

**A Paraphrased Latin Version of  
Euclid's *Optica*  
A Text of *De visu* in MS Add.17368,  
British library, London**

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## I Introduction

The present article provides for the first time a paraphrased version of Euclid's *De visu* which is a Latin translation of his *Optica*. It has already been acknowledged that there are many versions of it. Out of thirty-three MSS now known to be extant, Lindberg[1975] distinguishes seven versions according to the difference of *incipits* for definition 1 and proposition 1. Moreover, he notices that some manuscripts have alternative enunciations for one or more of their propositions, which are drawn from *De radiis visualibus*, the most popular Arabo-Latin translation of Euclid's work. Our manuscript of *De visu*, which is contained in MS Add.17368, British library, London, belongs to this category of manuscripts. In order to investigate the historical significance of our manuscript, let us begin by picking out those manuscripts of this type from Lindberg[1975], which may be arranged in chronological sequence.

### **MSS of *De visu* with alternative enunciations drawn from *De radiis visualibus***

- **Version 1 (6 MSS out of 21)**

- #1. London, British Museum, MS Add. 17368, fols.60r-69r. 12th c.
- #2. Oxford, Bodleian Lib., MS Auct. F. 5.28, fols. 17(57)r-24(64)r. 13th c.
- #3. Oxford, Bodleian Lib., MS Corpus Christi Coll. 251, fols. 1r-7v. 13th c.
- #4. Venice, Bibl. Naz. Marciana, MS Zanetti Lat. 332 (Valentinelli XI.6), fols.242r-251v. 13th c.
- #5. Erfurt, Wissensch. Bibl., MS Ampl. Q.385, fol. 210r. 14th c. Fragment
- #6. Leeuwarden, Provinciale Bibl., MS B.A.Fr.57, fols.59r-68r. 15th c.

- **Version 2 (1 MS out of 1)**
  - #7. Cambridge, Gonville and Caius Coll. Lib., MS 504/271, fols. 86v-93v. 13th c.
- **Version 3 (1 MS out of 1)**
  - #8. Milan, Bibl. Ambrosiana, MS T.91 sup., fols. 39r-49r. 13th c.
- **Version 5 (2 MSS out of 2), edited by Aimar (also entitled *De linea visuali*)**
  - #9. Oxford, Bodleian Lib., MS Corpus Christi Coll. 283, fols. 163r-165v. 12th-13th c.
  - #10. Seville, Bibl. Colombina, MS 7.6.2, fols. 43(44)v-54(55)r. 13th c.
- **Version 6 (6 MSS out of 7), edited by Witelo (?), fl.1270.**
  - #11. Milan, Bibl. Ambrosiana, MS R.47 sup., fols. 133r-148r. 13th c.
  - #12. Florence, bibl. Riccardiana, MS 885, fols. 132r-143v. 14th c.
  - #13. Paris, Bibl. Nat., MS Lat. 7366, fols. 90v-97v. 14th c.
  - #14. Vatican, Bibl. apost., MS Vat. Lat. 3102, fols. 37v-50r. 14th c.
  - #15. Paris, Bibl. Nat., MS Lat. 10252, fols. 159(154)v-172(167)r. 1476.
  - #16. Vienna, Osterr. Nat. bibl., MS 5303, fols. 32r-41v. 15th-16th c.
- **Version 7 (1 MS out of 1)**
  - #17. Lisbon, Bibl. Nac., MS Geral 2299, fols. 160v-161v. 14th c. Incomplete.

The above list shows us that the manuscripts containing enunciations drawn from *De radiis visualibus* amount to half the total number of manuscripts of *De visu*, and include probably the oldest one. Moreover, the Latin translation of *Optica* was made from Greek, not from Arabic, as is clearly shown by the publication of the Latin text in Heiberg[1895] and Theisen[1972][1979]. The coexistence of Greco-Latin and Arabo-Latin texts in the oldest manuscript is so significant that it invites us to look at it closely and publish its text separately.

Let us start with the contents of British Library, Add. 17368. This is a composite manuscript consisting of six codices [A-F] written by different hands at different dates and of the following treatises [1-8]<sup>1</sup> :

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<sup>1</sup>We owe the detailed philological information of this manuscript to Prof. Charles Burnett, the Warburg Institute, Univ. of London. He kindly informed us that treatise [2] was identified by Fritz Saaby Pedersen, but we have had no time to consult his study.

