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Excavation Problems in Babylonian Mathematics: Susa Mathematical Text No. 24 and Others

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I Introduction

The main purpose of the present article is to offer an interpretation of the second problem of Susa mathematical text No.24, which has not yet been deciphered.[1] In order to do so it is necessary for us to get acquainted with the technical expressions of "excavation problems" of Babylonian mathematics, to which our problem of Susa evidently belongs. In Babylonian mathematics there are many problems concerning digging a hole, a cistern, a canal and the ground for the foundation of a building, all of which may be called "excavation problems".[2]

From a mathematical point of view most of these problems are comparatively easy, but some of them have not been completely understood because of obscure words probably denoting something solid or because of certain scribal mistakes in the texts.

Before discussing the Susa mathematical text No.24, I will cite three Old Babylonian excavation problems in order to show their typical expressions which also occur in the second problem of Susa No.24. I will, in passing, propose several new readings of the texts.

II The first example from BM 85200 + VAT 6599

The large clay tablet BM 85200 + VAT 6599 must have originally contained 30 problems, of which 22 remain in good condition.[3] All the problems begin with

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[1]E. M. Bruins et M. Rutten, *Textes mathématiques de Suse* [=TMS], (1961), pp. 117–122.
For the general character of the Susa mathematical texts, see my paper: K. Muroi, Quadratic Equations in the Susa Mathematical Text No.21, *SCIAMVS*, Vol.1 (2000), pp. 3–10.

[2]Cf. K. Muroi, Small Canal Problems of Babylonian Mathematics, *Historia Scientiarum*, Ser.2, Vol.1 (1992), pp. 173–180.

[3]O. Neugebauer, *Mathematische Keilschrift-Texte* [=MKT], I,II(1935), III(1937). MKT I pp. 193–219. MKT III, pp. 54–55. F. Thureau-Dangin, *Textes mathématiques babyloniens* [=TMB] (1938),

the same word "túl-sag" and concern the calculations of the volume, the sides, and the depth of túl-sag. Neugebauer, who published the text, translated túl-sag as "Keller, Brunnen, od.dgl.", and Thureau-Dangin as "une cave". Thus neither scholar translated "sag" which may correspond to the Akkadian adjective *rēštûm* "uralt".[4] So I propose a tentative translation "a very old well" for túl-sag in the following translation.[5]

BM 85200 + VAT 6599 problem 14, Obverse II

Transliteration

1. túl-sag *ma-la* igi uš *ma-la* igi-bi sag *ma-la* igi {ug[u] igi-bi di[rig]}
2. GAM-*ma* 1[6 saḫar-ḫi-a ba-z]i uš sag *ù* GAM en-nam
3. za-e igi 12 d[u$_8$-a 5 *ta*]-*mar* 5 [*a-na* 16] *i-š*[*i* 1,2]0 *ta*-<*mar*>
4. 1,20 igi igi 1,2[0 du$_8$-a 45 *ta-m*]*ar* 40$^{\text{sic}}$ igi-bi [16 G]AM
5. *ne*-[*pé*]-*šum*

Translation

1. A very old well. The length is equal to a certain number (igi), the width is equal to its reciprocal (igi-bi). {The amount by which} the number {exceeded the reciprocal}
2. is the depth.[6] [The soil (of the well)], 1[6, was remov]ed. What are the length, the width, and the depth?
3. You, m[ake] the reciprocal of 12, [(and) you] see [0;5]. Multi[ply] 0;5 [by 16], (and)

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pp. 11–21.

[4]W. von Soden, *Akkadisches Handwörterbuch* [=AHw] Band I (1985), Band II (1972), Band III (1981), p.973.

[5]Perhaps túl-sag is the logogram of *šitpu* "pit, excavation, well, excavated soil" (A. L. Oppenheim and others, *The Assyrian Dictionary* [=CAD], Vol.17, Š part 2 (1992), p.194), "Ausschachtung, Mulde" (AHw, p.1200).

In lexical lists we can find túl=*šitpu* but not túl-sag=*šitpu*. In addition we have a strong evidence that sag of túl-sag is an adjective:

1. túl "a well"
2. túl-sag "a··· well"
3. túl-kù-ga "a pure well".

These are the first three lines of BM 78206, Reverse II, which lists various wells and rivers. See, E. Reiner, *Materials for Sumerian Lexicon* XI, 1974, p.144.

[6]For the peculiar usage of GAM in the meaning of "depth", see my paper:

K. Muroi, The Area of a Semicircle in Babylonian Mathematics: New Interpretation of MLC 1354 and BM 85210 No.8, *Sugakushi Kenkyu* No.143 (1994), pp. 50–60.

you <see> [1;2]0.
4. 1;20 is the number. [Make] the reciprocal of 1;2[0], [(and) you s]ee [0;45]. 40$^{\text{sic}}$ is the reciprocal. [16 is the de]pth.
5. (Such) is the pro[ced]ure.

