[eometrie]. Ergo CAM et EGT sunt maiores EBT, ita quod uterque. Si ergo EGT eque maior est EBT [*addidi*: que eque minor est CAM]vel minus est quam CAM, cum EGT et AGT sint equales duobus rectis per primum G[eometrie], erint AGT et EAM equales duobus rectis vel maiores. Ergo AM et BGT concurrere tunc est inpossibile per primum G[eometrie]. Si vero EGT est magis maior EBT quam GAM sit eodem, tunc TGE maior est GAM. Ergo BGT et AM concurrunt per premissa et primum G[eometrie]. Et hoc erat propositum.

[VII(H16)] In planis speculis unumquodque conspectorum secundum cathetum a conspecto videtur.

Verbi gratia: sit speculum planum ABC [Figure 7], oculus E, conspectum vero sit D, a quo ducatur chatetus ad punctum speculi A scilicet DA, protrahatur idem DA in continuum et directum donec concurrat cum EB radio incidenti ad punctum G. Concurrunt enim: quia angulus DAB rectus, ergo angulus BAG est similiter rectus

[VI] IN CONCAVE MIRRORS, IF AN EYE IS PLACED BETWEEN THE CENTER AND THE CIRCUMFERENCE, REBOUNDED VISUAL RAYS WILL SOMETIMES CONVERGE, AND SOMETIMES DO NOT CONVERGE.

For example: Let there be a mirror ABCD [Figure 6], and an eye E, from which let rays, EB and EA, be [respectively] emitted to points B and A on the mirror and be reflected[respectively] to T and M. I say that if we extend AE up to C, the circumference of the mirror, and also EB to D, and if AC is either [longer] than or equal to BD, then BT and AM will converge; but if [AC is] [smaller] than [BD], then they will sometimes converge and sometimes not.

Now this is proved as follows: First let us assume that AC be equal to BD, then [argue] as follows: because AC is equal to BD, ∠BAC = ∠CBD; but ∠LAM and ∠ABT are [respectively] equal to each [of the foregoing equation] by *Elm*-III; therefore by *Elm*-I, ∠DBT = ∠EAM; but ∠EGT > ∠DBT; therefore ∠EGT > ∠EAM; but by *Elm*-I, ∠EGT+∠AGT = 2∠R; therefore by *Elm*-I, ∠AGT+∠GAM < 2∠R; therefore by *Elm*-I, BGT and AM converge.

Similarly if chord AC is longer than chord BD, ∠EBT > ∠EAM by the preceding of this book. Therefore BGT and AM converge by the preceding of this book.

Further let it be that AC be smaller than BD. Then ∠BAC and ∠LAM are [respectively] smaller than ∠CBD and ∠ABT by *Elm*-III; therefore ∠CAM > ∠TBD by *Elm*-I and -III; but ∠TGE > ∠EBT by *Elm*-I; therefore ∠CAM and ∠EGT are both larger than ∠EBT; therefore if ∠EGT is equally larger than ∠EBT [which is equally smaller than ∠CAM] [that is, ∠EGT = ∠CAM] or smaller than ∠CAM [that is, ∠EGT < ∠CAM], it will be that ∠AGT+∠EAM ≥ 2∠R, since ∠EGT+∠AGT = 2∠R by *Elm*-I; therefore in this case, it is impossible, by *Elm*-I, for AM and BGT to converge. But if ∠EGT−∠EBT > ∠GAM−∠EBT, then ∠TGE > ∠GAM; therefore BGT and AM will converge by *Elm*-assumptions and -I. And this is what was proposed.

[VII(H16)] IN PLANE MIRRORS EVERY OBJECT OF SIGHT IS SEEN ALONG A PERPENDICULAR DRAWN FROM AN OBJECT.

For example: Let there be a plane mirror ABC [Figure 7], and an eye E, and let an object of sight be D, from which let a perpendicular DA be drawn to point A on the mirror, and let the same DA be extended continuously and directly until it meets the [extended] incident ray EB at point G. The reason why they meet is: because ∠DAB [is] right, ∠BAG is also right

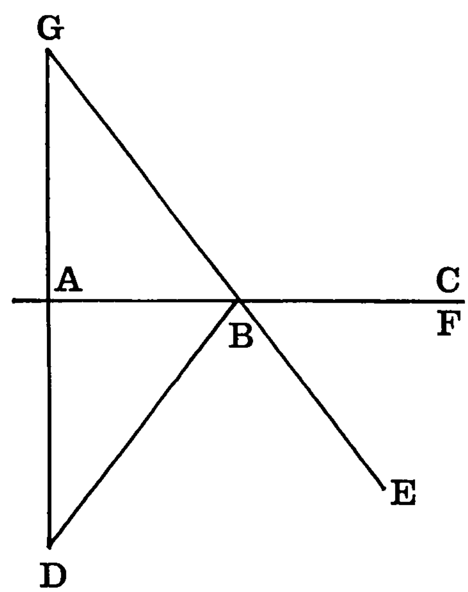

Figure 7

per primum G[eometrie]. Sed angulus EBF minor recto ex eodem et primo huius cui EBF angulus ABG equus est per contrapositionem; ergo anguli BAG et ABG sunt minores duobus rectis per V^{tum} G[eometrie]. Ergo latera AG et BG concurrunt. Concurrant ergo ad G punctum. Inde sic: anguli EBF et DBA sunt equales ex primo huius. Sed angulus EBF equus est angulo ABG per contrapositionem. Ergo angulus DBA et angulus ABG adequatur per primum G[eometrie]. Sunt igitur duo [trianguli] ADB et ABG quorum duo anguli unius, scilicet DAB et DBA, duobus angulis alterius, scilicet BAG et ABG, invicem prout se respiciunt adequantur et latus AB commune, ergo reliquus angulus reliquo angulo et latera prout equales respiciunt angulos adequantur per primum G[eometrie]. Unde latera AD et BG [*lege* AG] relinquntur equalia. Ergo D videtur in G cum omnia visa recte videantur per premissa ad hunc librum. Hoc autem erat propisitum, quoniam G est in puncto eiusdem linee que cathetus AD ad speculum.

[VIII(H17)] In convexis speculis secundum eam que est a conspecto in centrum spere ducta recta linea uterque conspectorum videtur.

Verbi gratia: sit convexum speculum ABC [Figure 8], oculus E a quo ad punctum speculi quod est B inmitatur radius EB videns D per reflexionem. Adiectis igitur T et D recta linea, dico quod in eadem linea TD scilicet videatur D.

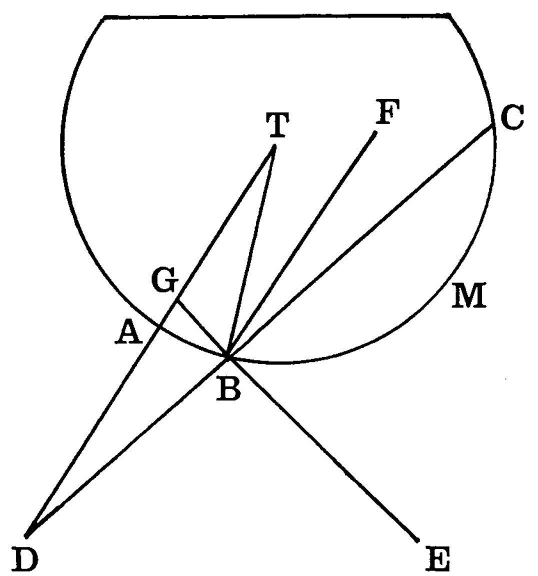

Figure 8

Quod si quis alibi videri posse vel ad centrum vel inter centrum et circonferentiam vel in peryferia etiam asserat, ad inpossibile deducatur.

Ponatur ergo primo in centro videri, B et T linea adiectis sic: TB a centro a[d]circonferentiam[*sic*] protenditur, ergo TB est semidiameter. Ergo facit angulos equales cum periferia alternatim per collectarium. Et ex TB et BE est una linea ex ypothesi. Ergo anguli extra circonferentiam alternatim equantur ex eodem collectario. Ergo angulus CMBE equus est angulo ABE. Sed idem CMBE equus est angulo ABD per primum huius. Ergo ABE angulus equus est angulo ABD, quod inpossibile revincitur per primum G[eometrie].

Si dicatur quod vel inter centrum et periferiam, id est in F, vel in periferia, id est in C, idem et prorsus eodem modo sequitur inconveniens. Ergo nec in centro nec inter centrum et partem speculi versus oculum sumptam potest nec in peryferia radius directe protractus incidere. Ergo incidet in TD lineam, id est ultra. Et hoc by *Elm*-I; but by the same and the first [proposition] of this book, ∠EBF < ∠R, and by opposite angles, ∠EBF = ∠ABG; therefore by *Elm*-V, ∠BAG + ∠ABG < 2∠R; therefore sides AG and BG meet. Let them meet at point G. Then [argue] as follows: by the first [proposition]of this book, ∠EBF = ∠DBA; but by opposite angles, ∠EBF = ∠ABG; therefore by *Elm*-I, ∠DBA = ∠ABG; wherefore because two [triangles], △ADB and △ABG, are congruent, —whose two angles of the one, ∠DAB and ∠DBA, are each other equal to two angles of the other, ∠BAG and ∠ABG, as long as they correspond, and the side AB [is] common—, then by *Elm*-I the remaining angle and the sides that correspond to equal angles are [respectively] equal to the remaining angle [and the corresponding sides]. Wherefore it remains that the sides AD and [AG] are equal. Therefore D is seen at G, because by the assumptions of this book all objects of sight are seen directly. This is what was proposed, because G is at the point of the same line as the perpendicular to the mirror.

[VIII(H17)] IN CONVEX MIRRORS EVERY OBJECT OF SIGHT IS SEEN ALONG THE STRAIGHT LINE DRAWN FROM THE OBJECT TO THE CENTER OF THE SPHERE.

For example: Let there be a convex mirror ABC [Figure 8], and an eye E from which the ray EB, that sees D by reflection, be emitted to point B on the sphere. Then, by connecting T and D by a straight line, I say that D would be seen on the same line TD.

Now if someone asserts that it may be seen somewhere else either at the center or between the center and the circumference or on the circumference, he would be brought to the impossible.

Then first let us assume that it may be seen at the center. Connect T and D by a line, [and argue] as follows: Stretch TB from the center to the circumference, then TB is the radius; therefore by *the [Mathematical] Collections*, it would form equal angles with the circumference on both sides; and by hypothesis one line is formed by TB and BE; therefore the angles on both sides outside the circumference would be equal by the same *Collections*; therefore ∠CMBE = ∠ABE; but by the first [proposition] of this book, ∠CMBE = ∠ABD; therefore ∠ABE = ∠ABD, which is refuted as impossible by *Elm*-I.

If it is objected that [it may be seen] either between the center and the circumference, *i.e.* at F, or on the circumference, *i.e.* at C, the contradiction would follow in quite the same manner. Therefore the ray, extended directly, can fall neither at the center nor between the center and the side of the mirror taken toward the eye nor on the circumference [of the same side]. Therefore it will fall on line TD, *i.e.* on the other side. And this [is] what was propositum, quod ex supposito manifestum est.

Sed nota quod semper incidet ultra lineam ubi conspectum tali loco ponatur; quod pars inmissi radii secans circulum sit equalis reflexo ad conspectum.

[IX(H18)] In concavis speculis unumquodque conspectorum secundum a conspecto in centrum spere ductam rectam lineam videtur.

Verbi gratia: sit speculum concavum ABCD [Figure 9a], oculus E. Erit igitur E vel in diametro speculi vel intra vel extra.

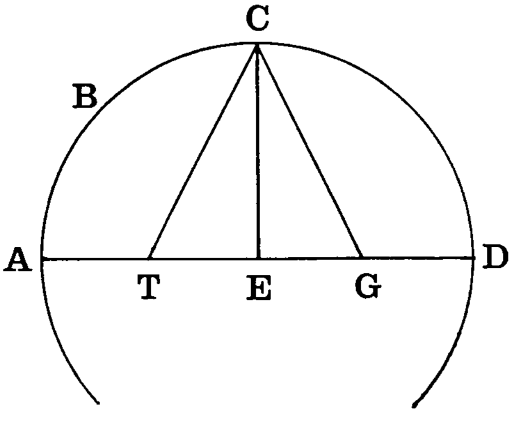

Figure 9a

Si in diametro, tunc vel in centro vel extra centrum. Et in centro ergo reflectitur per se. Ergo linea recta reflexa a conspecto ad centrum spere est radius reflexus. Sed in eo videtur res. Ergo in linea recta ducta a conspecto per centrum spere videtur. Videtur conspectum ut in speculo supposito. C videtur in E[*sic*].

Si vero oculus sit in diametro preter centrum ut in AD diametro in puncto G, derigaturque ad C radius, videbitur T in puncto G. Quia natura est concavi speculi quod res nunquam apparet ultra speculum, quare semper vel inter oculum et speculum vel oculus in linea eadem cum radio inmisso est inter punctum apparentie et speculum. Et tunc oculus videt ante et retro, cuius rei causam philosophis assignandam sufficientis pretermittiamus.

Quod autem in radio inmisso videatur res quantum ad apparentiam manifestum est intellecti. Sed quod in puncto ab E[u]clide dicto sectionis, scilicet linee protracte a conspecto per centrum ad radium inmissum, nec intellexi nec intelligi. Dicunt ergo quod ex natura speculi est concavi quod semper res videtur intra speculum et oculum, nunquam autem speculum apparebit intermedium. Quod etsi verum sit, non tamen ideo residuum constabit. Sed item ad subsequentia[?] fere omnia necesarium est. Et cum non habeamus demonstrationem contra, si ex natura speculi concavi dicatur contingere et etiam tanti actoris gratia tamquam verum sit admittatur, continget autem secundum hec rem aliquando non videri, quando scilicet radius inmissus et linea a conspecto per centrum ducta erunt equedistantes, quoniam tunc concurrentes esse non possunt.

Sed si aliter contingat, tunc videtur oculo, dico[?], vidente ab omini parte circa se. Verbi gratia: sit speculum ABC [Figure 9b], oculus E, D conspectum. Secundum predictam videbitur D in punto M. Quoniam linea DM reflectitur[*lege* protracta] a proposed, which is manifest from the assumption.

But notice that it will always fall on the other side of the line where the object of sight occupies the place; and that the part of the emitted ray which cuts the circle is the same [as the part of the ray] reflected toward the object.

[IX(H18)] IN CONCAVE MIRRORS EVERY OBJECT OF SIGHT IS SEEN ALONG THE STRAIGHT LINE DRAWN FROM THE OBJECT TO THE CENTER OF THE SPHERE.

For example: Let a concave mirror be ABCD [Figure 9a], and an eye be E. Then E is either on or inside or outside the diameter of the mirror.

If on the diameter, then [the eye lies] either on the center or outside the center. And on the center, then, [the visual ray] is reflected on itself. Therefore the straight line reflected from the object to the center of the sphere is the reflected ray. But the object is seen on it. Therefore it is seen on the straight line drawn from the object to the center of the sphere. The object of sight is so seen as on the placed mirror. C is seen on E [!].

But if the eye is so located on the diameter except for the center as at point G on diameter AD and let the radius be drawn to C, T will be seen at point G. Because it is the nature of a concave mirror that the object never appears beyond the mirror, wherefore [it appears]always either between the eye and the mirror, or the eye is between the point of appearance and the mirror on the same line as the emitted ray. And in this case the eye sees before and back, the assigning the cause of which let us leave to expert philosophers.

Well it is manifest to an intelligent person that [an object] is seen on the emitted ray as long as the appearance is concerned. But I did not understand and it cannot be understood that [an object may be seen] on the point of the intersection assigned by Euclid, *scilicet*, [on the point] of the line extended from an object of sight through the center to the emitted ray. Therefore people say that it is by the nature of a concave mirror that things are always seen between the mirror and the eye but the mirror will never intervene between them. Now even if this be true, nonetheless the rest will not hold. However, almost everything is necessary for [proving] the consequences. Since we do not have any proof against it, if it is said that this happens by the nature of a concave mirror, and moreover this should be admitted as true on account of great authority, nonetheless according to this it sometimes happens for things to be unseen, *scilicet* when the emitted ray and the line drawn from the object through the center are parallel, because in that case they cannot meet.

But if the situation happens another way, then [an object] is seen by the eye, I mean, by the observer from every side around him. For example: Let there be a mirror ABC [Figure 9b], an eye E, and an object of sight D. According to what was said before, D will be seen at point M. Since line DM [extended] from the

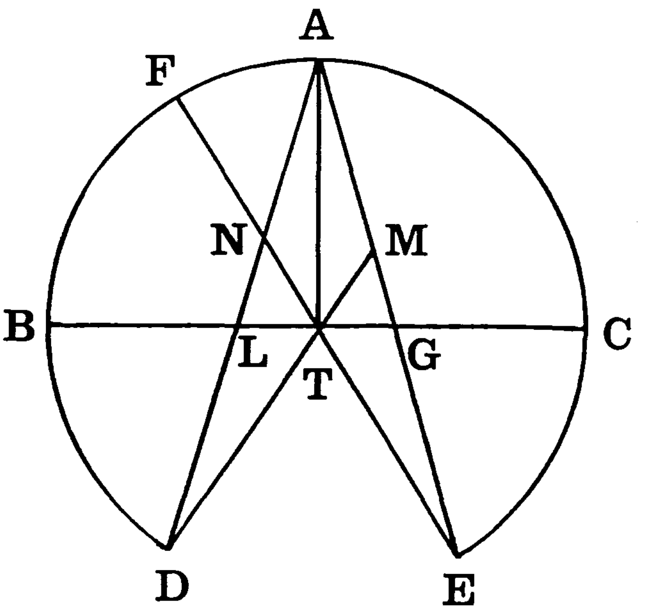

Figure 9b

conspecto per T centrum inmissum radium secans[*lege* secat] in puncto M, ergo D videtur in puncto M. Si vero E oculus videat N, apparebit N in oculo, quia linea recta protracta ab N per centrum secat radium inmissum in oculo. Item si G sit oculus et videat L, videt eandem in se. Si videt N, videt in E post se. Itemque existans oculus in CT semidiametro per A vel ABD punctum nichil videbit quod sit in quarta CTA ex primo huius. Item si intelligatur ABC sperice perfectum speculum sitque oculus E videns F, moto F versus E, quanto magis accedit ad oculum tanto videtur remotius, quoniam linea recta ducta a conspecto per centrum incidet in inferius punctum inmissi radii. Itemque e contrario si ab oculo moveatur per peryferiam ad ternimum diametri, quanto magis ab oculo recedit tanto propinquior videtur contraia ratione premisse rationis.

Sed consideratio[nes] reliqua[e] circa hanc propositionem omittantur, et quia verum[?] proba[n]tur et quia patent intelligenti, concesso quod dictum est. De lineis autem que non sunt equedistantes quod concurant[*sic*] cuilibet est manifestum.