- A [1] fols. 1r-19r: Bernard of Verdun, Astronomy
- B [2] fols.20r-35v: Astronomical Tables, related to the Toledan Tables
- C [3] fols.36r-51v: Johannes de Presorio, Astronomical Calendar
- D [4] fols.52r-59v: John Pecham, *Perspectiva communis*
- E { [5] fols.60r-69r: Euclid, *De visu*  
     [6] fol.69v: Archimedes, *De quadratura circuli*  
     [7] fols.70r-71r: Euclid, *De speculis*
- F { [\*] fols.71v-74r: blank  
     [8] fol.74v: Definitions of musical terms

Items [5], [6] and [7] all belong to the same codex, consisting of two quires of 8 folios each with one folio missing in the second quire. The first quire has the number ‘.xlii.’ written vertically at the edge of its last page, which may suggest that originally it was the 42nd quire or fascicle of a larger collection. Among those treatises, [4][6] and [7] are fully analyzed respectively in Lindberg[1970], Knorr[1990], and Takahashi[1992]. However, treatise [5] has not yet received full examination. In his collated edition of *De visu*, Theisen[1979] consulted the portion of our manuscript occasionally, but did not pay much attention to it, since his purpose was to produce the Greco-Latin translation of the text, not a paraphrased version of it. This is clearly evidenced by the fact that our manuscript does not appear in his final stemma of MSS, although he acknowledged that the manuscript was the oldest.<sup>2</sup> In passing, it is also worth noting that if version 1 as designated by Lindberg is meant to be a Latin translation of the Greek text, our manuscript does not belong to version 1, because this is a redaction made from version 1 as will be clear from our text below.

### I.1 The Status of our Manuscript in the Medieval Latin Optical Tradition

As stated above, our manuscript has a text of Greco-Latin and Arabo-Latin origins. It is convenient for our better understanding to survey beforehand the textual traditions in Greek, Arabic and Latin. For this purpose, it may be best to make

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<sup>2</sup>For his stemma, see Theisen[1979, 60]. For the date of codex E, we benefited greatly from Prof. William J. Coutenay, the University of Wisconsin-Madison, USA, who examined the reprint of the portion at our request: According his judgement, codex E may have been written either before the first half of the 13th century or even much earlier, although Clagett[1964, xxvii] ascribed it to “late 13 or early 14c.” Theisen[1979, 52] also admits its ascription to the 12th century, though on different reasons.

a correspondence table for the definitions (or postulates) and propositions of *Optica*. We benefited greatly from Rashed[1997] and Kheirandish[1999] for making our table.<sup>3</sup>

### Sigla

Greek Text <sup>4</sup>	<b>A:</b> the so-called genuine version of <i>Optica</i> [ed. Heiberg[1895], pp.1-121]
	<b>B:</b> the so-called Theonine version of <i>Optica</i> [ed. Heiberg[1895], pp.143-247]
Arabic Text	<b>M:</b> <i>Kitāb al-Manāẓir</i> , the Arabic Translation of <i>Optica</i> [ed. Kheirandish[1999]]
	<b>K:</b> the Version Commented by al-Kindī [ed. Rashed[1997]]
	<b>T:</b> the Redaction by al-Ṭūsī [cf. Rashed[1997]]
	<b>J:</b> the Redaction by Ibn Abī Jarrāda [cf. Rashed[1997]]
Latin Text	<b>V:</b> <i>De visu</i> , the Latin translation of <i>Optica</i> [ed. Theisen[1979]]
	<b>Our MS:</b> manuscript #1 listed above

In the following table of correspondences, the asterisk attached to the Latin proposition number indicates that it has an alternate enunciation drawn from *De radiis visualibus*, for whose text we consulted Theisen[1972]. Those enunciations are differentiated in our text usually as “*Habet alia translatio*”; the corresponding proposition of the *De radiis visualibus* is indicated in the edition that follows, for example, as [RV1]. For the Latin text V, our information is drawn from Theisen[1979] for proposition 10 and thereafter, and from our inspection of MSS #2 and #4, the chief witnesses of V, for propositions 1-9.

### Correspondence Table of Definitions (or Postulates) & Propositions of *Optica*

Greek Text		Arabic Text				Latin Text	Our Latin
A	B	M	K	T	J	V	MS #1
none	Introduction	none	none	none	none	none	none
Definition	Definition	Definition	Definition	Definition	Definition	Definition	Postulate
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	none	none	5	none	5	5
6	6	none	none	6	none	6	6
7	7	4	4	7	4	7	7
none	none	none	none	none	none	8	none
none	none	none	none	none	none	9	none

<sup>3</sup>See Rashed[1997, 10-13] and Kheirandish[1999, vol.II, xxxiii-xxxiv].

<sup>4</sup>For this designation of Greek texts, A and B, we follow Knorr’s proposal in Knorr[1994].