Commentary

If we denote the length, the width, and the depth of the well by x,y, and z, respectively, the equations presented in lines 1 and 2 would be:

$\overline{x} = y$ (that is $xy = 1$),
z (kùš) $= 12(x - y)$ (nindan, where 1 nindan = 12 kùš $\approx$ 6m),
$xyz = 16$ (volume-sar, 1 volume-sar = 1 nindan$^2\cdot$ kùš).

However, the equations actually solved in lines 3 and 4 are:

$\overline{x} = y$, $z = 12x$, $xyz = 16$,

because

$x = \overline{12} \cdot 16 = 0;5 \cdot 16 = 1;20$ (nindan) (line 3),
$y = \overline{x} = 0;45$ (nindan), $z = 16$ (kùš) (line 4).

In this example we must keep in mind the following two expressions:

"The amount by which the number exceeded the reciprocal$\cdots$", which has been erroneously inserted in line 1, and
"Make the reciprocal of 12 $\cdots$", which is typical of the calculation of the depth through the length or the width.

III The second example from YBC 8588

On the small clay tablet YBC 8588 only one excavation problem, whose statement is obviously defective, is written down.[7] Although Neugebauer and Sachs made clear its mathematical purport and presented a nearly satisfactory transliteration, a few points remain to be explained: their translation of Obverse lines 2 and 3 was tentative and they gave neither transliteration nor translation to the first half of Bottom line 2 and the second half of Reverse line 1.

[7]O. Neugebauer and A. Sachs, *Mathematical Cuneiform Texts* (1945) [=MCT], pp. 75–76.

YBC 8588

Transliteration

Obverse 1. ki-lá 1,30 uš 30 sag

2. *i-na iš-te-en ka-la-ak-ki-im*

3. 9 *ka-la-ak-ku*

4. *šu-up-lum* en-nam

5. 1,30 uš gar-ra 30 sag gar-ra

6. *i-na iš-te-en ka-la-ak-ki-im*

7. 9 *ka-la-ak-ku ša iq-bu-ú*

8. *iš-te-en ka-la-ak-kum ša iq-bu* éš-gàr

9. 15 éš-gàr gar-ra

10. 9 *ka-la-ak-ku ša iq-bu* erín-ḫi-a

11. 9 erin-ḫi-a gar-ra-*ma*

12. 15 éš-gàr *a-na* 9 erín-ḫi-a

13. íl 2,15 saḫar-ḫi-a

14. 1,30 uš *a-na* 30 sag íl 45 a-šà

15. igi 45 a-šà *pu-ṭur-ma* 1,20

16. 1,20 *a-na* 2,15 saḫar-ḫi-a

17. íl 3 *šu-up-l*[*um*]

Bottom

1. *šum-ma* 1,30 uš 30 sag

2. [3 *šu-up-lum*] 15 éš-gàr

Reverse

1. 9 erín-ḫi-a [*iq-bu-ni-kum*]

2. 1,30 uš *a-na* 30 sag íl 45

3. 45 a-šà *a-na* 3 *šu-up-lim*

4. íl 2,15 saḫar

5. igi 15 éš-gàr *pu-ṭur-ma* 4

6. 4 *a-na* 2,15 saḫar íl 9

7. 9 erín-ḫi-a *it-ta-di-kum*

Translation

Obverse

1. A hole (dug in the ground). 1;30 is the length. 0;30 is the width.

2. In the one hole

3. (there are) 9 holes.

4. What is the depth?

5. Put down 1;30, the length. Put down 0;30, the width.

6. In the one hole

7. (there are) 9 holes which they said.

8. One hole (of the nine) which they said is the work quota (per man-day).

9. Put down 0;15, the work quota.

10. 9 holes which they said are (the work quota of) the workers.

11. Put down 9, the workers, and

12,13. multiply 0;15, the work quota, by 9, the workers. (The result) 2;15 is the volume.

14. Multiply 1;30, the length, by 0;30, the width, (and the result) 0;45 is the area.

15. Make the reciprocal of 0;45, the area, and (the result is) 1;20.

16,17. Multiply 1;20 by 2;15, the volume, (and the result) 3 is the dep[th].

Bottom

1,2. If 1;30 is the length, 0;30 the width, [3 the depth], 0;15 the work quota (per man-day),

Reverse

1. (and) [they say] 9 workers [to you],
2. multiply 1;30, the length, by 0;30, the width, (and the result is) 0;45.
3,4. Multiply 0;45, the area, by 3, the depth, (and the result) 2;15 is the volume.
5. Make the reciprocal of 0;15, the work quota, and (the result is) 4.
6. Multiply 4 by 2;15, the volume, (and the result is) 9.
7. It has given you 9 workers.