[X(H7)] Altitudines et profunditates a planis speculis reverse videntur.

Verbi gratia: sit planum speculum ABC [Figure 10], oculus E, ATF altitudo perpendicularis ad planum speculum ABC, quoniam de talibus actor intelligit. Emittaturque radius in speculum videns T per punctum B, sed et alius videns F per C. Protractis igitur EB et EC donec concurrant cum catheto ad G et D, sic T videtur in G per 6^{am} [*lege* 7^{am}] propositionem huius. Eadem ratione F videtur in D ex eodem. Ergo TF videtur GD. Sed punctum summum, id est F, videtur inferius, et T item videtur superius, cum ATF sit altitudo ad ABC respectu E. Similiter et de qualibet parte poterit probari.

object through center T cuts the emitted ray at point M, D is seen at point M. However, if an eye E sees N, N will appear at the eye, because the straight line extended from N through the center cuts the emitted ray at the eye. Similarly if let G be an eye and let it see L, it sees the same on itself. If it sees N, it sees [N] behind itself at E. Furthermore let the eye exist on the radius CT, [then] by the first [proposition] of this book, it will see, by point A or any point on ABD, nothing that lies on the quarter-circle CTA. Moreover, if we think ABC to be a perfectly spherical mirror and let eye E see F, and if we move F toward E, the more it approaches toward the eye, the further it becomes seen, because the straight line drawn from the object through the center will fall on the lower point of the emitted ray. Furthermore on the contrary, if we move it from the eye to the extremity of the diameter along the circumference, the more it recedes from the eye, the nearer it becomes seen by the reason contrary to the preceding one.

But let us omit the considerations of the rest concerning this proposition, partly because they will be proved true and partly because they are evident to an intelligent person, once conceded what was said. However, concerning the lines that are not parallel, it is manifest to anybody that they meet.

[X(H7)] HEIGHTS AND DEPTHS ARE SEEN REVERSED IN PLANE MIRRORS.

For example: Let there be a plane mirror ABC [Figure 10], an eye E, and a height ATF perpendicular to the plane mirror ABC, because the author considers this situation. And let a ray and another one be emitted toward the mirror, seeing T by point B and F by C. Then let us extend EB and EC until they meet the perpendicular at G and D, so T is seen at G by the [seventh] proposition of this book. By the same reasoning F is seen at D by the same. Therefore TF is seen [as] GD. However, the highest point F is seen lower, and T is seen higher, since ATF is the height to ABC in regard to E. Similarly it is possible to prove concerning any part [of the height].

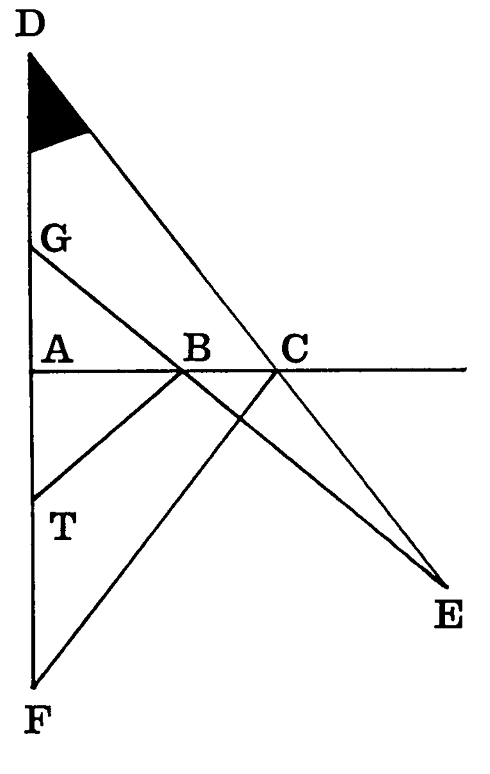

Figure 10

Itemque si intelligatur ATF profunditas visa ab E oculo per ABC speculum, erit F inferius punctum quam T. Et tunc videbitur in D, et T in G per 6^{am} [*sic*] huius, ergo F altius quam T et TF, GD. Ergo TF videtur reversa. Similiter et quelibet profunditas.

[XI(H8)] Altitudines et profunditates a convexis speculis reverse apparent.

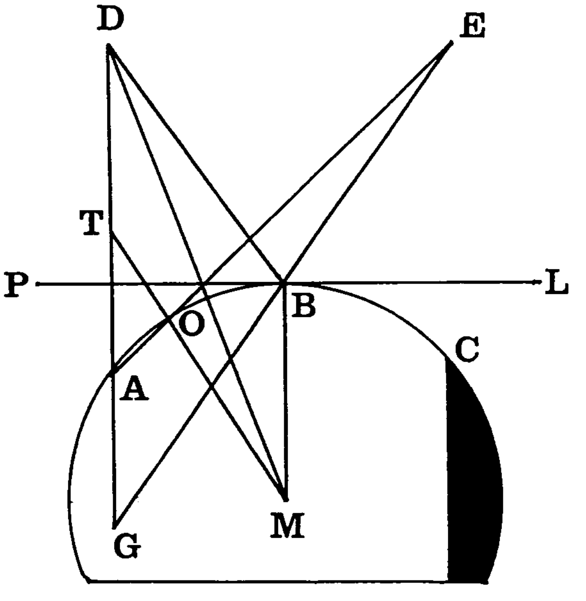

Figure 11a

Verbi gratia: sit oculus E, speculum convexum ABC, altitudo visa ATD [Figure 11a] sitque illa perpendicularis ad contingentem speculum secundum punctum terminantem medietates speculi, quoniam de talibus fit sermo. Videatur ergo D per B et T per O. Cum DTA perpendicularis sit ad lineam predicto modo contingentem, videbitur eorum uterque secundum lineam ductam a se in centrum speculi sicut determinavimus ex suppositione VII^{eme} [*lege* VIII^{vi}] propositionis huius. Ergo videbitur in TAG uterque visorum, T scilicet in A, et D in G per VI^{am} [*lege* VII^{am}] huius et suppositionem VII^{me} [*sic*]. Sed A est superius G. Ergo T videtur superius D. Sed T est inferius D in altitudine. Ergo quod superius est inferius apparet et econtrario. Ergo altitudo TD apparet reversa, quia eodem est de quibuslibet punctis similiter supectis.

Sit item in ABC [Figure 11b] speculo ATD profunditas perpendicularis ad lineam PL predicto modo contingentem, videaturque D per punctum speculi B, et T per O. Sic: T videtur per O in A, et D per B in G ex VI^{to} [*lege* VII^{mi}] huius et supposito VII^{mi} [*lege* VIII^{vi}]. Sed G est altius A. Ergo D videtur altius quam T. Sed D est inferius quam T. Ergo inferius videtur superius et econverso. Ergo profunditas TD est vera reversa. Cum de quibuslibet punctis similiter supectis, idem inveniri et eodem modo necesse sit. Et hoc erat propositum ostendere.

Furthermore if we think ATF to be a depth which is seen by the eye E by means of mirror ABC, point F will be lower than T. And in that case since it will be seen at D, and T at G by the [seventh] [proposition] of this book, F [is seen] higher than T and TF [is seen as] GD. Therefore TF is seen reversed. Similarly also for any depth.

[XI(H8)] HEIGHTS AND DEPTHS APPEAR REVERSED IN CONVEX MIRRORS.

For example: Let there be an eye E, a convex mirror ABC and a seen height ATD [Figure 11a], and let it be perpendicular to the tangent to the mirror at the point that divides the mirror into two halves, because discussion is concerned with this situation Therefore let D be seen by B, and T by O. Since DTA is perpendicular to the tangent line as said above, both of them will be seen on the line drawn from them to the center of the mirror, just as we have proved from the assumption of the [eighth] proposition of this book. Therefore by the [seventh] [proposition] of this book and the assumption of the [eighth], both objects will be seen on TAG, *scilicet* T at A and D at G. But since A is higher than G, T is seen higher than D. But since T is lower than D on the height, what is higher appears lower and *vice versa.* Therefore the height TD appears reversed, because the same reasoning applies to any points [of the height] seen in the same manner.

Similarly in the case of mirror ABC [Figure 11b] let there be a depth ATD which is perpendicular, as said above, to the tangent line PL. And let D be seen by point B on the mirror, and T by O. [Argue] as follows: by the [seventh] [proposition] of this book and the assumption of the [eighth], T is seen through O at A, and D through B at G. But since G is higher than A, D is seen higher than T. However, since D is lower than T, what is lower is seen [as] what is higher and *vice versa.* Therefore depth TD is [seen] truly reversed. Since [the same reasoning applies] to any points [of the height] seen in the same manner, the same conclusion will be found in exactly the same fashion. And this was the proposition that should be made clear.

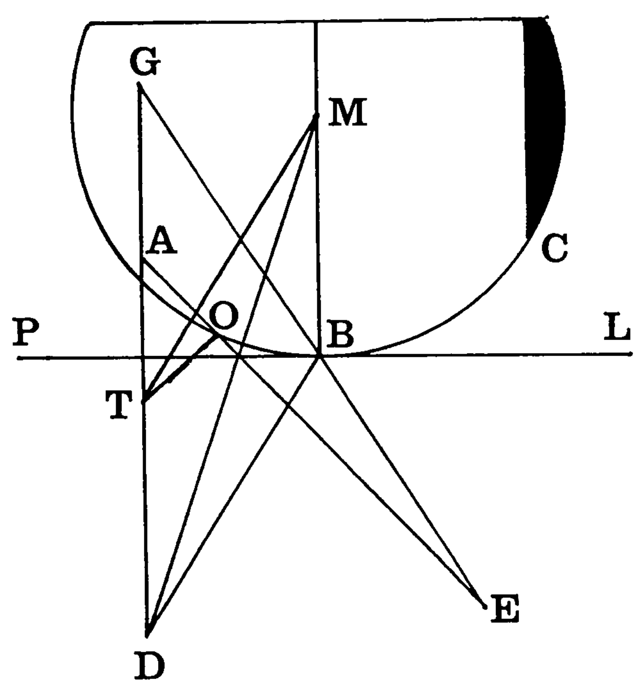

Figure 11b

[XII(H9)] Oblique longitudines a planis speculis, sicut in veritate se habent, ita apparere eas necesse est.

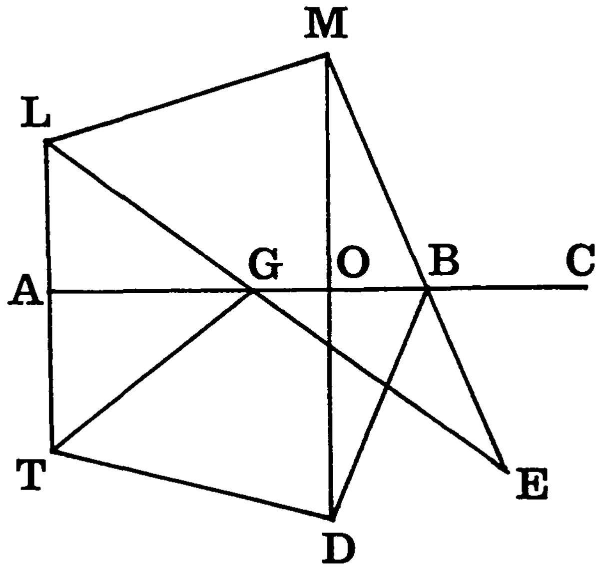

Figure 12a

Verbi gratia: sit planum speculum ABC [Figure 12a], obliquitas TD, oculus E. Sitque quod TD non sit equedistans speculo, quia quod in non equedistantibus probatur in equedistantibus est videri facilem. Videatur ergo D per B punctum speculi et T per G. Sit autem cathetus OD ad speculum DM, at vero AT TAL sit cathetus, protrahanturque EB et EG [in directum ?] donec concurrant in M et L cum cathetis. Inde sic: D videtur in M per 6^{am} [*sic*] huius et T in L ex eodem. Sed M propinquior est oculo quam L ut demonstrabitur. Ergo D videtur propinquior oculo quam L. Est in rei veritate scilicet quod D propinquior est oculo quam T. Ergo quemadmodum se habent D et T ita videntur. Similiter et de quolibet puncto et de quibuslibet supectis inter T et D se habet. Ergo obliquitas TD videtur sicut se habet.

Quod autem linea EM brevior sit quam linea EL [Figure 12a] sic astruere: linea AT et OD equedistant per primum G[eometrie]. Sed TD non equedistans AO ex dispositione speculi et T propinquior speculo quam D. Ergo DO[G:*del.*] maior quam TA. Sed DO adequatur OM et TA, AL similiter. Ergo OM est maior OB per V^{tum} G[eometrie]. Sed quemadmodum se habet OM ad AL ita se habet BM ad GL. Ergo BM longior GL, revincitur. Sed D videtur distare a speulo per BM et T per GL. Ergo D magis videtur distare a speculo quam T. Sed re vera ita est. Ergo quemadmodum se habet TD similiter videtur [*add. Figure 12b*].

Item EL maximo angulo opponatur in EML triangulo. Ergo est maximum latus. Ergo est maius EM per primum G[eometrie]. Id autem generaliter verum quod sicut distat oblique sic vera distare a speculis.

Notandum tamen quod non semper pars obliquitatis que videtur propinquior in rei veritate est propinquior. Im[m]o quandoque contingit econtrario. Sed que propinquior est speculo propinquior necessario apparet, et que remotior est et remotior apparet. Et hic est huius propositionis sensus.

[XIII(H10)] Oblique magnitudines a convexis speculis, quemadmodum vere, ita et apparere eas necesse est.

Verbi gratia: sit speculum convexum ABC, oculus E [Figure 13: *sic*]. TD sit

[XII(H9)] IT IS NECESSARY FOR OBLIQUE LENGTHS TO APPEAR IN PLANE MIRRORS JUST AS THEY ARE IN ACTUALITY.

For example: Let there be a plane mirror ABC [Figure 12a], an oblique length TD, and an eye E. And let it be that TD is not parallel to the mirror, because what is proved in the case of non-parallelism is easily understood in the case of parallelism. Therefore let D be seen through point B on the mirror and T through G. Moreover let the perpendicular OD to the mirror be DM, while the perpendicular AT be ATL, and extend EB and EG straightforwardly until they meet the perpendiculars at M and L. Then [argue] as follows: D is seen at M by the [seventh][proposition]of this book, and T at L by the same. But M is nearer to the eye than L as will be proved. Therefore D is seen nearer to the eye than L. Things are in their actuality, that is, in the sense that D is nearer to the eye than T. Therefore just as D and T actually are, so are they seen. Things are in the same manner concerning any one or other points seen between T and D. Therefore the oblique length TD is seen as it actually is.

Line EM is shorter than line EL [Figure 12a], to which you should add as follows: by *Elm*-I, lines AT and OD are parallel; but TD is not parallel to AO from the layout of the mirror and T is nearer to the mirror than D: therefore DO > TA; but DO = OM and also TA = AL; therefore by *Elm*-V, OM > OB [!]; but OM : AL = BM : GL [!]; therefore it is proved that BM > GL [!]; but D is seen to be distant from the mirror by BM, and T by GL; therefore D is seen more distant from the mirror than T; but this is what actually is; therefore just as TD actually is, it is so seen.

Further in triangle EML because EL is opposite to the largest angle, it is the longest side. Therefore by *Elm*-I, it is longer than EM. However, it is generally true that just as it is obliquely distant, it is seen so distant from the mirror.

It should be noted, however, that the oblique part which is seen nearer is not always nearer in its actuality. Rather sometimes it happens to the contrary. But what is nearer to the mirror necessarily appears nearer, while what is farther also appears farther. And this is the meaning of this proposition [Figure 12b: not used].

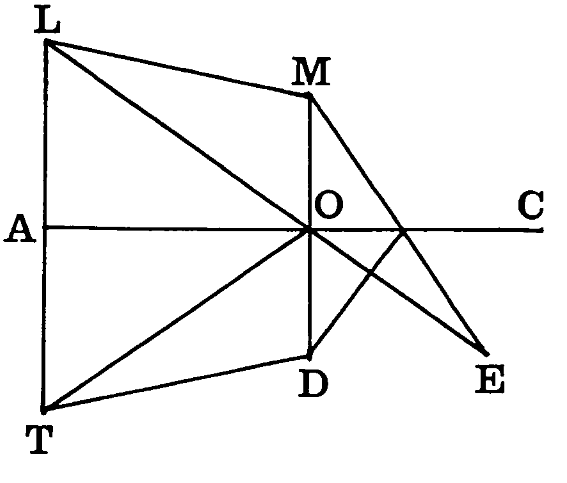

Figure 12b

[XIII(H10)] IT IS NECESSARY FOR OBLIQUE MAGNITUDES TO APPEAR ALSO IN CONVEX MIRRORS JUST AS THEY ACTUALLY ARE.

For example: Let there be a convex mirror ABC, and an eye E [Figure 13: *sic*].

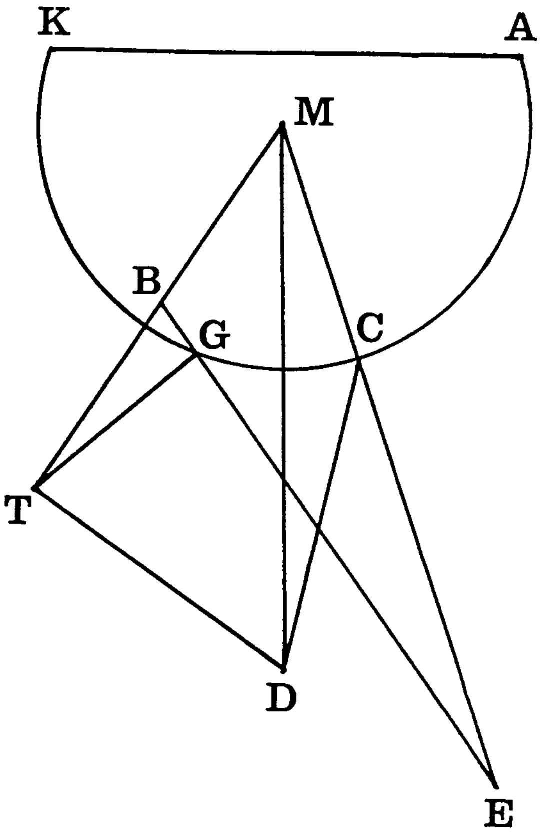

Figure 13

magnitudo conspecta. Centrum autem sit M sintque EC et EB[*lege* EG] radii inmissi, CD et GT radii reflexi, DM et TBM linee a conspectis in centrum. Inde sic: D videtur in M et T in B per 8 huius. Sed M videtur dexterior quam B. Ergo D videtur dexterior quam T. Sed D est dexterior quam T. Ergo quemadmodum se habet obliquitas TD ita et videtur. Et hoc modo debens[*lege* debet] intelligi precedentem propositiones[*sic*].