Greek Text		Arabic Text				Latin Text	Our Latin
A	B	M	K	T	J	V	MS #1
Proposition 1	Proposition 1	Proposition 1	Proposition 1	Proposition 1	Proposition 1	Proposition 1	Proposition 1*
2	2	2	2	2	2	2*	2*
3	3	3	3	3	3	3	3*
4	4	4	4	4	4	4*	4
5	5	5	5	5	5	5	5*
6a	6a	6	6	6	6	6*	6*
6b	6b	7	7	7	7	7*	7*
7	7	8	8	8	8	8*	8
8	8	9	9	9	9	9	9
9	9	10	10	10	10	10*	10*
10	10	11	11	11	11	11*	11
11	11	12	12	12	12	12*	12*
12	12	13	13	13	13	13*	13*
13	13	14	14	14	14	14*	14*
14	14	15	15	15	15	15*	15*
15	15	16	16	16	16	16*	16*
16	16	17	17	17	17	17*	17*
17	17	18	18	18	18	18	18*
18	18	19	19	19	19	19*	19*
19	19	20	20	20	20	20	20*
20	20	21	21	21	21	21	21*
21	21	22	22	22	22	22	22
22	none	23	23	23	23	23	23*
22 aliter 1	none	none	none	none	none	23 aliter 1	23 aliter
22 aliter 2	22	none	none	none	none	23 aliter 2	none
none	none	none	none	none	none	24	none
none	none	none	none	none	none	25	24*
23	23	24	24	24	24	26*	25*
24	24	25	25	25	25	27	26*
25	25	26	26	26	26	28	27*
26	26	27	27	27	27	29	28
27	27	28	28	28	28	30	29
28	none	29	29a	29	29	31	30* & 31
28 aliter	28	30	none	30	30	32	none
29	29	31	29b	31	31	33	32*
30	30	32	30a	32	32	34*	33*
31a	31	33	30b	33	33	35a	34
31b	none	none	none	none	none	35b	none
32	32	34	31	34	34	36*	35
33	33	35	32	35	35	37	36
34a	34	36	33	36	36	38	37a
34b	35a	37	34	37	37	39a	37b
34c	35b	none	35a	none	none	39b	37c
35 Enunc.	36 Enunc.	38 Enunc.	35b	38	38	40	38
35a	36abc	39	36a	39	39	40a	38a
35b	none	none	36b	none	none	40b	38b
35c	36d	40	37a	40	40	40c	38d
35d	36e	41	37b	41	41	40d	38c
36	37	42	38	42	42	41	39
37a	41	43a	39	43	43	42a	40
37b	none	none	none	none	none	42b	none
38	42	43b & 48	40 & 44	43b & 48	43b & 48	43	41
39a	38a	44	41a	44	44	44a	42
none	none	none	41b	none	none	none	none
none	none	none	41c	none	none	none	none
39b	38b	45a	42a	Part of 44	44	44b	none

Greek Text		Arabic Text				Latin Text	Our Latin
A	B	M	K	T	J	V	MS #1
none	none	none	42bc	none	none	none	none
40a	40a	45b	43a	47	47	45a	43
40b	40b	46	43b	46	46	45b	none
none	40c	47	43c	45	45	none	none
41	39	49	45	49	49	46	44
42a	none	none	none	none	none	47a	45a
42b	none	none	none	none	none	47b	none
42 aliter	43	50	50	50	50	47c	none
43	44	51	46	51	51	47d	45b
44	none	52	47	52	52	48	46
44 aliter	45	none	none	none	none	48 aliter	46 aliter
45	46	54	49	54	54	49	47
46	none	53	48	53	53	50	48
47	47	55	51	55	55	51	49
48	48	none	none	none	none	52a	50a
49	none	56	52	56	56	52b	50b
50	49	57	53	57	57	53	hereafter
51	50	58	54	58	58	54	lacking
52	51	none	none	none	none	55	
53	52	59	55	59	59	56	
54	53	none	none	none	none	57	
54 aliter 1	none	60	none	60	60	57 aliter 1	
54 aliter 2	none	none	none	none	none	57 aliter 2	
55	54	61	Part of 56	61	61	58	
56	55	62	none	62	62	59	
57	56	63	none	63	63	60	
58	57	64	59	64	64	61	

Some peculiar features of the Latin texts appear from the table. Let us focus our attention on the following: (1) additions of definitions 8 and 9 in *De visu*, and their absence in our MS, (2) the development of the last part of Prop.6 in the Greek text into an independent Prop.7, (3) the addition of Props.24 and 25 in *De visu*, and the absence of the former in our MS, (4) Prop.28 of the Greek text and its treatment in the Latin texts, and (5) appearances of some propositions with additional Arabo-Latin enunciations.

(1) It is beyond any doubt that definitions 8 and 9 were added for the first time in the Latin text, since there are no corresponding ones in the Greek and Arabic texts. These read as follows:

Def. 8: Omnes visus equeveloces esse.

Def. 9: Non sub quocumque angulo rem videri.

Out of seventeen manuscripts listed above in the preceding section, eight are now available to the authors: #1, #2, #3, #4, #7, #8, #9, and #11; these practically exhaust the earlier manuscripts. These additional definitions are found in #3, #4,

#7, #8, and #11.<sup>5</sup> As for manuscript #2, which serves as the chief witness of *De visu* in Theisen[1979], although it lacks the first eight and a half propositions and begins in the middle of the ninth proposition, we have every reason to believe that it may have originally had these two definitions, because #4 is so faithful to #2 in reproducing propositions and marginal glosses that #4 can be regarded as a reliable witness to the missing portion of #2.<sup>6</sup> Therefore, #1 and #9 are the only manuscripts that do not have the additional definitions and are in a sense faithful to the Greek text. This fact is significant for establishing the chronological sequence between the earlier manuscripts, because #1 and #9 are legitimately conjectured to be prior to the rest. Moreover, it may be useful to note that these two are thought of as having their origins in the twelfth century whereas the rest are of the thirteenth century or later, which accords well with our conjecture.