Commentary

The Akkadian word *kalakkum*, which is a loanword from the Sumerian ka-lá or ki-lá, means "excavation, storehouse, silo". [8] In mathematical texts it substantially denotes a rectangular parallelepiped or a truncated pyramid. In our text it denotes the former. Although Neugebauer and Sachs correctly remarked that it is "an excavation of some sort" in the vocabulary of MCT (p. 166), they did not translate it and gave a tentative translation of lines 2 and 3:

"in one *kalakkum* (there are(?)) 9 *kalakkū*".

The expression in question, *ina ištēn kalakkim* 9 *kalakkū*, is obviously an abbreviation of:

**ina ištēn kalakkim* 9 *kalakkū ibaššû* "There are nine holes in one hole".

We can find a similar expression in a mathematical text concerning equality of two volumes:

8 *i-na* 1,36 še-gur *i-ba-ši-i* "There is 8,0,0 (sìla) in 1,36 gur of barley", that is, "8,0,0 (sìla) equals 1,36 gur of barley", where 1 sìla $\approx$ 1 liter and 1 gur = 5,0 sìla.[9]

My restoration of Reverse 1 is based on the facts that the Akkadian verb *qabûm* "to say" occurs in Obverse lines 7,8 and 10, and that a similar expression also occurs in another mathematical text:

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[8]CAD, Vol.8, K (1971), p.62.

[9]K. Muroi, Two Harvest Problems of Babylonian Mathematics, *Historia Scientiarum*, Vol.5 (1996), pp. 249–254.

na-ap-ḫa-ar uš *li-iq-bu-ni-kum-ma*

"They should say the sum of lengths to you $\cdots$".[10]

Now, we can understand the text completely. In the statement of the problem (Obverse lines 1-4) the volume of a rectangular parallelepiped (= V), whose length is 1;30 nindan, the width 0;30 nindan, is said to be nine times a certain volume (= v) and then its depth is asked for. It is obvious that we need one more condition in order to solve the problem. The necessary condition, that is, $v = 0;15$ (volume-sar), is lengthily and awkwardly stated in the solution (Obverse lines 8-10).[11] The answer is obtained from a simple linear equation as follows:

$$V = 1;30 \cdot 0;30 \cdot z = 0;15 \cdot 9$$
$$0;45 \cdot z = 2;15$$
$$\therefore z = \overline{0;45} \cdot 2;15 = 1;20 \cdot 2;15 = 3(\text{kùš}).$$

The remaining lines (Bottom and Reverse) are for verification:

$$1;30 \cdot 0;30 \cdot 3 \cdot \overline{0;15} = 9.$$

We will see below that the *kalakkum* occurs in the Susa mathematical text No.24, lines 31, 33, and 37, in the sense of a rectangular parallelepiped.

IV The third example from BM 85196 No.15

The comparatively large tablet BM 85196, which resembles BM 85200 + VAT 6599 we have seen above both in style and terminology, contains 18 problems of miscellaneous contents.[12] The fifteenth problem, which concerns the repairs of a canal, has not been completely understood as yet because of a certain mistake in the final part of the solution. To the left of lines 19-23 which contain the problem there is a diagram with the relevant numbers (cf. Fig.1).

BM 85196, Reverse I, lines 19-35 (= No.15)

Transliteration

19. *a-ta-ap ta-ra-aḫ-ḫi pa-na-nu-ša*
20. 20 *mu-ḫu* 15 *sà-súm ù* 3 bùr 2,30 [*e-l*]*e-nu-um*
21. *aš-lu-uṭ* 1,40 ki-ta *aš-lu-uṭ a-ta-pa-*[*a*]*m pa-ni-*[*tam*]

[10] Taha Baqir, Another Important Mathematical Text from Tell Harmal, *Sumer* 6 (1950), pp. 130–148.

[11] The poor composition of a problem such as this is not rare in Babylonian mathematics. See IV below.

[12] MKT II, pp. 43–59. TMB, pp. 39–46.