[XIV(H11)] Altitudines et profunditates quecumque intra confluentiam visuum in concavis speculis reversa apparent quemadmodum in planis speculis et convexis, quecumque quidem extra confluentiam visuum quemadmodum sunt et apparent.

Verbi gratia: sit speculum concavum ABC [Figure 14: *sic*], OG sit oculo altitudo conspecta, E oculus, TF ydolum rei vise, radii inmissi EA et EB, reflexi vero AC[*sic*] et BL[*sic*], centrum speculi M. Linea ducta a centro per G ad punctum concavis sit EBF concurrens in punctuo F in quo videtur G. Linea vero a centro per O ad punctum concavis sit EAT concurrens in puncto T in quo videtur O per 8 huius. Videatur ergo G in F sub inferiori radio. Quare inferius per librum de visu. Eodem arguens O videtur superius. Sed econverso se habet veritas. Ergo OG videtur reversa. Propositi autem hec est pars prima, ergo prima propositi sic probatur.

Let a seen magnitude be TD. Moreover let the center be M, and let EC and [EG] be the emitted rays while CD and GT be the reflected rays, and DM and TBM be the lines drawn from the object to the center. Then [argue] as follows: by the eighth [proposition] of this book, D is seen at M, and T at B. But since M is seen more to the right than B, D is seen more to the right than T. However, D is more to the right than T. Therefore just as the oblique length TD actually is, so is it seen. And the preceding proposition should also be understood in this way.

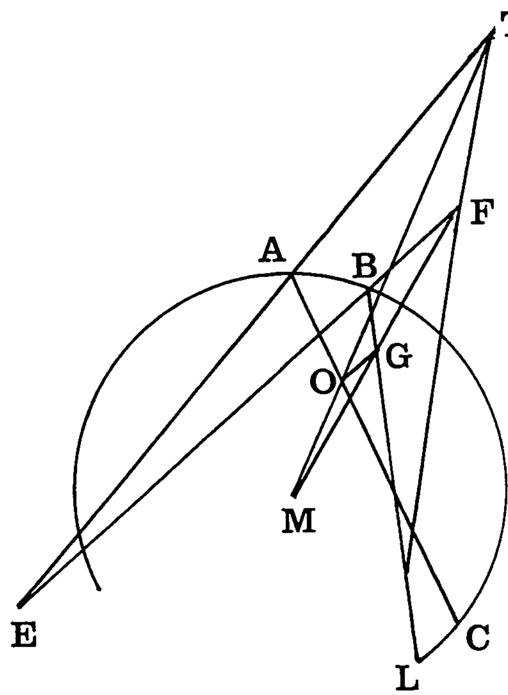

Figure 14

[XIV(H11)] IN CONCAVE MIRRORS, THOSE HEIGHTS AND DEPTHS WHICH ARE [AT SOME POINT] INSIDE THE CONFLUENCE OF VISUAL RAYS APPEAR REVERSED JUST AS IN PLANE AND CONVEX MIRRORS, AND THOSE WHICH ARE OUTSIDE THE CONFLUENCE OF VISUAL RAYS ALSO APPEAR JUST AS THEY ARE.

For example: Let a concave mirror be ABC [Figure 14: *sic*], OG be a height seen [by an eye], E the eye, TF be the image of the object of sight, the emitted rays be EA and EB, the reflected [rays] be AC and BL, and M be the center of the mirror. Let the line drawn from the center through G to the point on the concave [mirror] meet EBF at point F on which G is seen, while let [another] line [drawn] from the center through O meet EAT at point T on which O is seen by the eighth [proposition] of this book. Therefore G is seen at F by the lower ray, wherefore [seen] lower by the book *On Optics*. In the same way argue that O is seen higher. But the truth is to the contrary. Therefore OG is seen reversed. Moreover this is the first part of the proposition. Therefore it is thus proved.

Et nota quod intra triangulum reflexorum tantummodo dicitur esse visuum confluentiam et in spatio quod duntaxat ambiunt ubi se intersecant dicitur esse extra confluentiam visuum.

Si vero radii reflexi paralelli sint, nichil est intra confluentiam vel extra. Item nichil est altitudo ubi multo magis accedat ex una parte ad centrum quam ex alia neque intra confluentiam neque extra.

Quod autem idem eveniat in altitudinem extra visus[*sic*] confluentiam disposita manifestum. Ideo omittatur commentum, quia deest locus ad figuram construendam

Item dicunt quidam quod sub radiis reflexionum nichil videtur quotiensque sunt paralelli. Hoc autem argumentum ex hinc falsum est. Ad A pervenit radius. Ergo A videtur licet videatur obviare quiddam de libro de visu, ea scilicet ad que pervenit radius videri.

[XV (H12)] Oblique longitudines a concavis speculis quecumque quidem intra confluentiam visus, quemadmodum sunt ita et apparent similiter; quecumque vero extra, reverse.

Verbi gratia: sit speculum concavum ABC [Figure 15], obliqua maganitudo intra confluentiam visus sit TD, ydolum autem eius sit GF sintque EA et EB radii inmissi, reflexi vero AC et BL. Inde sic: EB radius est superior quam EA. Ergo quicquid videtur sub EO [*lege* EB] videtur altius quam quod videtur sub EA per librum de visu. Sed D videtur sub EO [*lege* EB] in F puncto protractis EBF et MDF ad concursum, M existente centro, per 8 huius. Ergo D videtur altius quam T per 1^{am} G[eometrie]et T videtur[?] inferius eisdem argumentis. Et T est inferius et D superius. Ergo quemadmodum se habet obliquitas TD intra visuum confluentiam ita videtur. Hec est prima pars propositi.

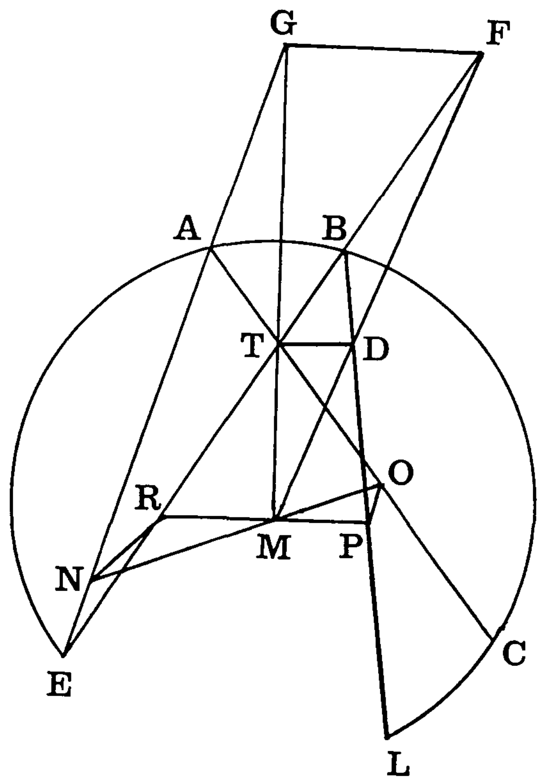

Figure 15

Quod autem obliquitates ille qui sunt extra confluentiam reverse apparent, sic astrue: indicato[?] quod in obliquitatibus illa pars dicitur esse inferior que magis accedit ad partem illam mediam circumferentie ex qua parte disponitur oculus; illa vero superior que magis recedit a medietate versus quam disponitur oculus, unde disposita obliquitate PO extra confluentiam AC et BL, dico quod P inferior est

And notice that to be [inside] the confluence of visual rays means only to be inside the triangle formed by reflected rays, while to be outside the confluence of visual rays means to be only in the space round which they [reflected rays] go after meeting each other.

However, if the reflected rays are parallel, nothing can be inside or outside the confluence. Moreover, there is no such height that does not approach more to the center on one side than on the other either inside or outside the confluence [!].

Further it is manifest from the layout that the same thing occurs in the height outside the confluence of visual rays. Thus let us omit [our] comment, because we have no space for a construction of a figure.

Moreover, there are some people who say that nothing is seen by the rays of reflection in any case in which they are parallel. But this argument is false for this reason: Because a ray comes to A, A is seen although this seems to be contrary to a certain part of the book *On Optics*, *scilicet* those to which a ray comes can be seen.

[XV(H12)] IN CONCAVE MIRRORS, THOSE OBLIQUE LENGTHS WHICH [LIE] INSIDE THE CONFLUENCE OF VISUAL RAYS APPEAR ALSO JUST AS THEY ARE; THOSE [LYING]OUTSIDE [APPEAR] REVERSED.

For example: Let a concave mirror be ABC [Figure 15], and let an oblique magnitude inside the confluence of visual rays be TD, its image be GF, the rays emitted be EA and EB while the reflected be AC and BL. Then as follows: Because ray EB is higher than EA, by the book *On Optics*, whatever is seen by [EB] is seen higher than what is seen by EA. But by the eighth [proposition] of this book, D is seen by [EB] at the convergent point of extended EBF and MDF, M being the center. Therefore by *Elm*-I, D is seen higher than T, and T is seen lower by the same arguments. And T is lower while D is higher. Therefore just as the oblique length TD actually is inside the confluence of visual rays, so is it seen. This is the first part of the proposition.

Further, those oblique lengths which are outside the confluence appear reversed, to which you should add as follows: Given[?] that in oblique lengths the part, which is nearer to the middle side of the circumference where the eye is placed [that is, which is nearer to the center ?], is said to be lower, while the part, which is farther away from the middle [the center ?] to the place of the eye, [is said to be] higher, thus if a length PO is placed outside the confluence of AC and BL, I say that P is the lower terminus, O vero superior. Protracta igitur a P linea per centrum M concurrens ad radium scilicet in quo videtur, id est, ad EB. In puncto R videtur P per 8 huius. Eademque ratione in puncto N videtur O protracta linea ab O per M concurrente ad radium EA sub quo videtur O in puncto N. Erit igitur ydolum PO NR. Sed P videtur in R et O in N ex premissis. Ergo P videtur in EB et O in EA. Sed EB altior est radius quam EA. Ergo P videtur sub altiori radio quam O. Ergo videtur altius per librum de visu. Similiter O videtur inferius ex eadem VI[*sic*]. Sed econverso se habet vera dispositio. Ergo obliquitas PO videtur eversa. Et hoc intelligendum est cum constantiis oculis, et habemus sic utrumque propositorum.

[XVI(H13)] Per plura plana specula idem videri est possibile.

Verbi gratia: sit speculum planum AB [Figure 16: *sic*], cui perpendiculare sit AC, cui at perpendiculare sit CD. Potest autem esse ut AB et AC et CD sint equalia et ut E oculus ita ponatur quod ad medium punctum scilicet G imittatur radius EG. Cum ergo EG radius ad equales angulos reflectatur, erit angulus EGB equus angulo AGF per primum huius. Sed angulus EGB medietas recti. Est enim angulus EBG rectus et latus EB equum est lateri BG. Sit ita ergo AGF medietas recti. Ergo AFG medietas recti. Ergo CFT medietas recti. Ergo CTF et DTM uterque medietas recti. Et ad DTM reflectitur radius EG ultimo ad M et videtur.

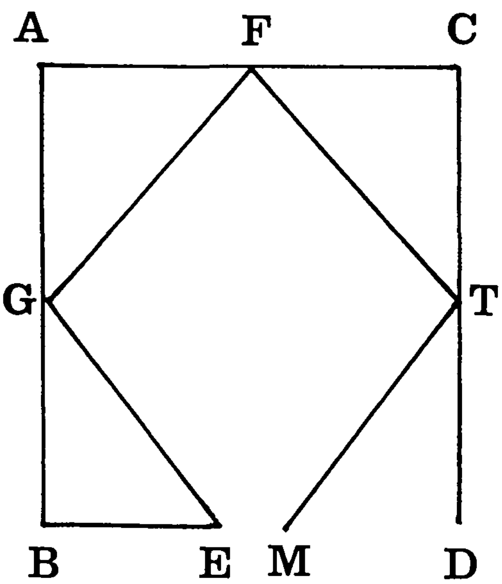

Figure 16

Item quoniam AB ad punctum G in duo equa dividatur, erit AG equalis GB, AF et FC et CT et TD, equalia invicem inter se. Et reflexiones possunt esse sicut predictum est. Vel saltem si proponatur quadratum equilaterum et in eodem inscribatur aliud quadratum similiter equilaterum et equiangulum, oportet tunc quod ita sit ut in predicto exemplo potest ostenderi. Vel melius in figura supposita.

Et nota quod dicit possibile est. Ideo opportet tibi querere dispositionem in qua contingit hoc.

Et nota quod res visa apparet in concursu primi radii et katheti a re visa ad speculum.

[XVII(H14)] Per quotcumque specula plana contingit idem videre; oportet vero ad hoc specul[o]rum poligonium equilaterum et equiangulum constituere.

Verbi gratia: sit speculum[*sic*] poligonium equilaterum et equiangulum ABCDFG [Figure 17], sitque oculus E, res visa sit K. Dividaturque latus speculum[*sic*] DF ad punctum E per equalia, similiter latus FG ad punctum O, similiter GA ad extremity while O the higher. Then let us extend a line from P through the center M until it meets the ray on which P is seen, *i.e.* until to EB. By the eighth [proposition] of this book, P is seen at point R. By the same reason O is seen at point N, if a line is drawn from O through M until it meets the ray EA by which O is seen at point N. Therefore the image of PO will be [seen as] NR. But P is seen at R and O at N by the preceding. Therefore P is seen on EB, and O on EA. But EB is the ray higher than EA. Therefore P is seen by the higher ray than O. Therefore it is seen higher by the book *On Optics*. Similarly O is seen lower by the same [fifth postulate of *On Optics*]. But the true arrangement is to the contrary. Therefore the length PO is seen reversed. And this should be understood in the case of the eyes being unchanged, and thus we have both of what were proposed.

[XVI(H13)] IT IS POSSIBLE FOR THE SAME OBJECT TO BE SEEN BY MEANS OF SEVERAL PLANE MIRRORS.

For example: Let there be a plane mirror AB [Figure 16: *sic*] to which let AC be perpendicular, and moreover let CD be perpendicular to the latter. However, it is possible to assume that AB, AC and CD be equal, and that the eye E be so placed that the ray EG be emitted to the middle point G. Then since the ray EG is reflected at equal angles, ∠EGB will be equal to ∠AGF by the first [proposition] of this book. But ∠EGB is a half of the right angle, because ∠EBG is right and the side EB is equal to the side BG. Thus since ∠AGF is a half of the right angle, ∠AFG is a half of the right angle. Therefore ∠CFT is a half of the right angle. Therefore ∠CTF and ∠DTM are both a half of the right angle. So the ray EB is reflected toward DTM and ultimately to M so that [M] is seen.

Moreover, because AB is divided at G into two equals, AG will be equal to GB, and AF, FC, CT and TD [will be] equal to one another. And reflection is possible [at each mirror]as said before. Or at least if we place an equilateral rectangle and inscribe in it another equilateral and equiangular rectangle of the same kind, then the same thing should happen as could be shown in the preceding example. Or [a proof is] more appropriate in the figure proposed.

Also notice that he [Euclid] says "it is possible." Therefore you should inquire into the arrangement in which this happens.

Notice also that the object of sight appears at the intersection of the first ray and the perpendicular [drawn] from the object of sight to the mirror.

[XVII(H14)] IT HAPPENS TO SEE THE SAME THING BY MEANS OF MANY PLANE MIRRORS WHATEVER. FOR THIS PURPOSE, HOWEVER, IT IS REQUISITE TO CONSTRUCT AN EQUILATERAL AND EQUIANGULAR POLYGON OF MIRRORS.

For example: Let an equilateral and equiangular polygon of mirrors be ABCDFG [Figure 17], let an eye be E, and let an object of sight be K. Divide DF, the side of the mirror, at point E into two equals, and also similarly the side FG at point O, GA at

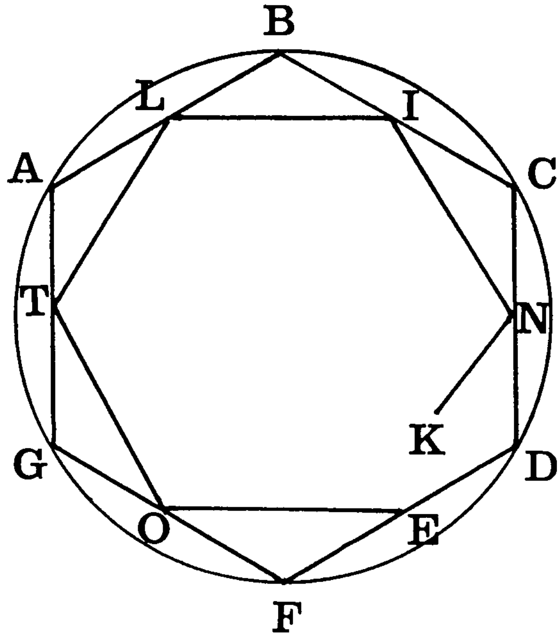

Figure 17

punctum T et quodlibet reliquorum similiter ad puncta L et I et N. Protrahaturque linee ab E in O, ab O in T et sic deinceps continuere usque ad K. Probantur linee ultimo protracte esse radii reflexionum, hoc modo: DF est eque GF ex dispositione poligonii, medietas medietati, ergo EF OF equale est. Consimili ratione OG et GT et TA et AL et LB et BI et IC et CN et ND et DE invicem adequatur, ex quo sic latera EF et OF equantur lateribus OG et GT et anguli EFO et OGT sunt equales ex ypothesy poligonii. Ergo basis EO basi TO adequatur per 1 G[eometrie], et angulus EOF angulo TOG sit equalis. Ergo anguli EOF et TOG sunt anguli reflexionum. Consimili ratione anguli GTO et ATL sunt anguli reflexi et sic usque dum reflectetur radius IN per punctum N ad K. Videtur K per plura plana specula et ita possibile est idem per plura plana specula videri. Quod propositum erat ostendere. Alia etiam potest adhiberi probatio, sed hec sufficiat.

[XVIII(H15)] Est autem et per concava specula et per convexa specula idem videri.