(2) The second half of Greek proposition 6 becomes proposition 7 in the Latin. This is also true for all the Arabic versions. Where does the enunciation of Latin Prop.7, which is absent in the Greek text, come from? The question is easy to answer. It comes from the Arabo-Latin version of *Optica*. The text is drawn from *De radiis visualibus*, and the wording is the same as the latter. Therefore, it is not strange to find used such words as “*latitudo*” and “*secundum visionem*” in Prop.7 which are not found in any of the other propositions of our text. This indicates that the translation of the Greek text into Latin may have been executed in an area where both Greek and Arabic texts were available. This impression will be reinforced in what follows.

(3) The *De visu* has the additional propositions numbered as 24 and 25 immediately before the Greek proposition 23[=V26]. Neither of these is found in the Arabic texts. Therefore these additions are unique to the Latin. Let us first look at Prop.25, which is found also in our manuscript. The enunciation reads: *Longior radius ad speram proveniens quasi linea contingens erit*. The provenance of this proposition is clear. This is nothing but a proposition of the Arabo-Latin text, namely [RV24]. The addition of this proposition to the Greco-Latin text is easy to understand, because it is closely related to the subsequent Greek proposition 23[=V26] so that the former would give a good introduction to the latter. The similarity of the figures in [V25] and [V26] reinforces our interpretation. In this connection it is worth mentioning that the internal cross referencing of propositions is not correct, giving consistently

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<sup>5</sup>MS #11 has the following additional comment to def. 8: Omnes visus equeveloces esse, qui scilicet secundum equales lineas deferuntur. Non autem sunt equeveloces qui secundum inequales lineas deferuntur.

<sup>6</sup>This is Theisen's judgement, with which we agree. See Theisen[1979, 57]. The close relationship between MSS #2 and #4 is also confirmed for *De speculis* which is customarily combined with *De visu* in the same codex. See Takahashi[1992, 78f and 99f].

the number less by one. For example, in Prop.33 our text gives a wrong reference to Prop.24[=V25] instead of the correct Prop.25[=V26], which is shown in our text as “per XXIII<sup>am</sup>[*lege* XXV<sup>am</sup>] presentis.” This is probably caused by the introduction of the Arabo-Latin proposition into the Greco-Latin version.

However, the addition of Prop.24 is not so easy to understand, since our manuscript #1 as well as #9 does not have it. The enunciation of the Latin proposition 24 reads as follows:

[V24] Si in eo plano in quo est oculus ponatur periferia, non tota apparet semicircumferentia.

The accompanying figure looks the same as that of the second alternative proof of the Greek proposition 22[=V23], the enunciation of which reads in our text as follows: *Si in eo plano in quo oculus est circuli periferia ponatur, ea circuli periferia recta linea apparet.* Once we have [V25], then [V24] would be unnecessary or redundant, because the latter would be automatically deduced from the former. And our manuscript #1 as well as #9 does not have [V24]. It may be that some later incompetent scholar added this on account of its close similarity to the second alternative proof of [V23].

At any event, it is very important that [V25], which derives from *De radiis visualibus*, is included in the Greco-Latin versions, including the earliest copies of our #1 and #9. This fact strengthens once again our conjecture that the Latin *De visu* may have been produced at the same place where the Arabo-Latin version was produced, or at least available, at an earlier phase of transmission.

(4) As is evident from the correspondence table, proposition [V31] of the Latin text corresponds to proposition [A28] of the Greek text, while the alternative proof of the latter corresponds to proposition [V32] of the former. What surprises us most is that the enunciation of [V32] was originally the last part of [A28], insisting that the same conclusions which are proved for the sphere seen by *two* eyes in [V27-29] can be drawn for the cylinder seen by *two* eyes. For convenience's sake, let us cite them:

[V31] Chilindro qualitercumque sub uno oculo viso minus hemichilindro videbitur.

[V32] Si sub duobus oculis chilindrus videatur, manifestum quoniam et in eo contingent que in sphaera.

However, the proof in [V32] is the same as the alternative proof of [A28] valid for the cylinder seen by *one* eye, so that there is no relevance between the enunciation and the proof of [V32]. In this regard, #9 follows the pattern of the Greco-Latin version. However, the author of our MS #1 was intelligent enough to make a new proof



of his own suitable for the wrongly inserted enunciation of [V32], thus successfully reducing the logical gap between them.

(5) We have come to the most important and final point of our argument. Some enunciations of Arabic origin are coexistent with the Greek ones. Let us recall the propositions with two enunciations of different origin. For our present purpose it suffices to show them in tabular form just for three chief texts: MS #2 which is the chief witness for establishing the Greco-Latin text in Theisen[1979], our MS #1, and MS #9 which seems to resemble our manuscript more closely than any other witness, as argued in section (1).