22. 1 kùš sukud[sic] daḫ-ḫa saḫar-ḫi-a *la-bi-ru-tum*

23. saḫar-ḫi-a gibil$_4$ *ù* 1 lú uš *pu-lu-uk*

24. za-e saḫar-ḫi-a sumun *a-mur* 20 *mu-ḫa-am* *ù* 15 *sà-súm* ul-gar-*ma*

25. 35 *ta-mar* 1/2 35 *ḫe-pé* 17,30 *ta-mar* 17,30

26. *a-na* 3 kùš bùr *i-ši* 52,30 *ta-mar* saḫar-ḫi-a *la-bi-ru-tum*

27. nigín-na saḫar-ḫi-a gibil$_4$ *a-mur* 2,30 *ša ta-aš-lu-ṭú* tab-ba 5 *ta-mar*

28. 1,40 *ša ta-aš-lu-ṭú* tab-ba 3,20 *ta-mar* 5 *a-na* 20 daḫ-ḫa

29. 25 *ta-mar* 3,20 *a-na* 15 daḫ-ḫa 18,20 *ta-mar* 18,20

30. *ù* 25 ul-gar 33[sic],20 *ta-mar* 1/2 [3]3,20 *ḫe-pé*

31. 16,40 *ta-mar* 16,40 *a-na* 4 kùš bùr *i-ši* 1,6,40 *ta-mar*

32. 52,30 saḫar-ḫi-a sumun 1,6,40 dúr-gal saḫar-ḫi-a ba-zi 14,[10 *ta-mar*]

33. saḫar-ḫi-a 14,10 en-nam 14,10 gar-ra 10[sic] gín éš-gàr

34. 56,40 gar-ra 56,40 1 lú uš *i-ṣ*[*a-ba-at*]

35. *ki-a-am ne-p*[*é-šum*]

2, 30	20	2, 30
1, 40	15	1, 40
	3	
	1	

Fig. 1

Translation

19. A small canal with sloping sides. Formerly

20. 0;20 was the top width, 0;15 the base width, and 3 the depth. [At] the top

21-23. I cut off 0;2,30. At the base I cut off 0;1,40. I enlarged (literally: added) the former canal by 1 kùš of height[sic] (a mistake for depth). (What are) the old volume (and) the new volume? And then divide the length (for) one man.

24. You, look at the old volume. Add together 0;20, the top width, and 0;15, the base width, and

25. you see 0;35. Halve 0;35, (and) you see 0;17,30.

26. Multiply 0;17,30 by 3 kùš of the depth, (and) you see 0;52,30. (This is) the old volume.

27. Return. Look at the new volume. Double 0;2,30 which you cut off, (and) you see 0;5.

28. Double 0;1,40 which you cut off, (and) you see 0;3,20. Add 0;5 to 0;20, (and)

29. you see 0;25. Add 0;3,20 to 0;15, (and) you see 0;18,20.
30. Add 0;18,20 and 0;25 together, (and) you see 0;33[sic],20. Halve 0;33,20, (and)
31. you see 0;16,40. Multiply 0;16,40 by 4 kùš of the depth, (and) you see 1;6,40.
32. Subtract 0;52,30, the old volume, (from) 1;6,40, the large foundation's volume, (and) [you see] 0;14,[10].
33. The volume (of the soil removed anew) is 0;14,10. What should I put (to) 0;14,10 (in order to obtain) 10[sic] (a mistake for 15) gín of the work quota?
34. Put down 0;56,40. 0;56,40, one man ta[kes] the length.
35. Such is the pro[cedure].

Commentary

In lines 21,27, and 28 the Akkadian verb *šalāṭum* "einschneiden" (AHw, p. 1147), "to split off, to split, to cut",[13] occurs in the meaning of "to cut off (the side of a hole)". It should be contrasted with another verb *šuppulum* "niedrig machen, vertiefen" (AHw, p. 1169), "to lower, make lower, to excavate"[14], which occurs not here but in other excavation problems. For example:

ki ma-ṣí ù-ša-pí-il "How deep did I excavate ?"[15]

Both *šalāṭum* and *šuppulum* occur in the Susa mathematical text No.24, lines 35 and 31, respectively.

In line 32 a problematic word, dúr-gal "the large dúr", occurs. The logogram dúr must stand for *išdum* "base, foundation (of a building, wall, gate etc.)"[16] or *šubtum* "site, foundation of a building, built-over area of a building plot"[17] or *šuršum* "root, base, foundation (of buildings, mountains, etc.)"[18]. And the same word, dúr-gal, certainly occurs in another excavation problem in the meaning of "a large foundation". [19] But in our problem it substantially denotes the new cross-section

[13] CAD, Vol.15, S (1984), p. 94. But the translation suggested in CAD, Vol.17, Š part 1 (1989), p. 240, is not correct: "I added (?) 2,30 at the top (of the bank)" and so on.

[14] CAD, Vol.17, Š part 1 (1989), p.422.

[15] IM 54478, Obverse line 5. See, Taha Baqir, Some More Mathematical Texts, *Sumer*, Vol.7 (1950), pp. 28–45.

[16] CAD, Vol.7, I/J (1960). p.235.