Verbi gratia: describantur specula concava AB scilicet et BC et CD et DF et FA [Figure 18a], et sit oculus E et inmittat radium ad G punctum AB, deinde ad aliquod punctum BC et sic donec perveniat ad punctum T in speculo DF. Item alius radius ab eodem E ad punctum AB speculi inter[*sic*] A et G reflectaturque in eodem speculo ad aliquod punctum et sit quotiens poterit reflecti donec reflectatur in punctum F. Erit TF visus sub angulo AEG. Probetur ergo quod AGE equus est angulo MGB: protracto scilicet speculo plano contingente AB speculum in puncto G et sit ideo RS, alio contingente a puncto M, alio a puncto N, deinde a centro O protracta recta linea ad G et recte aliis ad M et N et T et E. Sic: latera que sunt point T, and all the sides of the rest at points L, I and N. Extend the lines from E to O, from O to T, and so on, continuously up to K. The lines stretched to the end are proved to be the rays of reflection in this way: By the layout of the polygon, DF = GF; because the half [of the former is equal] to the half [of the latter], EF = OF; by the same reasoning, OG = GT = TA = AL = LB = BI = IC = CN = ND = DE, from which the sides EF and OF are consequently equal to the sides OG and GT, and angles ∠EFO and ∠OGT are equal by the hypothesis of the polygon; therefore the base EO is equal to the base TO by *Elm*-I, and ∠EOF is equal to ∠TOG; therefore the angles, ∠EOF and ∠TOG, are angles of reflection; in the same manner ∠GTO and ∠ATL are angles of reflection, and so on until the ray IN will be reflected through point N to K; K is seen by the several plane mirrors, and thus it is possible for the same thing to be seen by the several plane mirrors. This was the proposition that should be made clear. Another proof can also be supplied, but this would be enough.

[XVIII(H15)] IT IS POSSIBLE FOR THE SAME THING TO BE SEEN BY MEANS OF CONCAVE AND CONVEX MIRRORS.

For example: Let us describe concave mirrors, *scilicet* AB, BC, CD, DF, and FA [Figure 18a], and let an eye be E, and let a ray be emitted [first] to point G on AB, next to some point on BC and so forth until it arrives at point T on the mirror DF. Moreover, [let] another ray [be emitted] from the same E to point[s] on the mirror AB, [*i.e.*] A and G, and let it be reflected by the same mirror to another point and so on as many times as it can be reflected until it may be reflected toward point F. TF will be seen by the angle ∠AEG. Then it will be proved that ∠AGE = ∠MGB: that is, let us draw a plane mirror tangent to the mirror AB at point G and let it be RS, while [let us draw] another tangent at point M and still another at point N, and then from the center O let us draw a straight line to G and another lines [respectively] to M, N, T, and E. [Argue] as follows: the sides, EO and GO, are equal to the sides,

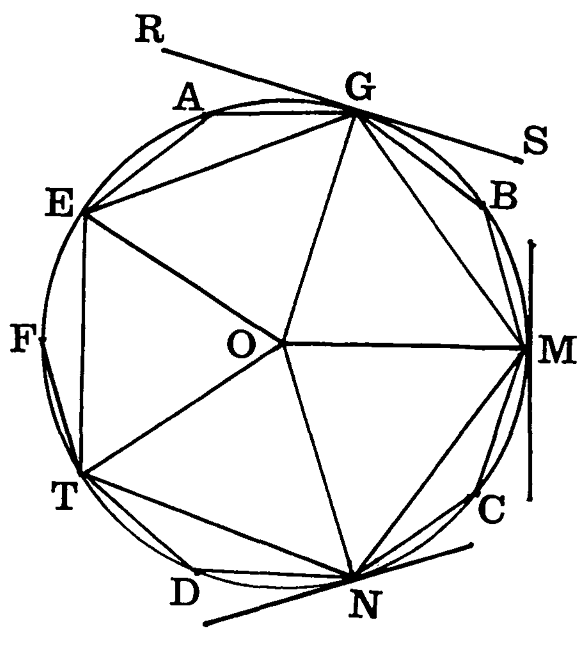

Figure 18a

EO et GO sunt equalia lateribus GO et OM per centri rationem et basis EG equus est basi GM ex dispositione. Sic ita ergo angulus EGO rectilineus equus est angulo OGM rectilineo per primum G[eometrie]. Sed angulus OGR equus est angulo OGS per tertium et primum G[eometrie] et angulus AGR contingens est equus angulo BGS contingente. Ergo demptis illis equalibus AGO equus erit angulo BGO per primum G[eometrie]. Sed angulus EGO et angulus MGO recti[linei] equi sunt ex premissis. Ergo demptis illis equalibus angulus AGE curvilineus equabitur angulo BGM curvilineo. Quare idem anguli [anguli:*del.*] anguli reflexionum possunt esse. Consimili ratione probetur de cetris angulis donec fiat reflexus incidens in T.

Eodem modo probetur quod radius EA reflectatur totiens ad angulos equales experveniat in F et erit arcus TF visus sub angulo AEG rectilineo. Ex collectionibus hoc liquet intelligenti quod aliquid potest videri sub aliquanto angulo et sub maiori. Non tamen videtur maius tam ipso quam oculo existente in moto, licet hoc videatur esse contra librum de visu. Notandum quod potest aliquid videri secundum predictam dispositionem sub aliquanto angulo et adhuc sub maiori et adhuc sub maiori et adhuc et adhuc et adhuc, non tamen sub aliquanto et sub infinito[?]maiori quod[*lege* angulo]. Ex hoc est quod oportet rem videri in concursu radii et linee a centro per conspectum ducte.

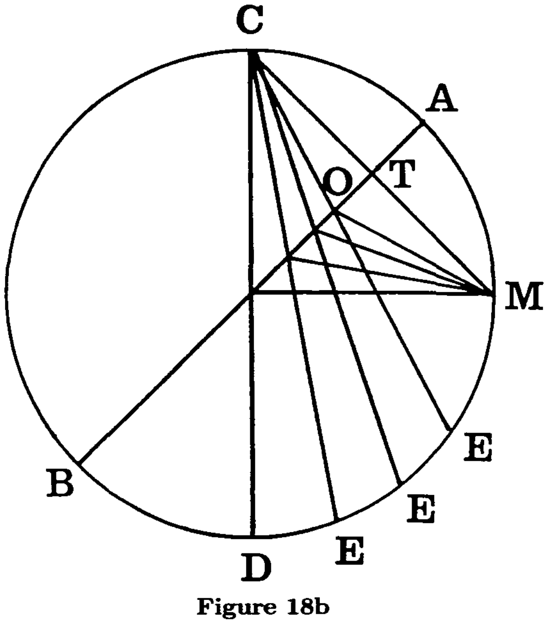

Figure 18b

Item hoc argumentum falsum est: alius oculus aliquid videat et videbit illud a remotiori loco et adhuc a remotiori et adhuc et adhuc, ergo visum videbitur mutare locum. Ad cuius oppositum videndum describatur circulus ABDE [Figure 18b] in quo ponatur AB speculum planum diameter secans CM lineam ad rectos sitque oculus E a quo inmitatur radius ad speculum in puncto O qui in continuum protractus cum CM concurrat in C. Dico igitur eius radii reflexionem esse in M. Quod sic probetur: CTM[?] linea recta ad rectos AB secat scilicet CM ortogonaliter et est diameter circuli, ergo secat eorundem[?] per equalia per 3 G[eometrie]. Ergo CT et MT sunt invicem equalia. Ergo TO sumpta utrium communiter OT et TO, MT et CT adequantur et anguli CTO et MTO sunt equales ex ypothesy. Ergo basis CO equa est basi MO et angulus COT equus angulo MOT per 1 G[eometrie]. Sed angulus COT equus est angulo EOB ex eodem. Ergo angulus MOT equus est angulo EOB. Ergo anguli MOT et EOB sunt anguli reflexi. Ergo radius EOC reflectitur in M. Comsimili ratione si ponatur E, DC diameter, et ab eo inmittatur radius qui in continuum protractus cadat in C, GO and OM, because they are radii, and the base EG is equal to the base GM by the arrangement; so that the rectilinear angle ∠EGO is equal to the rectilinear angle ∠OGM by *Elm*-I; but the angle ∠OGR is equal to the angle ∠OGS by *Elm*-III and -I, and the horn angle ∠AGR is equal to the horn angle ∠BGS; therefore by taking away these equals, ∠AGO will be equal to the angle ∠BGO by *Elm*-I; but by the preceding the rectilinear angles, ∠EGO and ∠MGO, are equal; therefore by taking away these equals, the curvilinear angle ∠AGE will be equal to the curvilinear angle ∠BGM; wherefore these angles can be the angles of reflection. By the same reasoning, proof can be executed concerning other angles, so that the reflection which arrives at T may occur.

By the same way it will be proved that the ray EA can be reflected at equal angles as many times as it can reach F, and so the arch TF will be seen by the rectilinear angle ∠AEG. It will be clear to an intelligent person from the *Collections* that anything can be seen both under a certain degree of angle and under the larger [angle]. However, the object is seen neither larger than itself nor than the eye being in motion, although this seems to be contrary to the book *On Optics*. It should be noted that anything can be seen according to the aforesaid arrangement under a certain degree of angle and under the increasingly more and more larger angles as well, but not under a certain degree of angle and under the infinitely larger angle as well. Now it is concluded from this that the object must be seen at the concourse of the ray and the line drawn from the center to the object of sight.

Moreover, the following argument is false: if an eye sees something and will see it from a farther and increasingly farther places, then the object of sight will seem to change its place. In order to make clear its contrary, let us describe a circle ABDE [Figure 18b] in which let us place a plane mirror AB, [which is also] its diameter, cutting the line CM at right angles. And let the eye be E from which a ray is emitted toward the mirror at point O and let us extend it continuously until it meets CM at C. Then I say that the reflection of the ray arrives at M. Now let us prove as follows: [let] CTM [be] a perpendicular straight line, *scilicet* AB cuts CM perpendicularly and is the diameter of the circle, then it cuts them[?] into equals by *Elm*-III; then CT = MT; then taken TO commonly as OT and TO, because MT = CT and ∠CTO = ∠MTO by the hypothesis, CO = MO and ∠COT = ∠MOT by *Elm*-I; but by the same, ∠COT = ∠EOB; therefore ∠MOT = ∠EOB; therefore the angles ∠MOT and ∠EOB are the angles of reflection; therefore the ray EOC is reflected to M. By the same reasoning, if we place [the eye] E, DC being the diameter, and from E let us emit a ray and extend it continuously so that it may fall on C, reflectetur in M. Et sic cum CM sit kathetus ad AB speculum, semper M videbitur in C. Et ita semper in eodem puncto et semper videbitur sub maiori dum oculus sit D per 3 G[eometrie]. Ostendimus predictum argumentum contrarium esse verum.

Restat igitur secundam partem astuere scilicet per plura convexa contingat idolum videri. Quod astruas: descripto circulo ABNEGC [Figure 18c] inscriptoque in eo exagono equilatero et equiangulo ABNEGC, posito vero M centro a quo linee ad A et B et C et G dirigantur quibus punctis apponantur convexa specula scilicet A et B et C et G a quibus punctis protrahantur iterum contingentes A et B et C et G. Inde sic: latera CM et AM sunt equalia lateribus AM et BM et bases AB et AC sunt equales ex primo G[eometrie] et dispositione. Ergo BAM angulus equus est CAM angulo rectilineo per 1 G[eometrie]. Sed angulus RAM equus est angulo SAM per 3 G[eometrie]. Ergo demptis CAM et SAM[*lege* BAM] equalibus rectilineis RAC rectilineus SAB rectilineo adequatur. Sed angulus TAR et angulus DAS sunt equi. Ergo additis illis angulis RAC et SAB fient anguli COAT et BIAD equales. Ergo erunt anguli reflexionum. Eodemque modo probetur de reflexionibus ceteris precedentibus illam et sequentibus. Dum radius inmissus ab E speculo convexo in N et sic N per plura convexa contingit videri. Sed quocumque loco sit res et per quotcumque specula appareat semper in concursu sicut in uno solummodo.

[XIX] In planis speculis dextra sinistra apparant [*sic*] et sinistra dextra et imago equalis rei vise et distantia a speculo ad imaginem equalis distantie a re visa ad speculum.

Verbi gratia: sit speculum planum ABC [Figure 19a], E sit oculus, TD res visa, radii inmissi EC et EO reflexi vero radii CD et OT. Kathetus vero a T ad speculum sit TAF, a D autem sit DBG. Protrahatur radius EO ad punctum F concurrens in katethum TAF. Similiter EC concurrat in katethum ABG[*lege* DBG] ad punctum G. Erit igitur FG ydolum TD rei vise. Est

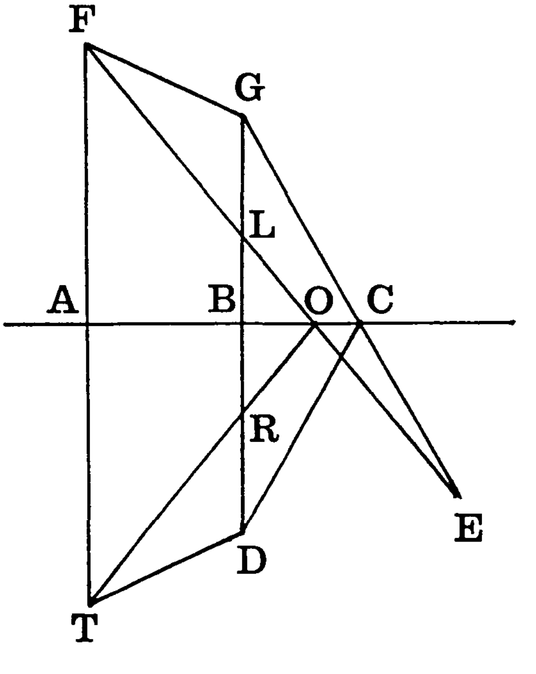

Figure 19a

then it will be reflected to M. Then because CM is the perpendicular to the mirror AB, M will always be seen at C. Thus it will always be seen at the same point and always under a certain amount of angle until the eye reaches D by *Elm*-III. We have shown that the foregoing argument is contrary to the truth.

Therefore it remains for us to add the second part, *scilicet* that it is possible to see the image by the several convex mirrors. Now you should add [as follows]: Let us describe a circle ABNEGC [Figure 18c] and inscribe in it an equilateral and equiangular hexagon ABNEGC, assuming M the center from which let us draw lines to A, B, C, and G to which points let us attach convex mirrors, *scilicet* to A, B, C, and G, and from those points let us, moreover, draw the tangents to A, B, C, and G. Then [argue] as follows: by *Elm*-I and the arrangement, CM = AM, AM = BM, and AB = AC; therefore by *Elm*-I, the angle ∠BAM is equal to the rectilinear angle ∠CAM; but by *Elm*-III, ∠RAM = ∠SAM; therefore by taking away the equal rectilinear angles, ∠CAM and [∠BAM], the rectilinear ∠RAC is equal to the rectilinear ∠SAB; but ∠TAR = ∠DAS; therefore by adding to these the equal angles, ∠RAC and ∠SAB, it will be that ∠COAT = ∠BIAD; therefore they will be the angles of reflection. By the same manner proof will be executed concerning other reflections, whether preceding or following it. As long as a ray is emitted from E to N by convex mirror[s], N can consequently be seen by several convex mirrors. But whatever place an object may be placed and [be seen] by any number of mirrors, it should always appear at the convergence, solely as in a single place.

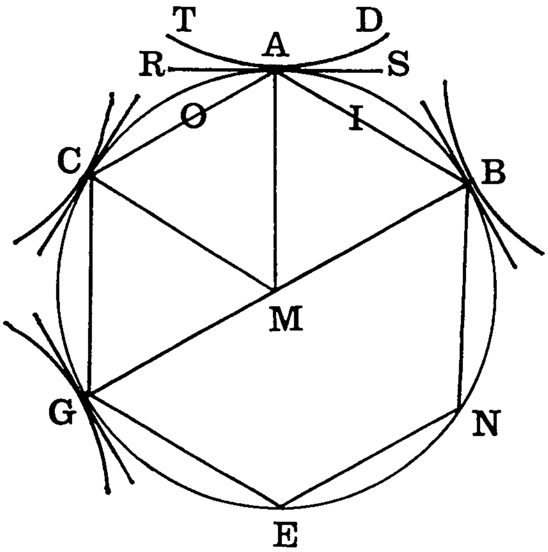

Figure 18c

[XIX] IN PLANE MIRRORS RIGHT APPEARS [TO BE] LEFT, AND LEFT [APPEARS TO BE] RIGHT. AND THE IMAGE IS EQUAL [IN SIZE] TO THE OBJECT OF SIGHT, AND THE DISTANCE FROM THE MIRROR TO THE IMAGE IS EQUAL TO THE DISTANCE FROM THE OBJECT OF SIGHT TO THE MIRROR.

For example: Let a plane mirror be ABC [Figure 19a], an eye be E, TD be an object of sight, and the emitted rays be EC and EO while the reflected rays be CD and OT. Moreover, let the perpendicular [drawn] from T to the mirror be TAF, while the one from D be DBG. Let the ray EO be extended to point F, converging toward the perpendicular TAF. Similarly let EC converge toward the perpendicular [DBG] at point G. Then FG will be the image of the object of sight TD. However, autem D dextrum, T sinistrum. EO vero dexter radius, EC sinister. Cum ergo D videatur in G per 6 huius eademque ratione T in F videtur, T sub dexteriori radio et D sub sinisteriori. Ergo T videtur dextrum et D sinistrum. Sed D est dextrum et T sinistrum. Ergo dextrum apparet sinistrum et sinistrum dextrum. Et hec est prima pars propositi.

Quod autem distantie a speculo et ad ydolum et ad rem visam sint equales, sic astrue: AF est equale TA per 6 huius. Similiter BG BD adequantur ex eodem. Ergo coniunctum[?] AF et BG adequantur TA et BD. Sed AF et BG sunt distantia speculi ad ydolum et TA et BD sunt distantia speculi ad visum. Ergo distantia speculi ad visum equa est distantie speculi ad ydolum. Et hec est secunda pars propositi.

Quod autem ydolum sit equale rei vise, sic astrue: latera BL et BO [*addidi*: sunt equalia et] angulus OBL equus est angulo OBR et angulus BOL BOR per reflexionem et contrapositionem. Ergo reliqui anguli et reliqua latera sunt equalia. Ergo angulus BLO equus est angulo BRO et latus BR lateri BL et latus OL lateri OR per primum G[eometrie]. Sed angulus GLF equus est angulo BLO. Ergo angulus GLF equus est angulo BRO. Sed idem BRO equatur angulo DRT. Ergo angulus GLF equus est angulo DRT. Sed item eisdem argumentis linea TO equalis linee FO. Ergo demptis OL et OR scilicet equalibus ab illis equabuntur LF et TR. Similiter BG et BD sunt equales. Ergo demptis equalibus ab illis scilicet BL et BR equabuntur GL et DR per 1 G[eometrie]. Inde sic: TR est equale LF et GL DR. Ergo TR et DR sunt equales GL et FL et angulus GLF equus est angulo DRT ex premissis. Ergo basis TD equatur basi FG per 1 G[eometrie]. Sed TD est res visa et FG ydolum. Ergo res visa est equalis ydolo. Et hoc est tertium propositum.