**Propositions with additional Arabo-Latin enunciations (\*) and without them (/)<sup>7</sup>**

Prop. # in V	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
MS #2	/	*	/	*	/	*	*	*	/	*	*	*	*	*	*	*	*	/	*	/
MS #1	*	*	*	/	*	*	*	/	/	*	/	*	*	*	*	*	*	*	*	*
MS #9	*	*	*	*	*	*	*	*	/	*	*	*	*	*	*	*	*	*	*	*

Prop. # in V	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37 ~ 61
MS #2	/	/	/	/	/	*	/	/	/	/	/	/	/	*	/	*	///
MS #1	*	/	*	N	*	*	*	*	/	/	*	N	*	*	/	/	///
MS #9	*	*	*	N	*	*	*	*	*	*	*	N	*	*	*	*	///

where N means a proposition that has no counterpart in V.

The table above shows that Theisen's following description is not correct: "Their appearance in so many of the *Liber de visu* manuscripts is probably due to the influence of *L* (our MS #2) rather than to the wide circulation of the Arabo-Latin version. That this is a sound conclusion is clear from the fact that in the other *Liber de visu* manuscripts the only alternate forms of the enunciations are those already found in *L*."<sup>8</sup> Although it may be right to assume that the Arabo-Latin version did not enjoy a wide circulation, the coexistence of two different enunciations in a proposition may reasonably be considered to be a result of the earlier existence of such amalgamated texts as #1 and #9.

In addition, there is an interesting difference between MS #2 on the one hand and MSS #1 and #9 on the other. In MS #2 alternate enunciations are written down as marginal glosses, whereas in MSS #1 and #9 they are incorporated in the text.

In order to avoid confusion, we have to point out that the proofs given in #1 and #9 are so different that they belong to distinct versions. The following examples taken at random will amply show this:

<sup>7</sup>A short remark is necessary for Props.V37~V61: MS #2 has all of them, whereas MS #1 lacks Props.V53~V61 and MS #9 lacks Props.V37~V45.

<sup>8</sup>See Theisen[1979, 58].

MS #1	MS #9
<p>[1] Nullum visorum simul videtur totum. Habet alia translatio:[RV1] In eodem instanti non videri plura.</p> <p>[V1] Esto enim visum quidem AD, oculus vero esto B, a quo incidant visus BA, BG, BK, BD. Igitur quoniam in distantiam feruntur incidentes visus, non quidem incidunt continue ad AD. Quare fieret et in AD spatio ad quem visus non incident. Non ergo videbitur simul totum AD. Videtur autem videri visibus velociter transportatis.</p> <p>[3] Unumquodque visorum habet longitudinem spatii quo facto iam non videbitur. Alia translatio habet: [RV3] Cuiuslibet visibilis per elongationem terminari visum.</p> <p>Esto oculus B, res visa GD. Dico quod GD in aliquo spatio factum iam non videbitur. Fiat enim GD in intermedio spatio visuum in quo K. Igitur cum a B ad K visus non accidat, non videtur GD. Quia ad quod visus non accidunt, illud non videtur.</p> <p>Vel aliter [RV3]: quanto enim res magis removetur, sub minori angulo videtur, qui tam diu diminuitur donec linee eum continentes velud in unam redigantur. Linee vero terminus est punctus, qui cum sit invisibilis, quod videbatur propinquius iam non videtur remotius. Unumquodque ergo visorum habet longitudinem spatii quo facto iam non videbitur.</p>	<p>[1] Nullum visorum simul videtur totum. In alia translatione habetur:[RV1] In eodem instanti non videri plura.</p> <p>Cuius hec est ratio. Sit visus B, oculus A, a quo incidant visus C, D, E, F ad 4 diversas notulas B. Cum ergo illi radii in ordine progrediantur, prius concidit unus in rem visam postea alius. Non ergo simul videbitur totum B. Videtur autem videri visibus velociter transportatis.</p> <p>[3] Unumquodque visorum habet longitudinem spatii quo facto iam non videtur. Alia translatio: [RV3] Cuiuslibet visibilis per elongationem terminari visum.</p> <p>Quanto enim res magis removetur sub minori angulo videri qui tam diu diminuitur donec linee illum angulum continentes concurrant et in unam redigantur. Linee vero terminus est punctus qui cum sit invisibilis qui videbatur propinquius iam non videtur remotius ut docet prior dispositio.</p>

Let us summarize our argument so far. The lack of Defs. 8 and 9 as well as Prop.V24 strongly suggests that our MS #1 along with MS #9 belongs to the earliest phase of the Euclidean optical tradition in the Latin West, well before the standard Latin text became established. Moreover, our MS has many Greco-Latin propositions accompanied by those Arabo-Latin enunciations included in the text itself, not in marginal glosses. And we find in our MS a new Prop.V25, which is not in the Greek text, but derives from the Arabo-Latin translation of *Optica*. These facts seem to indicate that the Greek text itself may have been translated and studied in an intellectual atmosphere where the Arabic text or its translation were ready at hand. One of the most probable candidates for such translating activity is twelfth-century Sicily, where Greek, Arabic and Latin were officially used.<sup>9</sup> One of us has proposed a hypothesis of positing “the Sicilian school of translators” in his study of the Latin traditions of *De speculis*.<sup>10</sup> In our understanding, our MS of *De visu*, a companion volume to *De speculis*, indicates the same line of interpretation.