[17] CAD, Vol.17, Š part 3 (1992), p.172.

[18] CAD, Vol.17, Š part 3 (1992), p.363.

[19] BM 85194, Obverse I, line 19, MKT I, p.143.
See further MKT II, p.45, where Neugebauer reads the words in question as ŠU[sic]-GAL saḫar-ḫi-a or KU (=dúr)-GAL saḫar-ḫi-a "dem Gesamt(?)-Volumen", and TMB, p.44, where Thureau-Dangin reads them as šu[sic]-gal *epiri*[ḫi-a] "le total général de la terre". We have no example of *šu-gal "the total sum".

ABCD of the enlarged canal and does not concern “the foundation” at all (cf. Fig.2).

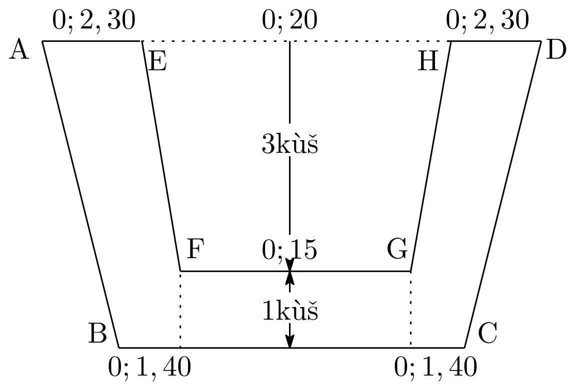


Fig. 2

The word seems to be out of place in line 32 of our text. We expect here saḫar-ḫi-a gibil$_4$ “the new volume” instead of dúr-gal saḫar-ḫi-a “the large foundation’s volume”. In addition to this there are several mistakes in the text.

1. What is asked for in the statement of the problem is the volume, but what is calculated in the solution is the area. This seems to be a mistake, although it is also possible that the length of the canal is assumed to be 1 nindan or 1,0 nindan.
2. In line 22 sukud “height” occurs for bùr “depth”. We expect here 1 kùš bùr *ù-ša-pí-il* “I excavated 1 kùš deep” instead of 1 kùš sukud[sic] daḫ-ḫa “I added 1 kùš high[sic]”.
3. There is a mistake in calculation in lines 29 and 30:

   $$0;18,20 + 0;25 = 0;33,20.$$

   The scribe of this problem continued his work without noticing the error.
4. The work quota 10 gín in line 33 is a mistake for 15 gín. The work quota is not given in the statement but suddenly occurs in the solution just as in YBC 8588 discussed above. This mistake has long prevented us from understanding the final answer 0;56,40.

Now we can understand the text completely. See Fig.2. Firstly the area of the cross-section of the old canal, whose upper width, lower width and depth are 0;20 nindan ($\approx$ 2m), 0;15 nindan ($\approx$ 1.5m), and 3 kùš($\approx$ 1.5m), respectively, is calculated:

$$0;20 + 0;15 = 0;35 \quad (1/2)\cdot 0;35 = 0;17,30$$
$$0;17,30 \cdot 3 = 0;52,30. \qquad \text{(lines 24–26)}$$

Since the result 0;52,30 is called “the old volume”, it is necessary for us to assume its unit is not *area*-sar but *volume*-sar. Secondly the area of the cross-section of the enlarged canal is calculated:

$$0;2,30 \cdot 2 = 0;5 \quad 0;1,40 \cdot 2 = 0;3,20$$
$$0;20 + 0;5 = 0;25 \quad 0;15 + 0;3,20 = 0;18,20. \qquad \text{(lines 27–29)}$$
$$0;18,20 + 0;25 = 0;33^{\text{sic}},20$$
$$(1/2) \cdot 0;33,20 = 0;16,40$$
$$0;16,40 \cdot 4 = 1;6,40. \qquad \text{(lines 29–31)}$$

The result must be in *volume*-sar. Lastly the difference between the old volume and the new volume is calculated, and then it is divided by the work quota to obtain the length which one worker undertakes per day:

$$1;6,40 - 0;52,30 = 0;14,10, \qquad \text{(line 32)}$$
$$0;14,10/0;15 = 0;56,40. \qquad \text{(nindan, lines 33 and 34)}$$

The expression of the division, which is carried out as if 0;15 were not a regular number, is awkward.

V The Susa mathematical text No.24

The Susa mathematical text No.24 consists of two excavation problems, the first of which is on the obverse of the tablet and the second on the reverse. Since the statements of both problems are almost lost, it is very difficult to reconstruct the equations treated in these problems. Bruins and Rutten nearly made clear the mathematical purport of the first problem, but their transliteration and translation need a number of corrections. I shall present improved ones below. As to the second problem they could not succeed in deciphering the text. Therefore I shall present its mathematical interpretation for the first time with my own transliteration and translation.