Item alia dispositio [Figure 19b]: sit ABCK speculum, E oculus, TD visum et erit GF ydolum. Kathetus a D ad speculum sit DKG. Kathetus a T ad idem sit TCF. Radii inmissi EB et EO reflexi BT et OD. Protrahatur OD in continuum ad lineam TF. In puncto citra F probatur cadere necessario, quia si caderet in F, esset CF equale CL et ex hoc pars toti, quod est inpossibile. Adiectis igitur OF et OT lineis rectis, sic: FR est equale TL et OR et OL, itemque TO est OF, quia CF et CO equantur CT et CO et anguli contenti sub illis equi, ex quo sequitur FOR angulus TOL adequari. Quare et angulus GOF angulo DOT adequatur. Unde manifestum est TD visum ydolo FG adequari.

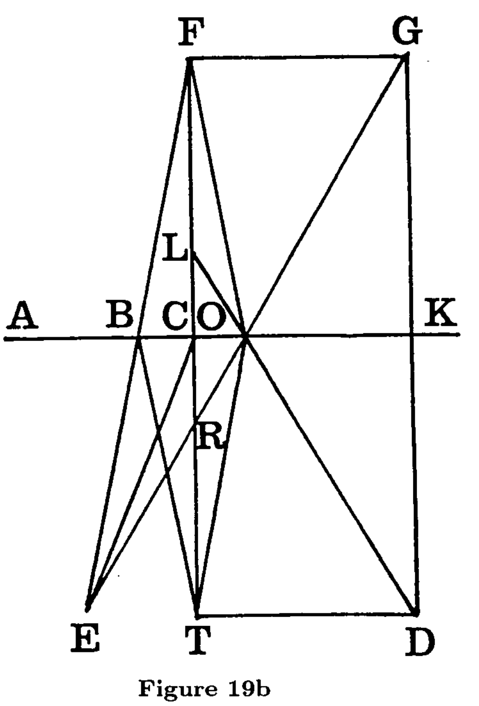

Figure 19b

D is to the right while T to the left. But EO is the right ray [!] while EC [is] the left [!]. Then since D is seen at G by the sixth [proposition] of this book and T is seen at F by the same reason, T [is seen] under the ray more to the right, while D under [the ray] more to the left [!]. Therefore T is seen to the right while D to the left [!]. But D is to the right while T to the left. Therefore right appears to be left while left to be right. And this is the first part of the proposition.

Moreover, the distances from the mirror both to the image and to the object of sight are equal, to which add as follows: by *Elm*-VI, AF = TA; also by the same, BG = BD; therefore by addition, AF + BG = TA + BD; but AF and BG are [respectively] the distance from the mirror to the image while TA and BD are [respectively] the distance from the mirror to the object; therefore the distance from the mirror to the object is equal to the distance from the mirror to the image. And this is the second part of the proposition.

Further, the image is equal to the object of sight, to which add as follows: BL = BO, ∠OBL = ∠OBR, and ∠BOL = ∠BOR by reflection and opposite angles; therefore the remaining angles and sides are equal; therefore ∠BLO = ∠BRO, BR = BL, and OL = OR by *Elm*-I; but ∠GLF = ∠BLO; therefore ∠GLF = ∠BRO; but the same ∠BRO is equal to ∠DRT; therefore ∠GLF = ∠DRT; but further by the same arguments, TO = FO; therefore by taking away the equals, OL and OR, from these, LF = TR; in a similar manner, BG = BD; therefore by taking away the equals, BL and BR, from these, GL = DR by *Elm*-I; Then as follows: TR = LF and GL = DR; therefore, because TR and DR are equal [respectively] to FL and GL and ∠GLF = ∠DRT from the preceding, by *Elm*-I TD = FG; but TD is the object of sight while FG the image; therefore the object of sight is equal to the image. And this is the third proposition.

Furthermore another arrangement [Figure 19b]: Let there be a mirror ABCK, an eye E, and an object TD, then GF will be the image. Let a perpendicular [drawn] from D to the mirror be DKG while a perpendicular [drawn] from T to the same be TCF. [And let] the emitted rays be EB and EO while the reflected be BT and OD. Let us extend OD continuously to the line TF. This is proved to fall necessarily on the point before F, because if this were to fall on F, CF would be equal to CL, and from this a part [would be equal to] the whole, which is impossible. Then connect O F and O T by straight lines, [and argue] as follows: FR = TL, OR = OL, and moreover TO = OF because CF and CO are [respectively] equal to CT and CO and the angles contained by these are equal, from which ∠FOR is concluded to be equal to ∠TOL; wherefore ∠GOF = ∠DOT. Therefore [!] it is manifest that the object TD is equal to the image FG.

[XX] In convexis speculis sinistra dextra apparent et dextra sinistra distantiamque a speculo ydolum minorem habet.

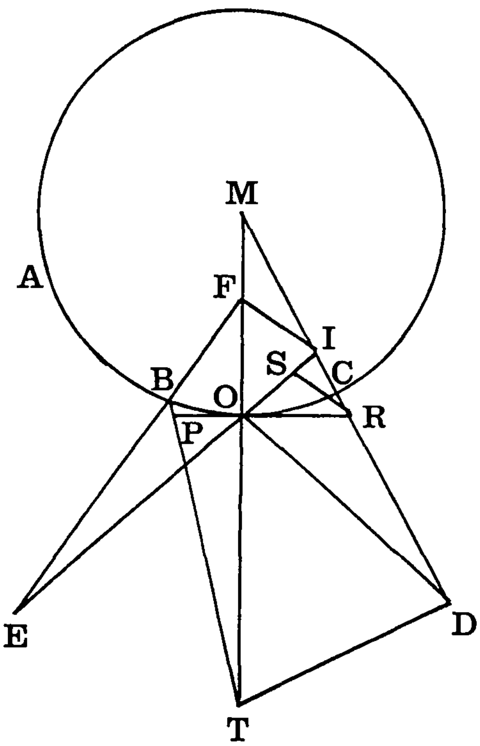

Figure 20

Verbi gratia: sit speculum convexum ABC [Figure 20], oculus vero E, radii inmissi EB et EO, reflexi vero BT et OD, linea a T ad centrum TOM, a D vero ad centrum DCM. Protrahantur radii [dum? concidant:*lege* inmissi] dum concidant in TOM [et] DCM in punctum F et I. Et erit TD visum, IF vero reflexum id est ydolum. Deinde a puncto reflexionis, O scilicet, protrahatur speculum planum PR incidens in DCM ad R punctum. Inde sic: dextra radius est EI et sinistra EF. Dextra autem est T et sinistra D. Sed T videtur sub EF, et D sub EI. Ergo dextrum videtur sinistrum et econverso.

Amplius angulus EOP est equalis angulo DOR. Sed idem IOP[*lege* EOP] angulus est equalis angulo IOR per contrapositionem. Ergo angulus DOR est equus angulo IOR per 1^{am} G[eometrie]. Inde sic: triangulus IOD cuius angulus IOD dividitur in duo equalia per OR lineam secantem IRD in puncto R. Ergo que est proportio DR ad IR ea est CO[*lege* DO] ad OI. Ergo conversim que est CO[*lege* DO] ad OI ea est DR ad IR per 6 et 9 G[eometrie]. Sed DO maius est OI ut probabitur. Ergo DR est maius IR. Ergo DC est maius IC per locum a maiori. Eisdem argumentis probetur quod TO est maius OF. Et iunge. Ergo DC et TO sunt maiora IC et OF. Ergo distantia TD ad ABC speculum maior est quam distantia ydoli IF ad idem ABC.

Quod autem DO sit maius OI, indirecte sic astruas: non est maius, ergo est equale vel minus. Si est equale, sic procede: linea DO est equale OI, ergo su[m]pta OR communiter latera DO et OR sunt equa lateribus OI et OR et anguli sub illis equis lateribus contenti equantur per premissa. Ergo basis basi et reliqui anguli prout equa respiciunt latera adequantur. Ergo anguli ORI et ORD adequantur per primum G[eometrie]. Ergo uterque rectus ex eodem. Quare angulus ORI rectus erit. Sed angulus MOR rectus est per 3 G[eometrie]. Ergo triangulus MOR duos rectos habet angulos, quod est inpossibile per primum G[eometrie]. Reliqua[*sic*] ergo latus DO lateri OI equale esse non possibile.

Erit igitur DO minus OI. Ergo potest [OI] recindi ad quantitatem DO per 1^{am} G[eometrie]. Recindatur ergo in puncto S, inde sic: latera SO et OR sunt equa lateribus OD et OR, et anguli sub illis contentis adequantur ex premissis. Ergo reliqui

[XX] IN CONVEX MIRRORS LEFT APPEARS [TO BE] RIGHT, AND RIGHT [APPEARS TO BE] LEFT. AND THE IMAGE HAS SMALLER DISTANCE FROM THE MIRROR THAN [THE OBJECT DOES].

For example: Let there be a convex mirror ABC [Figure 20], an eye E, the emitted rays EB and EO, the reflected BT and OD, a line [drawn] from T to the center TOM, and [a line drawn] from D to the center DCM. Let us extend the [emitted] rays until they meet TOM [and] DCM [respectively] at points F and I. Then TD will be the object while IF the reflected, *i.e.* the image. Then from the point of reflection, *scilicet* O, let us draw a plane mirror PR which [is tangent to ABC and] falls on DCM at point R. Then as follows: the right ray is EI while the left one EF. However, T is to the right while D to the left. But T is seen by EF while D by EI. Therefore right is seen to be left and *vice versa.*

Furthermore $\angle$EOP = $\angle$DOR. But the same [$\angle$EOP] = $\angle$IOR by opposite angles. Therefore by *Elm*-I, $\angle$DOR = $\angle$IOR. Then as follows: the triangle IOD whose angle $\angle$IOD is bisected by the line OR that cuts IRD at point R; therefore DR : IR = [DO] : OI; then by the conversion of proportion, [DO] : OI = DR : IR by *Elm*-VI and -IX; but DO > OI as will be proved; therefore DR > IR; therefore DC > IC because of their size; by the same arguments it will be proved that TO > OF; add [them], then DC + TO > IC + OF; therefore [!] the distance of TD concerning the mirror ABC is larger than the distance of the image IF concerning the same ABC.

You should add as follows the indirect proof that DO > OI: if not, then the former is either equal to or smaller than the latter; if equal, proceed as follows: DO = OI, then with OR taken common, the sides DO and OR are equal to the sides OI and OR, and the angles contained by the equal sides are equal by the assumptions; therefore the base is equal to the base and so are the remaining angles as long as they correspond to the equal sides; therefore by *Elm*-I, $\angle$ORI = $\angle$ORD; therefore both are right by the same; wherefore $\angle$ORI will be right; but $\angle$MOR is right by *Elm*-III; therefore the triangle MOR would have two right angles, which is impossible by *Elm*-I. Therefore it becomes impossible for the side DO to be equal to the side OI.

Therefore it will be that DO < OI. Then by *Elm*-I, [OI] can be cut away by the quantity DO; therefore let it be cut away at point S, then as follows: the sides SO and OR are equal to the sides OD and OR, and the angles contained by them are equal by the assumptions; therefore the remaining angles of anguli triangulorum ROS et ROD prout respiciunt equalia latera sunt equales. Ergo angulus ORS equatur angulo ORD per primum G[eometrie]. Sed ORM est maior angulo ORS. Ergo angulus ORM est maior angulo ORD. Ergo angulus ORM est obtusus. Sed angulus MOR rectus ex premissis. Ergo triangulus ORM habet duos angulos duobus rectis maiores, quod est inpossibile ex 1° G[eometrie]. Ergo latus DO neque minus nec eque erit OI.

Ergo utrum propositorum verum est.

[XXI] In convexis speculis ydolum minus apparet re cospecta.

Esto verbi gratia speculum convexum ABC [Figure 21], oculus E, TD visum, radii inmissi EB et EC, reflexi vero BT et CD. Protrahatur ergo speculum NBLGZ planum ad puncta relexionum scilicet B et C inmittaturque radii ad illis puncta que sunt L et G sub quibus videatur TD. Si poterit fieri inde sic: angulus ABE maior est angulo NBE. Sed idem ABE equus est angulo CBT curvilineo. Ergo angulus CBT curvilineus maior est angulo NBE. Ergo multo magis LBT rectilineus maior est NBE. Item NBE maiores est angulo BLT[*lege* BLE], quia extrinsecus. Sed idem BLT[*lege* BLE] equatur CLT per reflexionem. Ergo NBT[*lege* NBE] maior est CLT. Sed CLT maior est angulo LBT, quia extrinsecus. Ergo NBT[*lege* NBE] maior est LBT, cuius contrarium probatum est. Relinquitur ergo radium sub quo videtur T per speculum planum citra angulum BTM posse intercipi, quoniam sequatur inpossibile. Eodem modo neque radius sub quo videtur D in angulo predicto potest intercipi. Incidens radius sub quo T [videtur] in N extra angulum predictum et alius ad punctum G videtur. Ergo TD per planum speculum sub maiori angulo quam per convexum. Ergo maius videtur per planum speculum TD quam per convexum. Sed quicquid videtur per planum videtur tantum quantum ipsum est [per] penultimam huius. Ergo TD videtur tantum quantum ipsum est per planum et videtur minus per convexum quam per planum. Ergo TD videtur minus quam ipsum sit per convexum. Et hoc erat propositum ostendere.

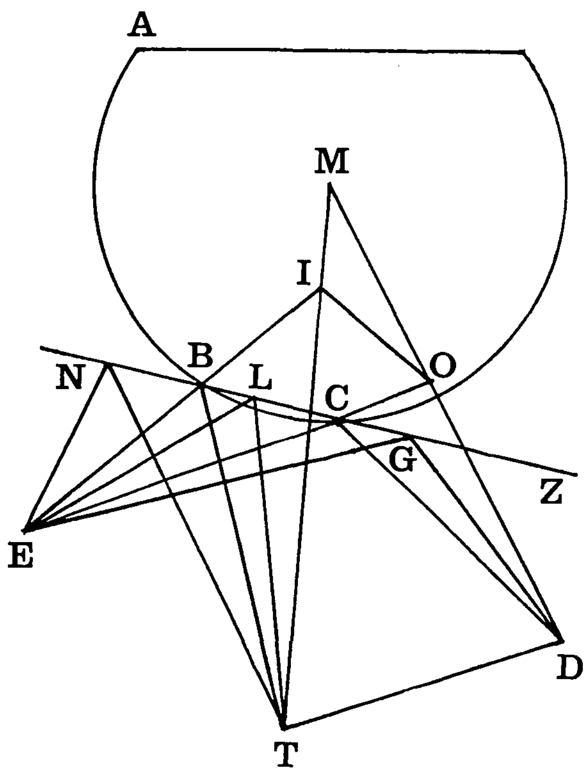

Figure 21

triangles ROS and ROD as long as they correspond to the equal sides are equal; therefore by *Elm*-I, ∠ORS = ∠ORD; but ∠ORM > ∠ORS; therefore ∠ORM > ∠ORD; therefore ∠ORM is obtuse; however, ∠MOR is right by the assumptions; therefore the triangle ORM would have two angles larger than two right angles, which is impossible by *Elm*-I.

Therefore the side DO will neither smaller than nor equal to OI. Therefore each of what were proposed is true.

[XXI] IN CONVEX MIRRORS IMAGE APPEARS SMALLER THAN THE OBJECT OF SIGHT.

Let there be, for example, a convex mirror ABC [Figure 21], an eye E, an object of sight TD, the emitted rays EB and EC, and the reflected [rays] BT and CD. Then let us draw a plane mirror NBLGZ at the points of reflection, B and C, and emit the rays to points L and G under which TD may be seen. If this can happen, then [argue] as follows: ∠ABE > ∠NBE; but the same ∠ABE is equal to the curvilinear angle ∠CBT; therefore the curvilinear angle ∠CBT is larger than ∠NBE; therefore the rectilinear ∠LBT is, *a fortiori*, larger than ∠NBE; moreover, ∠NBE > ∠[BLE] because of exterior [angle]; but the same ∠[BLE] is equal to ∠CLT by reflection; therefore ∠[NBE] > ∠CLT; but ∠CLT > ∠LBT because of exterior [angle]; therefore ∠[NBE] > ∠LBT, the contrary of which is already proved. Therefore it remains that the ray by which T may be seen by the plane mirror can be intercepted on the nearer side than angle ∠BTM, because otherwise the impossible would entail. In the same manner the ray under which D may be seen cannot be intercepted on the aforesaid angle. The ray under which T [is seen] falls on N outside the aforesaid angle, and another [D] is seen at point G. Therefore TD [is seen] under the larger angle by the plane mirror than by the concave [mirror]. Therefore TD is seen larger by the plane mirror than by the concave [mirror]. But by the penultimate [proposition] of this book, whatever is seen by the plane [mirror] is seen just the same size as it is. Therefore by the plane [mirror] TD is seen just the same size as it is, and it is seen smaller by the convex than by the plane. Therefore by the convex [mirror] TD is seen smaller than it is. And this was the proposition that should be made clear.

[XXII] A minoribus convexis speculis apparent ydola minora.

Verbi gratia: sit convexum minus ABC [Figure 22a], maius vero DFG, oculus E inmittans super utrumque tam ABC quam DFG radios. Dicet falsigrafus radium eundem ad utrumque incidere et ad idem punctum reflectere, scilicet radius inflexus EB per F reflexi FT et BT. Ad eius rei inprobationem protrahatur linea ab M centro utriusque ad F maioris speculi reflexionem MR scilicet dividens angulum EFT per equalia. Per equalia, quod sic probatur: anguli DFM et DFR sunt equales duobus rectis, similiter anguli MFG et GFR. Ergo demptis DFM et GFM equalibus, angulus DFR erit equalis angulo GFR per primum G[eometrie]. Sed angulus EFD est equus angulo GFT per 1 huius. Ergo demptis illis angulus EFR equabitur angulo TFR, quod erat propositum. Consimili quoque ratione angulus OBF equus est angulo OBG. Inde sic: angulus OBG[*lege* OBF] maior est angulo BFM, quia extrinsecus. Sed angulus BFM angulo EFR adequatur per contrapositionem. Ergo angulus OBG[*lege* OBF] est angulo EFR maior. Ergo angulus OBF[*lege* OBG] maior est angulo TFR per media, quia uterque totalium duobus equalibus petitur. Ergo totalis GBF maior est totali EFT, intrinsecus scilicet extrinseco maior, quod est inpossibile per 1 G[eometrie]. Relinquitur ergo predicta dispositione radios inflecti et reflecti posse.