## I.2 Some Salient Features of Our Paraphrased Version

Now let us consider the paraphrase itself. As is generally true for paraphrased versions made from any Latin translation of a Greek text, the enunciations are usually left substantially untouched, whereas the proofs are vitally changed. In this respect, our manuscript is somewhat different from the medieval stereotype of paraphrasing. To show this, we have made a table of comparisons for some propositions that have different readings from those of *De visu*, disregarding minor differences and omissions. By glancing at the table, one can easily realize that the sentences of our MS are more natural Latin than their counterparts, which sometimes become clumsy on account of the word for word translation technique of the twelfth century. See especially [V26], [V28], [V36], and [V43] and their paraphrasing in our MS. It is also worth noting that the transliterated word in [V41], “parespamini,” is correctly changed into “parespasmēni,” since the original word is “παρεσπασμένοι.” This probably means that the author of our text has had a chance to consult a Greek text and/or knew Greek.

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<sup>9</sup>Our conjecture is contrary to Theisen's. According to Theisen[1979, 52], with no reasonable reasons given, our manuscript #1 “was written in England between 1150 and 1170.” But such a too narrow specification of date and place seems to us less plausible. We hope that our proposal will open up a new possibility of historical investigation.

<sup>10</sup>See Takahashi[2001b].

### A Table of Comparison for Enunciations

<i>De visu in Theisen</i> [1979]	Our MS #1
[Postulate 7] Sub pluribus autem angulis visa perspicatius <u>videri</u> .	[Postulate 7] Sub pluribus autem angulis visa perspicatius <u>intueri</u> .
[V2] Equalium magnitudinum in distantia iacentium propius <om.> iacentia perspicatius videntur.	[2] Equalium magnitudinum in distantia iacentium propius <u>posita</u> iacentia perspicatius videntur.
[V3] Unumquodque visorum habet longitudinem spatii quo facto <u>non iam videtur</u> .	[3] Unumquodque visorum habet longitudinem spatii quo facto <u>iam non videbitur</u> .
[V8] In eadem recta existentes <u>magnitudines equales</u> non deinceps ad invicem posite <om.> inequaliter <u>sub</u> oculo distantes, <u>inequales</u> apparent.	[8] In eadem recta existentes <u>equales magnitudines</u> non deinceps ad invicem posite <u>et</u> inequaliter <u>ab</u> oculo distantes, <u>inequales</u> apparent.
[V22] Datam longitudinem quanta est <u>reperire</u> .	[22] Datam longitudinem quanta est <u>invenire</u> .
[V25] Longior radius ad speram <u>proveniens</u> quasi linea contingens erit.	[24] Longior radius ad speram <u>perveniens</u> quasi linea contingens erit.
[V26] Spere qualitercumque <u>vis</u> sub uno oculo minus hemisperio <u>semper</u> apparet, eaque visa spere pars sub circulo <u>periferia</u> apparet.	[25] Spere qualitercumque sub uno oculo <u>vis</u> minus <u>semper</u> hemisperio apparet, eaque visa spere pars sub circulo <u>contenta</u> apparet.
[V27] Oculo accedente propius spere minus erit quod <u>videtur</u> , videtur <u>autem magis</u> videri.	[26] Oculo accedente propius spere minus erit quod <u>videbitur</u> , <u>sed maius</u> videtur videri.
[V28] Spera a duobus <u>oculis</u> visa, si <u>diametros</u> spere equalis fuerit <u>recte</u> in qua a se invicem <u>oculi</u> distant, <u>hemisperium eius</u> videbitur <u>totum</u> .	[27] Spera a duobus <u>occulis</u> visa, si <u>diametro</u> spere equalis fuerit <u>recta</u> in qua a se invicem <u>occuli</u> distant, <u>totum eius hemisperium</u> videbitur.

[V30] Si oculorum distancia ea que in spera diametro minor fuerit, minus hemisperio videbitur.

[V32] Si sub duobus oculis chilindrus videatur, manifestum quoniam et in eo contingent que in spera.

[V36] Si ab oculo ad basim conii accidant radii, ab accidentibus vero radiis et contingentibus a contactu recte trahantur per superficiem conii ad verticem eius, per protractas vero et ab oculo ad basim conii accidentes epipeda educantur, in contactu autem eorum, hoc est, in communi sectione epipedorum oculus ponatur, visum conii per totum equale videbitur visu in parallelo epipedo subiacenti plano existente.

[V41] Curruum rote aliquotiens circulares apparent, aliquotiens parespamini.

[V43] Est locus ubi oculo transposito, eo vero quod videtur manente, semper equale quod videtur apparet.

[V49] Est aliquis locus communis a quo inequales magnitudines equales apparent.

[29] Si oculorum distantia <om.> sperae diametro minor fuerit, minus hemisperio videbitur.

[31] Si sub duobus oculis chilindrus videatur, patens est quoniam et in eo contingent que in spera.