Susa mathematical text No.24

Transliteration

Obverse

(The beginning of the tablet is lost.)

1. [··· ··· ···]-*ma* 9,22,30 [··· ···] *i-ší*
2. [··· ··· ··· 2]4(?) *ù* 2 *ma-na-at*
3. [··· ··· ···] sag(?) 2 zi a-šà
4. [··· ··· ···]···-*ma* uš 15 *wa-ṣí-ib* uš
5. [··· ··· ···3]0 [*a*]-*na* 9,22,30 *i-ší-ma*

6. [··· 4,41,15 *ta-ma]r tu-úr-ma* 45 uš *a-na* 2 sag
7. [*i-ší-ma* 1,30] *a-na* [9,2]2,30 *i-ší-ma* 14:3,45 *ta-mar*
8. [14:3,45 a-šà *sà-a]r-ru* [14:]3,45 [*a-n]a* 4,[4]1,15 a-šà
9. [*i-ší-ma* 1,5,55],4,41,15 *ta-mar re-[iš-k]a li-ki-il*
10. [15 *wa-ṣí-ib* uš *a-na* 2] sag *i-ší-ma* 30 *t[a-mar* 30 *n]a-sí-iḫ* sag
11. [30 *a-na* 3 *i-š]í-ma* [1],30 *ta-mar* [3]0 *i-na* 1,[30] zi
12. 1 *ta-mar* 1 *a-[na]* 9,22,30 *i-ší-ma* 9,2[2,30] *ta-mar*
13. *aš-šum* <1> *ki-ma* uš *qa-bu-ku* 1 a-rá [*a-na* 9],22,[30 daḫ]
14. 1,9,22,30 *ta-mar* 1[/2] 1,[9,2]2,30 *ḫe-pe* 34,41,15 *ta-mar*
15. 34,41,15 nigin 20:3,[13,21,3]3,45 *ta-mar*
16. *a-na* 20:3,13,21,33,[45] 1,5,55,4,41,15 daḫ
17. 21:9,8,26,15 [*ta-mar*] *m[i-n]a* íb-si 35,37,30 íb-si
18. 34,41,15 *ta-ki-i[l-ta]-ka* [*a-na* 3]5,37,30 daḫ
19. 1,10:18,45 *ta-m[ar] m[i-na a-na]* 14:3,45
20. a-šà *sà-ar-ri* gar *šà* [1,10:1]8,45 *i-na-ad-di-na*
21. 5 gar 5 dagal an-ta 1/2 [5 *ḫe-pe* 2,30 *ta-mar* 2,30 *a-na*]
22. 30 dirig daḫ 3 *ta-mar* 3 dagal ki-ta [igi 12 *šà* dagal an-ta]
23. ugu dagal ki-ta *i-te-ru le-q[é]* 10 *ta-mar* [30 *ù* 10 ul-gar]
24. 40 *a-na* 12 *šu-up-li i-ší* 8 *ta-mar* [5 dagal an-ta]

Bottom

25. *ù* 3 dagal ki-ta ul-gar 8 *ta-mar* [1/2 8 *ḫe-pe*]
26. 4 *ta-mar* 4 *a-na* 8 *šu-up-li i-ší-[ma]*
27. 32 *ta-mar* igi 32 *pu-ṭú-úr* 1,5[2,30 *ta-mar*]
28. 1,52,30 *a-na* 24 saḫar *i-š[í* 45 *ta-mar* 45 uš][20]

Reverse

30. za-e(?) ··· [···
31. *ú-šà-pí-i[l] i*(?)-[*na*(?) *k]a-la-ak-[ki-im* ···
32. 2-kam *ta-a[d]-di-in* 2-*tu* [··· ···
33. a-šà *ka-la-ak-ki* gal ul-[gar ···
34. *a-na* tùn *šà a-ta-ap pa-[n]a-n[im]* da[ḫ ···
35. *a-na* tur *aš-l[u-u]ṭ* ul-gar saḫar ··· [··· ··· 1,15]
36. za-e 1,15 ul-gar *a-na* 13 *šà-[la-šé-ra-t]i i-ší-ma* 16,[15]
37. 10 *šà ka-la-ak-ku* ugu *ka-la-a[k-ki i-t]e-ru* nigin 1,40 *ta-mar*
38. 1,40 *i-na* 16,15 zi 16,[13,20] *ta-mar* igi 10 dirig *pu-ṭú-úr*
39. 6 *ta-mar* igi 12 *šu-up-li pu-ṭú-úr* 5 *ta-mar* 5 *a-na* 6 *i-ší*
40. 30 *ta-mar* 30 *ta-lu-ku* 30 *ta-lu-ka a-na* 16,13,20 *i-ší-ma*

[20]In line 29 remnants of a few cuneiform signs are visible, but they must go with the beginning of the reverse, and therefore belong to the second problem. So I omit line 29 in my transliteration and translation.