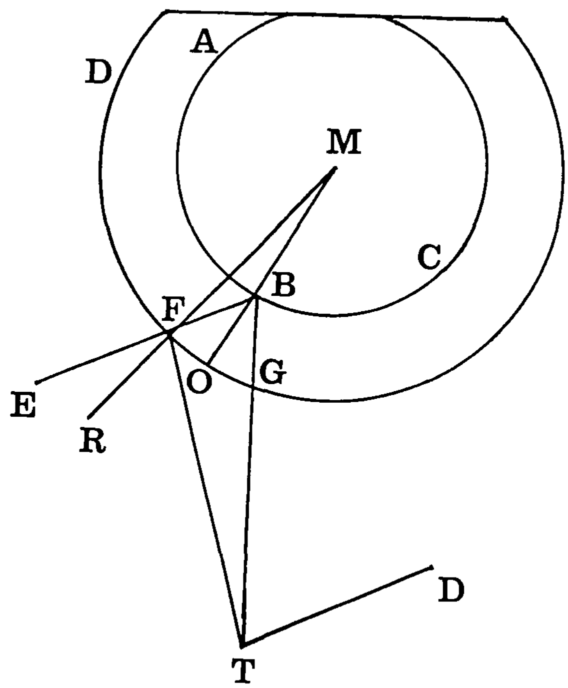

Figure 22a

Proponet igitur aliter falsigrafus ut incidant secundo loco radii EF et EG [Figure 22b] reflectanturque in T a maiori speculo scilicet DFG, ad minus autem speculum ABC incidat EB extra angulo scilicet FEG, reflectaturque idem ad T. Quod si est, cum argumentis eadem ratione extra eum angulum possit inmitti, videtur visum sub maiori angulo sub[*del.*] per speculum ABC quam per speculum DFG, et sic contrarium propositi obtineres. Ideoque inprobetur. Protracta recta linea scilicet RS ad puncta reflexionum scilicet G et F. Inde sic: angulus TGS maior est angulo GFT, quia extrinsecus. Ergo est maior angulo GFT maiori curvilineo per locum a maiori. Sed angulus GFT maior curvilineus equus est

[XXII] IMAGES APPEAR SMALLER BY SMALLER CONVEX MIRRORS.

For example: Let a smaller convex [mirror] be ABC [Figure 22a], a larger be DFG, and an eye be E, emitting the rays on both ABC and DFG. An adversary would say that the same ray would fall on both [mirrors] and be reflected to the same point, *scilicet* that the ray be emitted through F [as] EB [and] be reflected [as] FT and BT. In order to refute this, let us draw a line MR from the center M of both [mirrors] to the reflecting point F on the larger mirror, dividing the angle ∠EFT into two equals. "Into two equals" is proved as follows: ∠DFM + ∠DFR = 2∠R, and also ∠MFG + ∠GFR = 2∠R; therefore by taking away the equal ∠DFM and ∠GFM, the angle ∠DFR will be equal to the angle ∠GFR by *Elm*-I; but ∠EFD = ∠GFT by the first of this book; therefore by taking away these, the angle ∠EFR will be equal to the angle ∠TFR, which was the proposition. By the same reasoning, ∠OBF = ∠OBG. Then as follows: ∠[OBF] > ∠BFM because of exterior [angle]; but by opposite angles, ∠BFM = ∠EFR; therefore, ∠[OBF] > ∠EFR; therefore by the "intermediation," ∠[OBG] > ∠TFR, because each of them is supposed to [be equal respectively] to [each of] the two angles [of the preceding inequality]; therefore the whole angle ∠GBF is larger than the whole angle ∠EFT, that is, the interior angle is larger than the exterior, which is impossible by *Elm*-I. Therefore it remains that in this layout the rays can[not] be emitted and reflected.

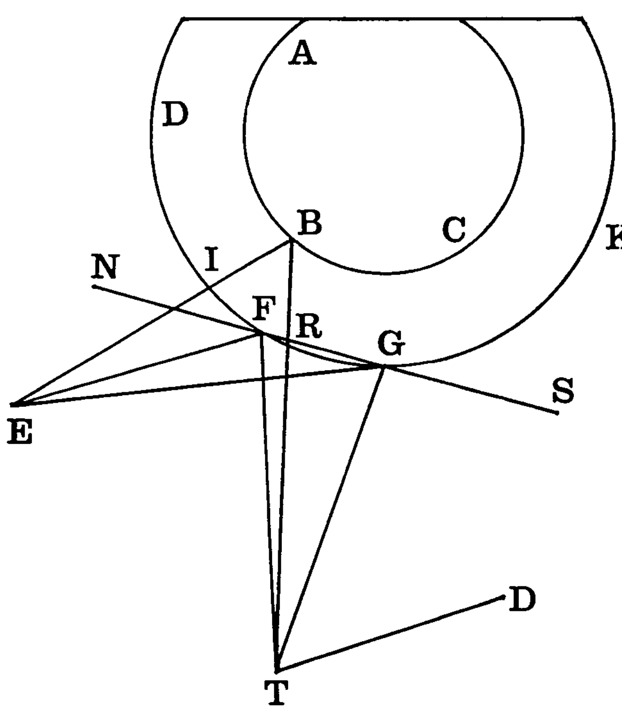

Figure 22b

Then the adversary would propose another possibility that the rays, EF and EG [Figure 22b], would fall on the other point[s] and be reflected to T by the larger mirror DFG, while EB would fall toward the smaller mirror ABC on the outside of the angle FEG and be reflected to T. Now if so, since it could be emitted on the outside of that angle by the same kind of argument, the object of sight would be seen under the larger angle by the mirror ABC than by the mirror DFG, so that you could obtain the contrary of the proposition. Therefore this should be refuted. Let us draw a straight line RS passing through the points of reflection, *i.e.* G and F. Then as follows: ∠TGS > ∠GFT because of exterior [angle]; therefore the latter is larger than the larger curvilinear angle ∠GFT because of its largeness; but the larger curvilinear angle ∠GFT is equal eodem EFD per 1 huius. Ergo a primo angulus TGS maior est eodem EFD. Ergo est maior angulo EFR[*lege* EFN] partis scilicet EFD. Sed angulus EFR[*lege* EFN] est maior angulo EGF, quia extrinsecus. Ergo idem est maior angulo EGF partis eius curvilinee. Sed EGF curvilineus est equalis TGR curvilineo per 1 huius. Ergo EFR[*lege* EFN] maior est eodem TGK, ergo est maior TGS, quo minor[rem] esse prius est demonstratum, et hoc est inpossibile. Relinquitur ergo quod predicto modo possiunt[*sic*] ad ABC et DFG specula radii inmitti et reflecti.

Ponet igitur aliter falsigrafus ut scilicet ad ABC et DFG specula.

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[XXIII] In convexis speculis ydola convexa apparent.

Verbi gratia: sit speculum convexum ABC [Figure 23: *sic*] et visum ab oculo E sit TDFG, oculus dispositus inter visum et speculum, inmittens radios EB et EO et EI et ER reflectanturque ad T et D et F et G. Linee per speculum et visa ad M centrum sint TBM et DOM et FIM et GRM. Videtur ergo T in B per 8 huius eademquoque ratione D in O, et F in I, et G in R. Quare BOIR erit ydolum rei vise id est TDFG. Sed BOIR convexum est. Quare ydolum convexum apparet, quod erat propositum.

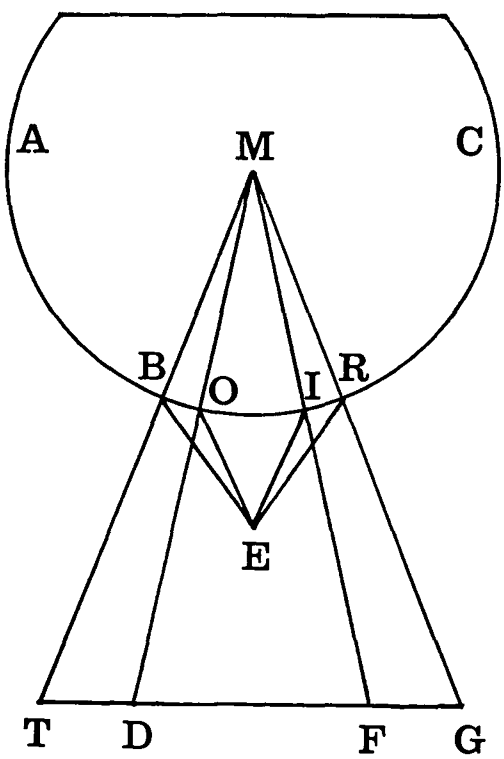

Figure 23

Sin autem radii a visis ad M centrum in speculo incidant, tantum per propositionem que asserit quod oblique longitudines sicut sunt apparent et quia[?] que sub propinquiori propinquiora appareret, convexum unde dicendum est. Proposita quacumque plana superficie dicendum quod videtur convexa nisi sit in eadem superficie oculus cum illa. Itemque ex hoc liquetur quod aliqua concava superficies oculo superposito videbitur plana.

[XXIV] Si super centrum ponatur oculus in concavis speculis, nichil videtur nisi oculus.

Verbi gratia: descripto quovis speculo concavo [Figure 24] quilibet radius inmissus [per]semidiametrum unde sequitur radium quemlibet inmissum per se reflecti per primum huius. Et per inpossibile: si enim alibi reflecteretur, haberemus partem toti equalem esse per probationes.

to the same ∠EFD by the first [proposition] of this book; therefore from the first [inequality], ∠TGS > ∠EFD; therefore the former is larger than the angle ∠[EFN] which is part of ∠EFD; however, ∠[EFN] > ∠EGF because of exterior [angle]; then the latter is larger than the part of its curvilinear angle, ∠EGF; but the curvilinear ∠EGF is equal to the curvilinear ∠TGR by the first [proposition] of this book; therefore, ∠[EFN] > ∠TGK > ∠TGS; [however] the first is already proved to be smaller than the last, so this is impossible. Therefore it remains that the rays can[not] be emitted and reflected toward the mirrors ABC and DFG in the aforesaid manner.

Then the adversary would suppose another way, *scilicet* that toward the mirrors ABC and DFG ...

{Then follows a wide blank space.}

[XXIII] IN CONVEX MIRRORS IMAGES APPEAR CONVEX.

For example: Let a convex mirror be ABC [Figure 23: *sic*], and let an object seen by the eye E be TDFG. Let the eye be placed between the object and the mirror, emitting the rays EB, EO, EI and ER, and let them be reflected [respectively] to T, D, F and G. Let us draw lines to the center M through the mirror and object-points, TBM, DOM, FIM and GRM. Then T is seen at B by the eighth [proposition] of this book [!] and by the same reason, D at O, F at I, and G at R [!]. Wherefore BOIR will be the image of the object TDFG. But BOIR is convex. Therefore the image appears convex, which was the proposition.

But if the rays fall on the mirror from the object-points to the center M, we should say [the image] to be convex, solely because of the proposition which asserts that oblique lengths appear just as they are and because that which [is seen] by the nearer rays appears nearer. Even if we place any plane surface, we should say that it be seen to be convex, unless the eye is on the same surface with it. Moreover it will be clear from this that any concave surface will be seen to be flat by the eye placed upon it [?].

[XXIV] IN CONCAVE MIRRORS IF AN EYE IS PLACED ON THE CENTER, NOTHING IS SEEN EXCEPT FOR THE EYE.

For example: Let us describe any concave mirror [Figure 24], [then if] any ray be emitted [along]a radius, it is accordingly concluded that any emitted ray would be reflected on itself by the first [proposition] of this book. Also [as follows] by the *reductio ad absurdum*: for if it should be reflected anywhere else, we would conclude by steps of proof that part would be equal to the whole.

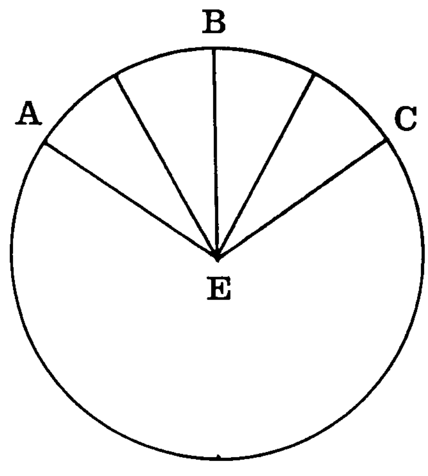

Figure 24

[XXV] In concavis speculis si super peryferiam ponatur oculus vel extra illam, non apparet oculus.

Verbi gratia: proposito ABC speculo concavo [Figure 25], oculo E existente in peryferia, aliquis radius inmissus erit diameter. Ergo reflectetur per se per primum[*lege* secundum] huius. Ergo oculus videtur, cuius contrarium hic proponit actor[*sic*]. Ad hoc dicendum quod diametrum radius non comprehenditur sub hic dicto. Immo sic intelligendum est: nisi reflectatur per diametrum. Et sit diameter ipse radius, tunc enim reflectitur per se ex precedenti. Vel dicatur quod si etiam reflectatur per se non tamen videtur, quia non perduci recta linea ad radium per centrum a conspecto. Sed dicet radium ipsum esse lineam ductam a conspecto ad centrum. Hoc solum circumscribere[?] oportet, quoniam diameter nunquam sit linea ducta a conspecto ad centrum. Ponatur ergo oculus in peryferia videns C non per diametrum, scilicet incidat ad T. Sunt igitur anguli ETA et CTB equales per primum huius. Ergo angulus ETB maior est angulo ETA. Ergo ET non potest reflecti per ET. Ergo E non videtur. Si vero dicat quod linea ducta a conspecto ad centrum reflectatur in E. Ergo eadem CME[*sic*] est diameter speculi. Ergo C videtur in C. Ibi enim est primus concursus radii lineeque ducte a conspecto per centrum ut manifestum est in EAB triangulo. Et sic habetur propositum.

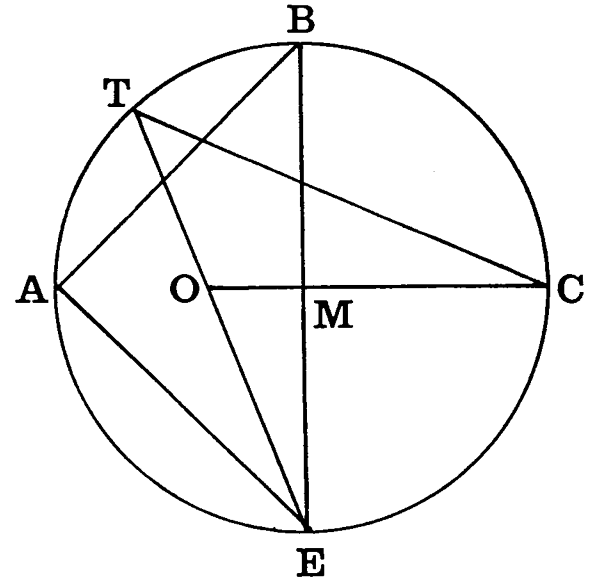

Figure 25

[XXVI] In concavis speculis si inmittens diametrum spere e centro ducas rectas lineas et in altera parte ponas oculum, nichil entium ex illa parte in qua est oculus videtur, hoc est neque eorum que in diametro sunt neque eorum que extra diametrum existunt.

Verbi gratia: sit speculum concavum ABC [Figure 26], oculus autem E in diametro CO. Ad primum E[*sic*] positionis[?], dico ergo quod nichil videt[ur] in quarta speculi que est AMO nec etiam in semidiametro OM videtur aliquid. Quod sic probatur: E oculus inmittit radium ad G[*lege* D]. Ergo non reflectitur in KMO, quia sic reflecteretur in quarta illa, verbi gratia, ad punctum S in semidiametro

[XXV] IN CONCAVE MIRRORS IF AN EYE IS PLACED EITHER ON THE CIRCUMFERENCE OR OUTSIDE IT, THE EYE NEVER APPEARS.

For example: Let there be a concave mirror ABC [Figure 25] and an eye E lying on the circumference, [then] some emitted ray can be a diameter. Therefore it will be reflected on itself by the [second proposition] of this book. Therefore the eye can be seen, whose contrary an advocate proposes here [though]. For this we should say that a diametrical ray is not dealt with in this case. Rather this should be understood as follows: "unless it is reflected along the diameter." For if the diameter itself were the ray, then it would be reflected on itself by the preceding. Or we should say that even if it were to be reflected on itself, nonetheless it would be unseen, because a straight line cannot be drawn out to the [emitted] ray from the object via the center [?]. But he would say that the ray itself could be a line drawn from the object to the center. We should confine only this case, because the diameter can never be a line drawn from the object to the center. Therefore let us place an eye on the circumference which sees C not by the diameter, *scilicet* let [a ray] be incident upon T. Then by the first [proposition] of this book, $\angle ETA = \angle CTB$; therefore $\angle ETB > \angle ETA$; therefore ET cannot be reflected along ET; therefore E is not seen. However, if someone insists that the line drawn from the object to the center would be reflected to E; then the same CME would be a diameter of the mirror; therefore C would be seen at C. For at that point is the first intersection of the ray and the line drawn from the object through the center, as is evident in the triangle EAB. And so the proposition is concluded.

[XXVI] IN CONCAVE MIRRORS, IF, BY INSERTING A DIAMETER OF A CIRCLE, YOU ERECT A LINE FROM THE CENTER [SO AS TO BISECT THE SEMICIRCLE] AND YOU PLACE THE EYE ON ONE SIDE, NOTHING OF THOSE IS SEEN WHICH ARE ON THE SAME SIDE AS THE EYE IS, *i.e.* WHICH LIE EITHER ON THE DIAMETER OR OUTSIDE IT.

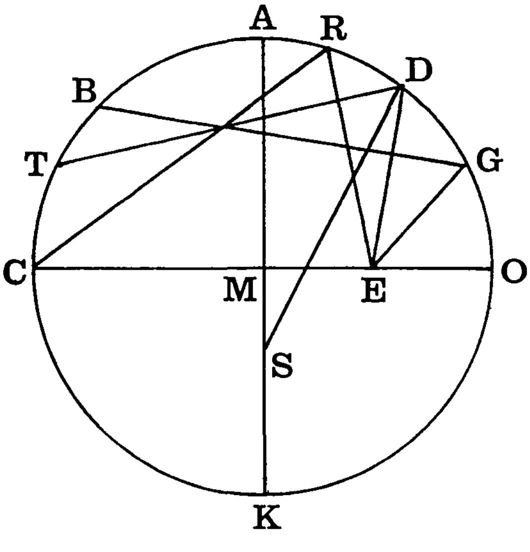

Figure 26

For example: Let there be a concave mirror AGD [Figure 26] and an eye E on the diameter CO. For the first part of the proposition, I say that nothing is seen on the quarter part of the mirror which is AMO and that anything is unseen on the radius MO. This is proved as follows: the eye E emits a ray to [D]; then it is not reflected toward KMO, because if it were reflected toward that quarter, for example, let it be reflected to point S on the radius

KM reflectatur, inde sic: angulus ADS equatur angulo GDS[*lege* GDE] per primum huius. Ergo angulus maioris portionis angulo minoris adequatur, quod est inpossibile per librum de corda et arcu. Idem autem sequitur ubicumque reflectatur ad IIIIam KMO. Reflectetur igitur EG ad GB[*lege* B] et ED ad T et ER ad C, et non in OMK possunt reflecti.

[XXVII] Si in concavis speculis in diametro ponantur oculi equaliter distantes a centro, neuter oculorum videbitur.