[35] Si ab oculo ad conii basim accidant radii, ab accidentibus vero radiis et contingentibus a contactu recte trahantur per superficiem conii ad verticem eius, per protractas vero et ab oculo ad basim conii accidentes educantur epipeda, in contactu autem eorum, hoc est, in communi sectione epipedorum oculus ponatur, visum conii per totum equale videbitur visu existente in epipedo parallelo subiacenti plano.

[39] Curruum rote aliquotiens circulares apparent, aliquotiens parespasmeni.

[41] Est locus ubi oculo transposito, eo <om.> manente quod videtur, illud quod videtur semper videtur equale.

[47] Est aliquis locus communis a quo magnitudines inequales apparent equales.

Now let us briefly examine the theoretical quality of paraphrases in our text. First let us look at the paraphrased proofs. They are in most cases rewritten after the manner of the Greek text, except for Prop.1 which is a faithful reproduction of the original proof.<sup>11</sup> However, we can find some proofs executed under the influence of the Arabo-Latin text, notably in the alternative proofs of Prop.3 and Prop.23, and to a lesser degree in Prop.30. Moreover, we can detect in Prop.48 the following enigmatic and redundant passages following each other:

- (1) “Coniungatur enim ab B super [D] recta BD, et dividatur in duo equalia ad punctum E, et protrahatur a puncto E perpendicularis EZ recte DB. Dico quoniam si super EZ oculus ponatur AB GD equalia apparent.”
- (2) “Lineetur enim ab B super D recta BD, et dividatur in duo equa ad E punctum, et ab E protrahatur perpendicularis EZ recte BD. Dico quoniam si super EZ oculus ponatur AB et GD equalia apparent.”

We do not know exactly why these passages appear consecutively. But we know that the underlined word in each passage is frequently used in the Greek-Latin and the Arabo-Latin texts respectively. The author’s careless mistake might invite one to the attractive idea that he was studying *Optica* with both texts at hand.

In this connection, it is worthy of note that our author mentions such mathematical instruments for describing or measuring figures as “*festuca*” [Props.7, 35, 37, 38, and 43], “*virga*” [Props.19, 20, 21, and 22], and “*pulvis*” [Props.35 and 38]. There is no mention of these instruments in the Greek text, since practical aspect of mathematics is totally absent in it. But we find in our text at least “*virga*” explicitly mentioned in the alternate enunciations in Props.19 and 20.

One of the main purposes of paraphrasing is to elucidate the intent of the author and the logical structure of propositions. This kind of activity can readily be found in our text, for which some examples below will suffice:

- [Prop.9] Nec auctor proponit determinate quod maior vel minor sit proportio ...
- [Prop.42] quod intellexit auctor dicens ...
- [Prop.12] Hec probatio est conversa prioris. [For similar expressions, cf. Props.15 & 17]
- [Prop.16] Sensus utriusque translationis est quod ...
- [Prop.18] Sensus huius propositionis est quod ...

The citation of theorems in the *Elements* is made typically in such forms as “per

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<sup>11</sup>The faithful reproduction in our text of Prop.1 is the cause that induced Lindberg[1975] to a misunderstanding that our text was of version 1.

III<sup>am</sup> XI<sup>mi</sup> Euclidis” or its abbreviations (*passim*). It is worth mentioning the citation of the Pythagorean theorem [*Elements*, I-47] with a nickname. In Props.7 and 38 our text refers to it as “*per dulcannon*,” not “*per dulcarnon*,” the usual and correct nickname of Arabic origin (literally ‘possessor of two horns’ [dhū’l qarnayn]<sup>12</sup>).

The indirect proof is introduced by such words as “*adversarius*” and “*falsigraphus*” in Prop.38. These words are so characteristic that they may give us some hint for identifying our author, as argued in Takahashi[2001a, 79].

We can count as one of the salient features of our author’s paraphrase the explanations of mathematical or optical terms of such kind as those listed below, which are very elementary:

BDF portio sive arcus [Prop.8]  
 ZTE superficiei, que dicitur sector [Prop.9]  
 perpendicularis sive cathetus quod idem est.[Prop.11][Cf. Prop.43]  
Axis piramidis est linea que est in medio piramidis ... [Prop.33]  
conus sive piramis quod idem est [Prop.36]  
epipedo circuli, sive superficiei circuli quod idem est [Prop.37]  
ad rectos angulos sive perpendiculariter quod idem est [Prop.38]  
ei que e centro id est semidiametro [Prop.39]  
paralella sive equidistans [Prop.43]  
 que est proportio visionis ad visionem, sive apparentie ad apparentiam, sive E anguli ad O angulum, quod idem est [Prop.9]  
 minor apparere putatur, sive minus videbitur videri quod idem est [Prop.36]

Moreover, our author employs a peculiar way of designating angles. The most typical one is, for instance, “*N angulus ONT trianguli*,” that is, “the angle N of triangle ONT.” This is found almost everywhere in our text, whereas the usual expression for an angle, “the angle ONT” (*angulus ONT*) in this case, is rarely found. In Prop.38(c), we can read “... *propter GZS angulum maiorem OZS angulo*,” accompanied by an astonishing comment: “*Semper per medias litteras intellige angulos*” (‘always understand angles by the middle letters’).