41. 8,6,40 *ta-mar* 10 [dir]ig nigin 1,40 *ta-mar* 1,40 *a-na* 13 *šà-la-šé-ra-ti*
42. *i-ší-ma* 21,[40 *ta-mar*] 21,40 *i-na* 8,6,40 zi
43. 7,45 *ta-*[*mar re-iš-k*]*a li-ki-il* 3[0 *ta-lu-k*]*a*
44. *a-na* 13 [*šà-la-šé-ra-ti*] *i-ší* 6,30 *t*[*a-mar* 30 *t*]*a-lu-ka*
45. *a-na ka-aiia-ma-*[*ni*] 2 tab-ba 1 *ta-mar* 1 *a-na* 6,30 daḫ
46. [7],30 *ta-mar* 1[3 *šà-l*]*a-aš-šé-ra-ti a-na* 3*-šu a-na ka-aiia-ma-ni*
47. *a-li-ik-ma* 3[9] *ta-mar* 7,30 *a-na* 39 daḫ 46,30 *ta-mar*
48. *mi-na a-na* 46,30 gar *šà* 7,45 *šà r*[*e-iš*]*-ka ú-ki-il-lu*
49. [*i-n*]*a-ad-di-na* [10] gar *re-iš-ka li-ki-il* 1/2 10 dirig *ḫe-pe*
50. [5 *ta*]*-mar* [5(?) gar(?)] 5 nigin 25 *ta-mar* 25 *a-na* 10
51. [*šà re-iš-ka ú-ki-il-lu*] daḫ 10,25 *ta-mar mi-na* íb-si
52. [25 íb-si 25(?) gar(?)] 5 *a-na* 25 *iš-te-en* daḫ 30 *ta-mar*
53. [*i-na* 25 2-kam zi 20 *t*]*a-mar* 30 gal 20 tur
54. [··· ··· ···] 2-kam *šà* 1 pa$_5$ IG(?)
55. [··· ··· ···] ··· MA

Translation

Obverse

1. [··· ··· ···], and 9;22,30. Multiply [··· ···]
2. [··· ··· ···2]4,0 (is the volume(?)), and 0;2 is the ratio (of the width to the length).
3. [··· ··· ···] width(?), subtract 0;2(?), and the area.
4. [··· ··· ···] ···, and the length. 15, the one which is to be added to the length,
5. [··· ··· ···]. Multiply [0;3]0 [b]y 9;22,30, and
6,7. [you see 4;41,15]. Return, and [multiply] 45, the length, by 0;2 of (the ratio of) the width (to the length), [and (you see) 1;30]. Multiply (1;30) by [9;2]2,30, and you see 14;3,45.
8,9. [14;3,45 is the fal]se [area]. [Multiply 14];3,45 [b]y 4;[4]1,15, the area, [and] you see [1,5;55],4,41,15. Let it hold you[r he]ad.
10. Multiply [15, the one which is to be added to the length, by 0;2] of (the ratio of) the width (to the length), and y[ou see] 0;30. [0;30] is the one which is subtracted from the width.
11. [Multip]ly [0;30 by 3], and you see[1];30. Subtract [0;3]0 from 1;[30], (and)
12. you see 1. Multiply 1 b[y] 9;22,30, and you see 9;2[2,30].
13. Since <1,0> as the length is said to you, [add] 1,0, the factor, [to 9];22,[30], (and)
14. you see 1,9;22,30. Halve 1,[9;2]2,30, (and) you see 34;41,15.
15. Square 34;41,15, (and) you see 20,3;[13,21,3]3,45.
16. Add 1,5;55,4,41,15 to 20,3;13,21,33,45, (and)
17. [you see] 21,9;8,26,15. W[ha]t is the square root ? 35;37,30 is the square root.
18. Add 34;41,15 (which was used in) your completing the square [to 3]5;37,30, (and)
19,20. you s[ee] 1,10;18,45. W[hat] should I put down [to] 14;3,45, the false area, which

will give me [1,10;1]8,45 ?

21. Put down 5. 5 is the upper breadth. [Halve 5, (and) you see 2;30].

22. Add [2;30 to] 0;30, the excess, (and) you see 3. 3 is the lower breadth.

23. Tak[e 1/12 of the amount by which the upper breadth] exceeded the lower breadth, (and) you see 0;10. [Add 0;30 and 0;10 together].