Verbi gratia: sit speculum concavum ABKC [Figure 27] cuius diametrum AC, M centrum, oculorum unus E reliquus O. Dico igitur quod nec O videt E, nec E O, nec E se, nec O se, quia neuter eorum est in centro.

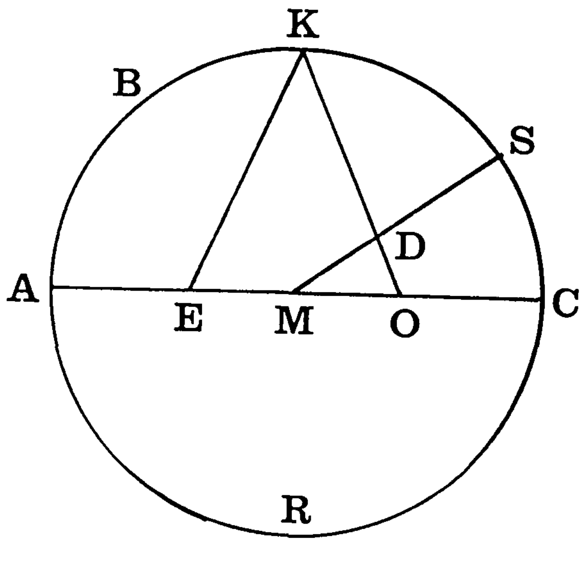

Figure 27

Si vero E oculus dicatur videri ab O, ergo linea ducta a conspecto in centrum ubi videtur E concurit[*sic*] cum radio sub quo videtur idem E. Sed nulla sic ducta concurrit ad radium reflexum vel inmissum nisi EC diameter, de quo dictum est in penultima propositione. Nunquam ipsum nunquam possibile esse lineam ductam a conspecto ad centrum. Ideoque dicet forsitan EMD esse rectam lineam ad concurrentem cum radio inmisso ad punctum D et totum E apparebit in D. Contra AEMDS est recta linea in circulo transiens per centrum M, ergo est diameter per tertium G[eometrie]. Ergo dividit circulum per equalia. Ergo portio AEMDS est equalis relique parti circuli. Ergo est maior portione ARC. Sed AC item est diameter. Ergo portio ARC equatur portioni ABKSC. Ergo portio ARC maior parte ABKSC que est AEMDS. Ergo ARC portio eodem est maior et minor, quod est inpossibile. Relinquetur ergo quod alter oculorum videat se vel alterum esse inpossibile.

[XXVIII] In concavis speculis si diametrum ducas et a centro ducas perpendicularem ad diametrum speculi quam dividat equedistans predicto diametro per equalia e quo perpendicularis ad diametrum dividat equedistantem per equalia et in equedistanti ponas oculos equedistanter a centro vel inter diametrum et equidistantem equidistanter a centro, neuter oculorum apparebit.

Verbi gratia: sit speculum concavum ABC [Figure 28a], oculus E et alter O, linea autem perpenduculariter ab eius diametro AMC et puncto M erecta, KM, then as follows: by the first [proposition] of this book, ∠ADS = ∠[GDE]; therefore the angle of larger portion would be equal to the angle of smaller [portion], which is impossible by the book *On the chord and the arc.* Moreover, the same thing would entail whenever it would be reflected toward the quarter KMO. Therefore EG will be reflected to [B], ED to T, and ER to C, and they cannot be reflected toward OMK.

[XXVII] IF IN CONCAVE MIRRORS THE EYES, EQUALLY DISTANT FROM THE CENTER, WERE PLACED ON THE DIAMETER, NEITHER OF THEM WILL BE SEEN.

For example: Let there be a concave mirror ABKC [Figure 27] whose diameter is AC, the center M, and one of the eyes E while the other O. Then I say that O does not see E, nor E does O, and neither E nor O does itself, because neither of them is at the center.

However, if the eye E is said to be seen by O, then a line drawn from the object to the center, on which E is seen, would meet the ray by which the same E is seen. But any line thus drawn would not meet the ray either reflected or emitted, except for the diameter EC concerning which we have already said in the penultimate proposition. It is impossible for it to be a line drawn from the object to the center. Therefore someone might say that EMD may possibly be a straight line which would meet the emitted ray at point D so that the whole E would appear at D. On the contrary, AEMDS would be a straight line within the circle that passes through the center; therefore it would be a diameter by *Elm*-III; therefore it would bisect the circle; therefore the portion AEMDS would be equal to the remaining portion of the circle; therefore it would be larger than the portion ARC; but AC is also a diameter; therefore the portion ARC would be equal to the portion ABKSC; therefore the portion ARC would be larger than the part of ABKSC, *i.e.* AEMDS; therefore the portion ARC would be [at the same time] larger and smaller than the latter, which is impossible. Therefore it will be left as impossible that one of the eyes would see either itself or the other.

[XXVIII] IN CONCAVE MIRRORS IF YOU DRAW A DIAMETER, AND FROM THE CENTER YOU DRAW SUCH A PERPENDICULAR TO THE DIAMETER OF THE MIRROR THAT A LINE PARALLEL TO THE FIRST DIAMETER BISECTS, AND AT THE BISECTION POINT LET THE PERPENDICULAR TO THE DIAMETER DIVIDE A PARALLEL LINE INTO TWO EQUALS, AND YOU PLACE [EITHER] AT EQUAL DISTANCE THE EYES EQUALLY DISTANT FROM THE CENTER OR BETWEEN THE DIAMETER AND THE PERPENDICULAR EQUALLY DISTANT FROM THE CENTER, THEN NEITHER OF THE EYES WILL APPEAR.

For example: Let there be a concave mirror ABC [Figure 28a], one eye E and the other O, while a line perpendicularly erected from its diameter AMC and point M, scilicet MRB, dividens ORE lineam per equalia ad punctum R ortogonaliter, in cuius terminis O et E oculi ponantur. Dico igitur quod nec alter videt se nec reliquum. Quod sic probatur: utraque MB et OE secat reliquam per equalia ortogonaliter. Ergo MR et RE latera sunt equalia OR et RB et angulus MRE equus est angulo ORB. Ergo basis basi et reliqui anguli prout respiciunt equalia sunt equales. Ergo angulus EMR adequatur angulo OBR. Ergo linee BO et EM sunt equedistantes per primum G[eometrie]. Ergo OM[*lege* EM] et EB[*lege* OB] nunquam concurrunt. Eadem ratione EM[*lege* OM] et OB[*lege* EB] equidistant. Ergo nunquam concurrunt. Sed non potest videri E ab O nisi concurrant OB et EM, nec O ab E nisi EB et OM concurrant per 8 huius. Non est autem dubium quin E inmissum ad B reflectatur ad O et econverso.

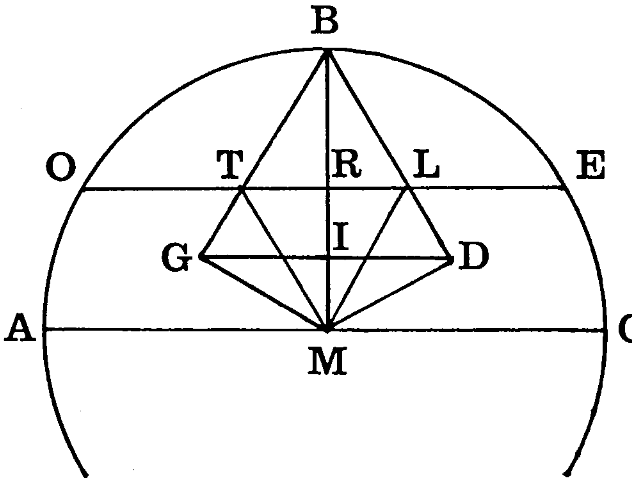

Figure 28b

Item in ABC speculo [Figure 28b] secat linea EO lineam MB per equalia ad punctum R. G vero et D oculis positis equedistanter a centro inter OE et centrum, et eisdem adiectis linea GID, patet igitur ex premissis lineam TB et TM et ML et LB invicem adequari et etiam angulum BMD maiorem esse angulo TBM, quia habet partem scilicet angulum BML angulo TBM equalem ex premissis. Sed anguli BGM et BMG et GBM sunt equales duobus rectis per primum G[eometrie]. Ergo anguli BGM et GMB et BMD sunt maiores duobus rectis. Ergo GB et MD nunquam concurrunt versus B et D protracte, quia anguli GMB et BMD faciunt angulum GMD. Quod autem ex eisdem argumentis habeas quod MG et DB nunquam concurrant manifestum in eandem partem. Sed non potest videri G et D nisi in concu[r]su illarum per 8 huius. Ergo non possunt videri.

Notandum tamen quod post oculum concurrunt, ideoque intelligendum est de oculo vidente tantum ante se. Si vero ita deponerentur oculi quod linea secans semidiametrum BM interesset[?] oculis et centro, uterque[?] oculorum videret tunc reliquum ut patet intelligenti.

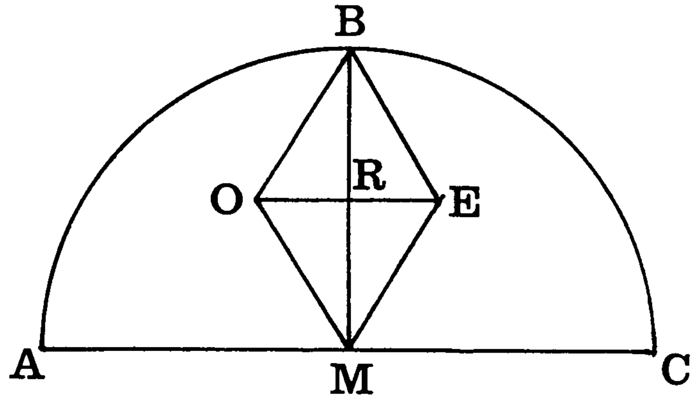

Figure 28a

scilicet MRB, which at point R bisects perpendicularly the line ORE, at whose extremities let us place the eyes O and E. Then I say that one of the two neither sees itself nor the other. This is proved as follows: since both MB and OE perpendicularly bisect each other, therefore MR = RB, RE = OR, and ∠MRE = ∠ORB; therefore the base is equal to the base and so are the remaining angles as long as they correspond to the equal sides; therefore ∠EMR = ∠OBR; therefore by *Elm*-I, BO//EM; therefore [EM] and [OB] never meet; by the same reason, [OM] // [EB]; therefore they never meet; but by the eighth [proposition] of this book, E cannot be seen by O unless OB and EM meet, nor O by E unless EB and OM meet. However, there can be no doubt that E incident on B should be reflected to O and *vice versa.*

Furthermore in the mirror ABC [Figure 28b] let the line EO bisect the line MB at point R. If we place the eyes G and D equally distant from the center between OE and the center, and connect them by the line GID, then it is evident from the assumptions that TB = TM = ML = LB and also that ∠BMD > ∠TBM, because the former has a part [of the latter], *i.e.* ∠BML which is equal to ∠TBM from the assumptions. However, ∠BGM + ∠BMG + ∠GBM = 2∠R by *Elm*-I. Therefore ∠BGM + ∠GMB + ∠BMD > 2∠R. Therefore, GB and MD, even if extended, never meet on the side of B and D, because the angles, ∠GMB and ∠BMD, make an angle ∠GMD. Now by the same arguments, you will clearly conclude that MG and DB never meet on the same direction. However, by the eighth [proposition] of this book, G and D cannot be seen except at the concourse of those lines. Therefore they cannot be seen.

It should be noted, however, that they meet behind the eye, and so this should be understood only in the case of the observer seeing before himself. However, if the eyes are so placed that the line cutting the radius BM is between the eyes and the center, then one of the eyes would see the other as is evident to an intelligent person [*Cf.* Prop. XXIX].

[XXIX(H28)] Si inter peryferyam et perpendicularem ponantur oculi dextra apparent sinistra, et sinistra dextra, et ydolum maius facie, spatium a speculo maius habens ydolum.

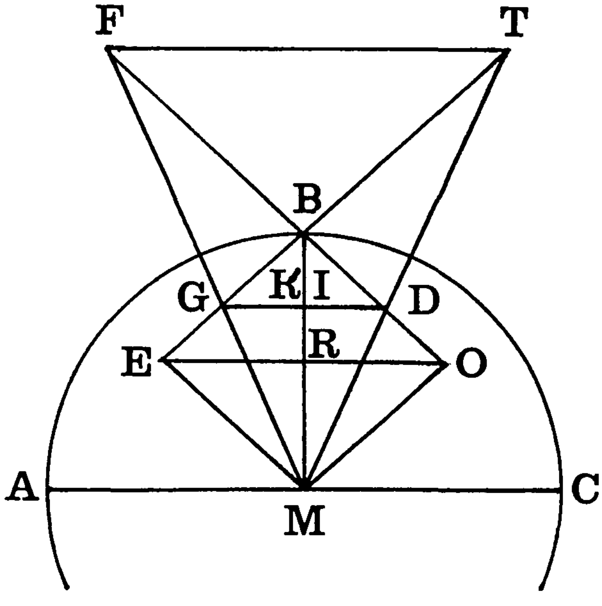

Figure 29a

Verbi gratia: sit speculum concavum ABC [Figure 29a], cuius diametrum AMC, M vero centrum, semidiametrum AB[*lege* BM] ad quem perpendicularis dividens per equalia sit OE. Dividit igitur unam et reliquam utramque per medium ad punctum R. Inter OE vero et peryferiam, OBE, ponantur G et D oculi. Dico quod dextra eis apparent sinistra et econverso etc. Quod sic probetur: latera GI et IM sunt equalia lateribus DI et IM, et anguli GIM et DIM invicem equantur, quoniam uterque rectus. Ergo basis GM basi DM equa fit et angulus IMG angulo IMD adequatur per 1 G[eometrie]. Sed item latus GM longius est lateri GB, quia longius est latere EM quod maius est GB. Ergo in triangulo GBM opponitur maiori angulo GM quam GB. Ergo angulus GBM maior est angulo GMB. Ergo idem GMB maior est angulo BMD, GMB equali per premissa. Sed anguli BGM et GBM et GMB sunt equales duobus rectis per 1 G[eometrie]. Ergo pro GBM posito se minori, scilicet BMD, erunt anguli BGM et GMB et BMD minores duobus rectis. Ergo linee GB et MD in illam partem protracte concurent[*sic*]. Concurrant igitur ad punctum T. Eadem ratione GM et BD in eam partem protracte concurrent in punctum F. Liquet igitur D videre [*lege* videri] in T, et G in F. Sed D est dexterius et G sinisterius. Ergo dextrum apparet sinistrum et e converso, que prima pars est propositi.

Ad secundum propositum astruendum sic [Figure 29b]: angulus ABM equatur angulo CBM. Sed item angulus ABG angulo CBD adequatur per primum huius. Ergo anguli GBI et DBI sunt equales. Sed item anguli GBF et TBD sunt equales. Quare anguli ABF et CBT adequantur, ex quo sequitur quod angulus MBF equatur angulo MBT, quia 3 anguli constituentes MBF 3 angulis constituentibus TBM sunt equales. Inde sic: duo trianguli sunt MBF et MBT quorum duo anguli unius, scilicet MBF et BMF, duobus alterius trianguli angulis, scilicet TBM et BMT, sunt equales. Ergo tertius scilicet angulus MFB angulo BTM in alio triangulo adequatur. Ergo trianguli MBF et MBT sunt similes. Ergo latera eorum prout respiciunt equos angulos sunt proportionalia. Sed latus MB commune est, id est

[XXIX(H28)] IF THE EYES [EQUALLY DISTANT FROM THE CENTER] ARE PLACED BETWEEN THE CIRCUMFERENCE AND THE PERPENDICULAR, THEN RIGHT APPEARS [TO BE] LEFT, AND LEFT [APPEARS TO BE] RIGHT, WHILE THE IMAGE IS LARGER THAN THE FACE AND HAS GREATER DISTANCE FROM THE MIRROR [THAN THE DISTANCE BETWEEN THE MIRROR AND THE FACE].

For example: Let there be a concave mirror ABC [Figure 29a] whose diameter is AMC, the center M, and the radius [BM] to which let a bisecting line OE be perpendicular. Then it divides each other equally at point R. Let us place the eyes G and D between OE and the circumference, OBE. I say that right appears to the eyes to be left and *vice versa.* This will be proved as follows: GI = DI and IM = IM, and ∠GIM = ∠DIM because both are right; therefore by *Elm*-I, GM = DM and ∠IMG = ∠IMD; but GM > GB, because the former is larger than the side EM which is larger than GB; therefore in triangle GBM, GM rather than GB is subtended by a larger angle; therefore ∠GBM > ∠GMB; therefore the same ∠GBM is larger than ∠BMD which is equal to ∠GMB by the assumptions; but by *Elm*-I, ∠BGM+∠GBM+∠GMB = 2∠R; therefore if in place of GBM we substitute a smaller angle than it, *scilicet* ∠BMD, it will be that ∠BGM+∠GMB+∠BMD < 2∠R; therefore the lines GB and MD will meet if extended in that direction; therefore let them meet at point T. By the same reasoning GM and BD, extended in the same direction, will meet at point F. Therefore it is clear that D is seen at T while G at F. However, D is more to the right and G [is] more to the left. Therefore [!] right appears to be left and *vice versa*, which is the first part of the proposition.

For the second part you should add as follows [Figure 29b]: ∠ABM = ∠CBM; but by the first [proposition] of this book, ∠ABG = ∠CBD; therefore ∠GBI = ∠DBI; but ∠GBF = ∠TBD; wherefore ∠ABF = ∠CBT, from which it is concluded that ∠MBF = ∠MBT, because three angles comprising ∠MBF are equal to the three angles comprising ∠TBM. Then as follows: we have two triangles, △MBF and △MBT, of which the two angles of the one, *scilicet* ∠MBF and ∠BMF, are equal to the two angles of the other, *scilicet* ∠TBM and ∠BMT; therefore the third angle, ∠MFB, is equal to ∠BTM in the other triangle; therefore △MBF and △MBT are similar; therefore their sides, as long as they subtend the equal angles, are proportional; but the side MB is common, that is,

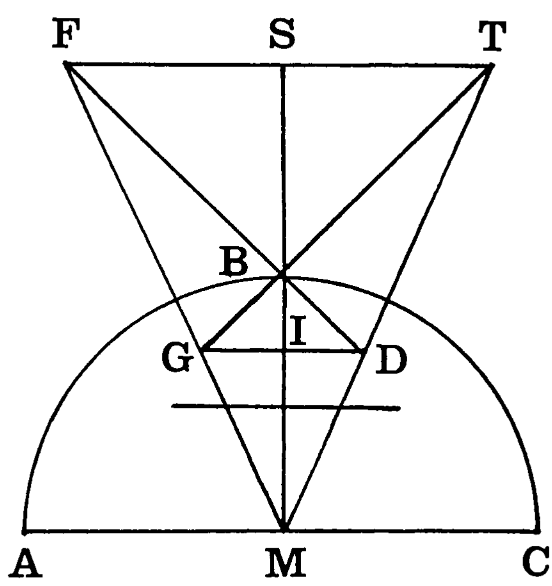

Figure 29b

sibi equale. Ergo latus BF equatur lateri BT et latus MF lateri MT. Sed trianguli sunt FMS et TMS quorum duo latera unius, id est MF et MS, duo lateribus alterius, id est TM et MS, adequantur et anguli sub illis equis continenti similiter equi sunt ex primis. Ergo bases FS et TS et angulus MSF angulo TSM invicem adequantur per 1 G[eometrie]. Ergo angulus MSF et TSM sunt uterque rectus per 1 G[eometrie]. Item trianguli sunt GIM et FSM quorum duo anguli unius duobus angulis alterius, scilicet SMF et MSF angulis GIM et GMI, adequantur ex premissis. Ergo tertius angulus scilicet MFS tertio angulo scilicet IGM adequatur. Trianguli igitur sunt MTF et MDG quorum duo anguli unius, scilicet MFT et TMF, duobus angulis alterius, scilicet DGM et DMG, adequantur ex premissis. Ergo trianguli MTF et MDG termino angulo equipollent et sunt similes per 6 Geometrum. Ergo que est proportio MG et[*del.*] ad GD, MF ad FT. Ergo permutatim que est proportio MG ad MF eadem est GD ad FT. Sed MG minus est MF quia eius pars. Igitur GD minus est FT. Sed GD res visa est et FT ydolum. Ergo ydolum maius est re conspecta. Et hoc est propositorum secundum.