Our final example concerns the phraseology used for the law of reflection in our text. In Prop.20 our text reads: *reverberatio non facit nisi pares angulos*. We know that the same phrase is often used in the text of *De speculis* in the same codex, namely Text III in Takahashi[1992]. As argued in Takahashi[2001b, 33-34], emphasis should be put on two words, “*reverberatio*” and “*pares*”, which are rarely found in medieval optical treatises. To our knowledge, the first word is found only in Ptolemy’s *Optica* which was translated probably in Sicily by Eugene the Emir in the

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<sup>12</sup>Cf. *Dictionary of Medieval Latin from British Sources*, Fascicule III, Oxford University Press, p.733.

middle of the twelfth century, while the second appears in *De radiis visualibus*, the Arabo-Latin rendition of Euclid's *Optica*. Here again we get an additional piece of evidence that suggests strongly that our text of *De visu* exhibits a unique outcome of cross-cultural intercourse between Greek, Arabic and Latin in Sicily.

### I.3 Editorial Procedures

In establishing the text, we followed the customary procedures.

The capitalization, punctuation and paragraphing are all ours. Capitalization is employed to represent geometrical magnitudes used in both the figures and the text, though our manuscript gives them in small letters. Figures are, of course, reproduced faithfully from our manuscript. There are some instances where original figures were incorrectly drawn. For such cases we put [*sic*] to the figures for drawing reader's attention.

Square brackets [ ] have been employed in our text to enclose our editorial insertions. The difference of proposition number between our text on the one hand, and Heiberg's Greek text A and Theisen's Latin text V on the other, is indicated, for instance, as [38(A35; V40)], which means that Proposition 38 of our text is the same as Proposition 35 of Greek text A and Proposition 40 of Latin text V. An alternate enunciation is shown, for example, as [RV1] which means that it is drawn from Proposition 1 of *De radiis visualibus*. A question mark [?] draws attention to a doubtful reading, and an exclamation mark [!] has been used as equivalent for *sic*, designating an irregular spelling, a dubious reasoning and so forth.

The following abbreviations and Latin terms have been employed.

<i>coniecimus ex</i>	we have conjectured from
<i>corr. (correxit)</i>	has corrected
<i>correximus ex</i>	we have corrected from
<i>del. (delevit)</i>	has deleted
<i>mg. hab. (in margine habet)</i>	has in the margin
<i>inter lineas hab.</i>	has between lines
<i>iter. (iteravit)</i>	has repeated
<i>lac. (lacuna)</i>	blank
<i>lege</i>	read
<i>post</i>	after
<i>scr. (scripsit)</i>	has written
<i>signo posito trans. (... transposuit)</i>	has indicated a change of position by means of a sign
<i>supra</i>	above



## Bibliography

- Clagett[1964], M., *Archimedes in the Middle Ages*, vol.1, The University of Wisconsin Press.
- Heiberg[1895], J. L., *Opera omnia*, eds.J.L.Heiberg and H.Menge, vol.7: *Optica, Opticorum recensio Theonis, Catoptrica cum scholiis antiquis*, Leipzig.
- Kheirandish[1999], E., *The Arabic Version of Euclid's Optics, Edited and Translated with Historical Introduction and Commentary*, 2 vols., Springer.
- Knorr[1990], W., "Paraphrase Editions of Latin Mathematical Texts: *De figuris ysooperimetris*," in *Medieval Studies*, vol.52, pp.132–189.
- Knorr[1994], W., "Pseudo-Euclidean Reflections in Ancient Optics: A Re-Examination of Textual Issues Pertaining to the Euclidean *Optica* and *Catoptrica*," in *Physis*, vol.XXXI, pp.1–45.
- Lindberg[1970], D. C., *John Pecham and the Science of Optics: Perspectiva communis*, Madison.
- Lindberg[1975], D. C., *A Catalogue of Medieval and Renaissance Optical Manuscripts*, Toronto.
- Rashed[1997], R., *L'Optique et la Catoptrique*, E.J.Brill, Leiden.
- Takahashi[1992], K., *The Medieval Latin Traditions of Euclid's Catoptrica: A Critical Edition of De speculis with an Introduction, English translation and Commentary*, Kyushu Univ.
- Takahashi[2001a], K., "A Manuscript of Euclid's *De speculis*: A Latin Text of MS 98.22 of the Archivo y Biblioteca Capitulares de la Catedral, Toledo," *SCIAMVS*, vol.2, pp.75–143.
- Takahashi[2001b], K., "Euclid's *De speculis*: Its Textual Tradition Reconsidered," in *Optics and Astronomy (Proceedings of the XXth International Congress of History of Science (Liège, 20–26 July 1997))*, Vol. XII, ed. G. Simon and S. Débarbat, Brepols, pp.29–41.
- Theisen[1972], W. R., *The Medieval Tradition of Euclid's Optics*, ph.D. dissertation (unpublished), Univ. of Wisconsin.
- Theisen[1979], W. R., "Liber de visu: The Greco-Latin Translation of Euclid's Optics," in *Medieval Studies*, vol.41, pp.44–105.