Tertium vero sic constabit: anguli MBF et SBF sunt equales TBM et TBF[*lege* TBS]per 1 G[eometrie]. Sed anguli MBF et MBT invicem equantur ex premissis. Ergo anguli SBF et TBS invicem equabuntur. Sed item anguli TBF et GBD adequatur per contrapositionem et anguli GBI et DBI item adequantur ex premissis. Ergo illi quorum scilicet GBI et DBI et SBF et TBS, quoniam quilibet eorum alterius illorum medietas est, adequantur, ideo quod quilibet cuilibet. Sunt igitur duo trianguli DBI et TBF quorum duo anguli unius duobus angulis alterius, scilicet TBS et TSB, DBI et DIB angulis adequantur. Ergo tertius angulus tertio angulo adequatur. Ergo illi trianguli sunt similes per 6 Geometrum. Ergo que est proportio BI ad ID ea est BF[*lege* BS] ad ST. Ergo permutatim que est proportio BI ad BF[*lege* BS] ea est ID ad FT[*lege* ST]. Ergo conversim que est TF[*lege* TS] ad ID ea est BF[*lege* BS] ad BI. Sed TF[*lege* TS] est maius quam ID per premissa. Ergo BF[*lege* BS] maius est BI. Sed BF[*lege* BS] est distantia speculi ad ydolum et BI distantia speculi ad conspectum. Ergo maior est distantia speculi ad ydolum quam eiusdem ad conspectum. Et hoc est ultimo loco propositum. Hoc autem est in[*iter.*] concavis speculis intelligendum et oculis equidistanter a centro positis.

[XXX(H28)] Si extra diametrum ponantur oculi, dextra apparent dextra et sinistra sinistra et ydolum minus facie intermedio faciei et speculi.

Verbi gratia: proponatur ABC cuius diameter AC [Figure 30] sitque concavum speculum sintque D et E oculi equidistantes ab O centro videntes ED rem per speculum ABC. Erunt inmissi radii DB a D oculo et EB ab E oculo quorum uterque equal to itself; therefore BF = BT and MF = MT; but we have triangles, △FMS and △TMS, of which the two sides of the one, MF and MS, are equal to the two of the other, TM and MS, and the angles contained under those equal sides are also equal by the preceding; therefore by *Elm*-I, FS = TS and ∠MSF = ∠TSM ; therefore by *Elm*-I, ∠MSF = ∠TSM = ∠R; Further we have triangles, △GIM and △FSM, of which the two angles of the one, ∠SMF and ∠MSF, are equal to the two angles of the other, ∠GIM and ∠GMI, by the preceding; therefore the third angle, ∠MFS, is equal to the third angle, ∠IGM; therefore we have triangles, △MTF and △MDG, of which the two angles of the one, ∠MFT and ∠TMF, are equal to the two angles of the other, ∠DGM and ∠DMG, by the preceding; therefore the triangles △MTF and △MDG have the remaining angles equal, so that they are similar by *Elm*-VI; therefore MG : GD = MF : FT; then by the permutation of proportion, MG : MF = GD : FT; but $MG < MF$, because the former is a part of the latter; therefore $GD < FT$; however, GD is the object of sight while FT the image; therefore the image is larger than the object of sight. And this is the second of what were proposed.

Moreover the third [part] will be established as follows: by *Elm*-I, ∠MBF = ∠TBM and ∠SBF = ∠[TBS]; but ∠MBF = ∠MBT by the preceding; therefore it will be that ∠SBF = ∠TBS; moreover ∠TBF = ∠GBD by opposite angles and ∠GBI = ∠DBI by the preceding; therefore, those [four] of preceding [six] angles, *scilicet* ∠GBI, ∠DBI, ∠SBF and ∠TBS, are equal, because any of those is a half of the other two, so that they are equal each other; therefore we have two triangles, △DBI and △TBF, of which the two angles of the one are equal to the two angles of the other, *scilicet* ∠TBS = ∠DBI and ∠TSB = ∠DIB; therefore the third angle is equal to the third angle; therefore these triangles are similar by *Elm*-VI; therefore BI : ID = [BS] : ST; then by the permutation of proportion, BI : [BS] = ID : [ST]; then by the conversion of proportion, [TS] : ID = [BS] : BI; but by the preceding, $[TS] > ID$; therefore $[BS] > BI$; however, [BS] is the distance of the mirror to the image while BI is the distance of the mirror to the object. Therefore the distance of the mirror to the image is larger than the distance of the mirror to the object. And this is the last proposition. This should be understood, however, in the case of concave mirrors and the eyes which lie at equal distance from the center.

[XXX(H28)] IF EYES ARE PLACED OUTSIDE THE DIAMETER, RIGHT APPEARS [TO BE] RIGHT WHILE LEFT [APPEARS TO BE] LEFT, AND AN IMAGE SMALLER THAN THE FACE [APPEARS] AT SOME POINT BETWEEN THE FACE AND THE MIRROR.

For example: Let there be ABC whose diameter is AC [Figure 30], and let it be a concave mirror. And let there be the eyes D and E which are at equal distance from the center O and see the object ED by the mirror ABC. It is possible for the rays DB and EB to be emitted from the eyes D and E, so that each of them will be per reliqum[*sic*] reflectetur quia per angulos equales. Est autem E dextrum, D sinistrum. Inde sic: E videtur, ergo videtur in concursu linee ducte a conspecto per centrum et radii inmissi sub cuis reflexione videtur. Ergo videtur in T per media. Eadem ratione D videtur in F. Sed T est dextrum respectu spere et F sinistrum. Ergo per media dextrum apparet dextrum et sinistrum similiter.

Secundum autem astrues per triangulos similes sicut in precedenti scilicet quod TF minus est ED. Si vero intelligendum sit de facie medio loco distanti inter conspectum et ydolum, verbi gratia ut de LR parte diametri, sic astrues: protracta linea a puncto B per centrum O ad punctum medium ED scilicet K, probabitur sicut in precedenti quod angulus DKB rectus est. Similiter angulus BOR rectus erit. Quare uterque eorum rectus equabitur ex 1° G[eometrie]. Erunt ergo duo trianguli BKD et BOR quorum duo anguli unius, scilicet BKD et DBK, duobus angulis alterius, scilicet BOR et OBR, equantur. Ergo et tertius tertio adequatur et trianguli sunt similes. Ergo que est proportio DB ad BR ea est DK ad OR per 6 Geometrum. Sed DB maius est BR quia totum ad illud. Ergo DK est maius OR. Eademque ratione EK est maius OL. Ergo ED maius est LR, et ita sive de conspecto sive de facie intermedia ydoli et conspecti habemus propositum verum.

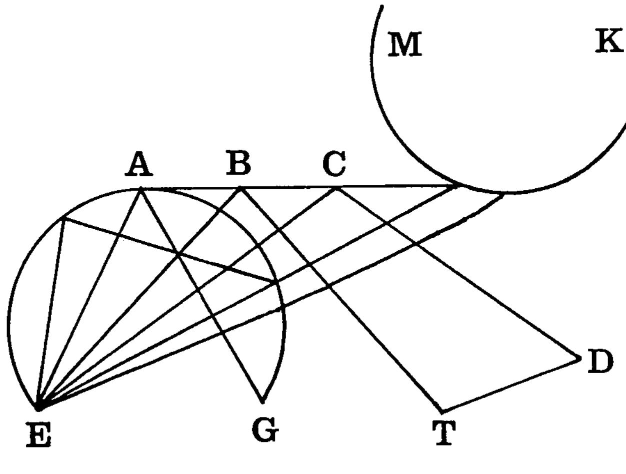

Figure 31

[XXXI(H29)] Possibile est speculum ita construi quod in eodem apparent plures facies, hee quidem maiores, ille vero minores, hee quidem propius ille vero longius, et hinc quidem dextre inde vero sinistre.

Verbi gratia: disponatur speculum planum ABC scilicet [Figure 31] cui superponatur convexum speculum MK, concavum vero EG, sitque ex his tribus speculis. Sitque[?] oculus E, TD visum et videatur TD per quodlibet illorum. Inde sic: TD videtur per ABC speculum planum, ergo eius ydolum eius[*iter.*] est ei equale per 18[*lege*19] huius. Item idem TD videtur per MK speculum convexum, ergo eius ydolum minus est re conspecta per 20 huius. Item TD videtur per speculum concavum CG[*lege* AG] scilicet, ergo eius ydolum maius est re conspecta per 28 huius.

reflected to the other because of their being equal angles. However, E is right while D is left. Then as follows: because E is seen, it is seen at the intersection of the line drawn from the object of sight through the center and the emitted ray under whose reflection it is seen. Therefore it is seen at T by the "intermediation." By the same reasoning, D is seen at F. However, T is right in regard to the spherical [mirror] and F is left. Therefore by the "intermediation" right appears right while left does in the same manner.

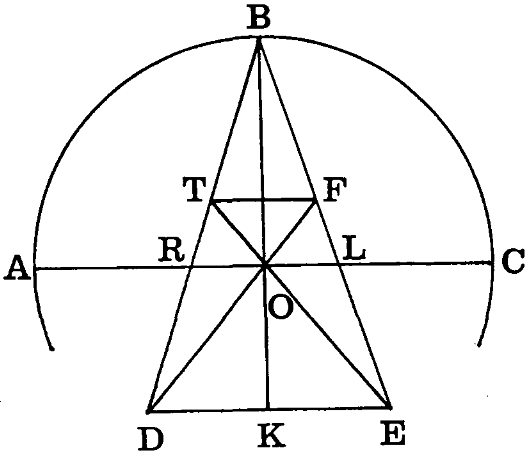

Figure 30

You should add the second part by [using] the similar triangles as in the preceding [proposition], *scilicet* that TF is smaller than ED. Now if understanding should be made concerning the face located at some place between the object of sight and the image, for example, concerning LR, the part of the diameter, then you should add as follows: Let us draw a line from point B through the center O to the middle point K of ED, then we can prove just as in the preceding that the angle ∠DKB will be right. The angle ∠BOR will also be right. Therefore, each of them, being right, will be equal by *Elm*-I. Then we will have two triangles, △BKD and △BOR, of which the two angles of the one, ∠BKD and ∠DBK, are equal to the two angles of the other, ∠BOR and ∠OBR. Therefore the third [angle] is equal to the third [angle], and those triangles are similar. Therefore by *Elm*-VI, DB : BR = DK : OR. But DB > BR, because the former is a whole to the latter. Therefore DK > OR. By the same reasoning, EK > OL. Therefore ED is larger than LR, so that we have the true proposition, whether concerning the object of sight or concerning the face located between the image and the object of sight.

[XXXI(H29)] IT IS POSSIBLE TO CONSTRUCT SUCH A MIRROR THAT THERE MAY APPEAR [AT THE SAME TIME] SEVERAL FACES IN IT, OF WHICH SOME ARE LARGER WHILE OTHERS ARE SMALLER; SOME ARE NEARER [TO THE MIRROR]WHILE OTHERS ARE FARTHER; AND SOME ARE RIGHTSIDE [FACES] WHILE OTHERS ARE LEFTSIDE.

For example: Let us place a plane mirror ABC [Figure 31], upon which let us place a convex mirror MK and a concave one EG. And let [us think the required mirror] be made of those three mirrors. Moreover, let there be an eye E and an object of sight TD, and let TD be seen by any of those mirrors. Then as follows: since TD is seen by the plane mirror ABC, its image is equal to it by the [nineteenth proposition] of this book. Further, since the same TD is seen by the convex mirror MK, its image is smaller than the object of sight by the twentieth proposition of this book. Further, since TD is seen by the concave mirror [AG], its image is larger than the object of sight by the twentyeighth [proposition] of this book.

Habemus ergo quod TD videtur equalis suo ydolo et maior et minor eodem. Quod autem ex illis tribus sit unum ex hypothesy quoniam est possibile.

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[XXXII(H30)] Concavis speculis ad solem positis ignis ascenditur.

Verbi gratia: sit speculum concavum ABC cuius diameter AC [Figure 32: *sic*], sol vero sit DEM, apponatur speculum ABC radio. Ergo incidens ab E in speculum ad punctum B reflectitur ad equales angulos per primum huius et transit per centrum O scilicet. Ergo reflectitur in se. Similiter quicumque radius procedens ab medietate solis versus speculum transit per centrum reflectitur in se. Sed infiniti sunt tales quorum quilibet reflectitur per centrum. Ergo infiniti radii sese repercutiunt in centro[*lege* centrum]. Consimili ratione cum a quolibet puncto medietatis solis que est PEZ procedant infiniti radii per diametrum AC ad speculum ABC, non erit invenire ABC punctum a quo non reflectantur infiniti neque aliquod punctum AC in quo non sese repercutiant infiniti propter equalitatem reflexionis angulorum. Sed quilibet radius solis est ignis. Ergo in quolibet puncto AC erunt infiniti puncti ignis. Sed non possunt esse infiniti puncti ignis in aliquo puncto quin ille punctus ascendatur[*lege* accendatur] vel sit ignis. Ergo quodlibet punctum AC ascendetur[*lege* accendetur]. Et hoc erat propositum ostendere. Hinc est habere quoniam si ponatur stupa in diametro speculi concavi sitque optime scaepsa[*sic*] sine dubio ascendetur[*lege* accendetur]. Sive ibi ponatur esche[*sic*] bien[*sic*] seche[*sic*] ascendetur[*lege* accendetur]. Et hec practica probata est plus logice quam specialiter. Hoc autem modo posses habere ignem de celo etsi nullus ignis neque aliquod igneum essent in terra. Huius autem consimile probavi per lapidem qui dicitur [*hab. lac.*]. Unde videtur haberi in convexsis[*sic*].

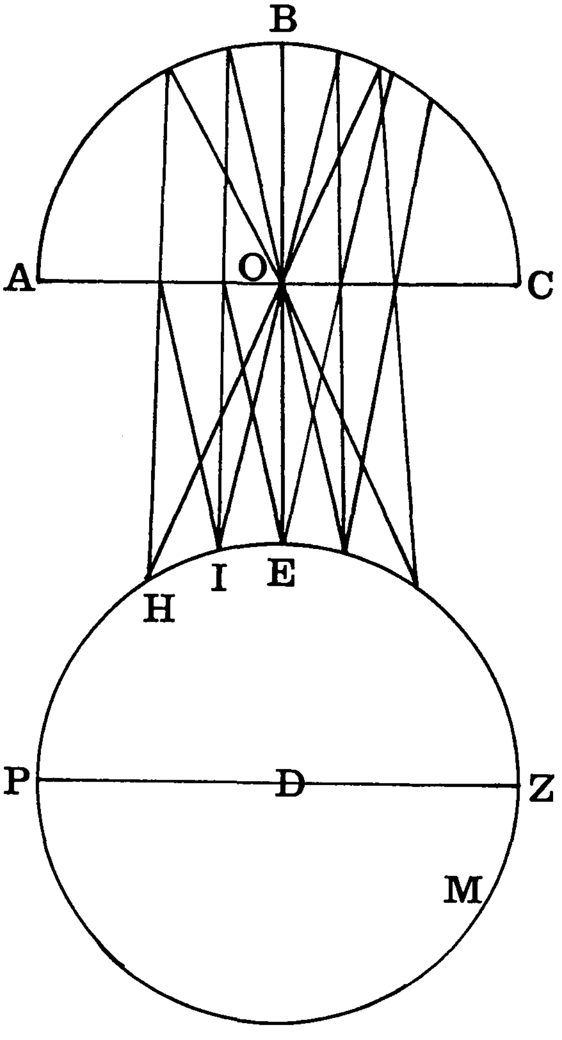

Figure 32

Explicit liber speculorum.

Therefore it is concluded that TD is seen [at the same time] equal to, larger and smaller than its image. Because it is possible to make one mirror from those three mirrors, ...

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[XXXII(H30)] FIRE IS KINDLED BY CONCAVE MIRRORS THAT FACE THE SUN.

For example: Let there be a concave mirror ABC whose diameter is AC [Figure 32: *sic*], the sun DEM, and let us face the mirror ABC to a ray. Then by the first [proposition] of this book, the ray incident from E toward the mirror upon point B is reflected at equal angles so that it passes through the center O. Therefore it is reflected on itself. By the same way any ray, that proceeds from the halfside of the sun toward the mirror and passes through the center as well, is reflected on itself. However, those are infinite which are reflected through the center. Therefore infinite rays rebound themselves toward the center. By the similar reasoning, since the infinite rays proceed from any point of the halfside of the sun, *i.e.* PEZ, to the mirror ABC through the diameter AC, it is impossible, on account of the equality of reflected angles, to find such a point of ABC that does not reflect infinite rays or to find some point of AC where infinite rays do not rebound themselves. However, any ray of the sun is [of] fire. Therefore at any point of AC there can be infinite points of fire. But there cannot be the infinite points of fire at such a point that that point may not be kindled or become fire. Therefore any point of AC will be kindled. And this was the proposition that should be made clear. It is concluded from this that if a flax is placed on the diameter of a concave mirror and let it be extremely dried[?], then it will without doubt be ignited. Or if a very dry food is placed there, it will be ignited. This practical knowledge is proved [here] logically rather than individually. By this method you can obtain fire from the heaven even if there is no fire nor fiery thing on the earth. I have examined a similar thing like this by a stone which is called [...]. Whence this seems possible also in the case of convex [mirrors].

Here ends the Book *On Mirrors*.

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