24. Multiply 0;40 by 12 of the (constant of the) depth, (and) you see 8.

25. Add together [5, the upper breadth], and 3, the lower breadth, (and) you see 8. [Halve 8, (and)]

26. you see 4. Multiply 4 by 8 of the depth, [and]

27. you see 32. Make the reciprocal of 32, (and) [you see] 0;1,5[2,30].

28. Multi[ply] 0;1,52,30 by 24,0, the volume, [(and) you see 45. 45 is the length].

Reverse

30. You(?). $\cdots$[$\cdots$

31. [$\cdots$ $\cdots$] I excavated. I[n(?)] the hol[e $\cdots$ $\cdots$

32. you gave the second [$\cdots$ $\cdots$]. A second time [$\cdots$ $\cdots$

33. I ad[ded $\cdots$ $\cdots$ and] the area of the large hole together [$\cdots$ $\cdots$

34. I add[ed $\cdots$ $\cdots$] to the depth of the for[mer] canal [$\cdots$ $\cdots$

35. I cut [off $\cdots$ $\cdots$] for the small. The sum of the volume [and $\cdots$ $\cdots$ is 1;15].

36. You. Multiply 1;15, the sum, by 13 of one thir[teenth], and (you see) 16;[15].

37. Square 0;10, the amount by which (the length of) the (large) ho[le ex]ceeded (the length of) the (small) hole, (and) you see 0;1,40.

38. Subtract 0;1,40 from 16;15, (and) you see 16;[13,20]. Make the reciprocal of 0;10, the excess.

39. (and) you see 6. Make the reciprocal of 12 of the depth, (and) you see 0;5. Multiply 0;5 by 6,

40. (and) you see 0;30. 0;30 is the product. Multiply 0;30, the product, by 16;13,20, and

41. you see 8;6,40. Square 0;10, [the ex]cess, (and) you see 0;1,40. Multiply 0;1,40 by 13 of one thirteenth,

42. and [you see] 0;21,[40]. Subtract 0;21,40 from 8;6,40, (and)

43. you [see] 7;45. Let it hold yo[ur head].

44. Multiply 0;3[0, the produc]t, by 13 [of one thirteenth], (and) yo[u see] 6;30.

45. Multiply [0;30, th]e product, by norm[al] (number) 2, (and) you see 1. Add 1 to 6;30, (and)

46. you see [7];30. Multiply 1[3 of one th]irteenth, by 3, by normal (number three),

47. and you see 3[9]. Add 7;30 to 39, (and) you see 46;30.

48. What should I put to 46;30 which gives me 7;45 that held your h[ead] ?

49. Put down [0;10]. Let it hold your head. Halve 0;10, the excess, (and)

50. [you] see [0;5. Put down 0;5(?).] Square 0;5, (and) you see 0;0,25.

51. Add 0;0,25 to 0;10 [that held your head], (and) you see 0;10,25. What is the square

root?

52. [0;25 is the square root. Put down 0;25(?).] On the one hand add 0;5 to 0;25, (and) you see 0;30,

53. [on the other hand subtract (0;5) from 0;25, (and) y]ou see [0;20]. 0;30 is the large, (and) 0;20 is the small.

54. [$\cdots$ $\cdots$ $\cdots$] the second $\cdots$ that of one canal $\cdots$.

55. [$\cdots$ $\cdots$ $\cdots$] $\cdots$ $\cdots$.

Commentary

First Problem

We cannot completely restore the system of simultaneous equations to be solved in the first problem. In particular, we cannot understand the meanings of the technical terms and calculations badly preserved in lines 1–14:

1. 2 *ma-na-at*[21] "0;2 is the ratio of $\cdots$" (line 2)

   It probably denotes $y/x = 0;2$. In fact y is obtained by the equation as follows.

   $$y = 0;2x = 0;2 \cdot 45 = 1;30. \qquad \text{(lines 6 and 7)}$$

   But we do not know what the width ($= y$) is, because similar words dagal an-ta ($= u$) "the upper breadth" and dagal ki-ta ($= v$) "the lower breadth" also occur in the text.

2. 15 *wa-ṣí-ib* uš "15, the one which is to be added to the length". (line 4)

   It probably concerns the calculation, $x + 15 = 45 + 15 = 1,0$, the result of which is mentioned in "Since 1,0 as the length is said to you" in line 13.

3. $0;30 \cdot 9;22,30 = 4;41,15.$ (lines 5 and 6)

   The last number is called "the area" in line 8.

4. $1;30 \cdot 9;22,30 = 14;3,45.$ (line 7)

   The last number is called "the false area" in line 8.

5. $15 \cdot 0;2 \cdot 3 - 0;30 = 0;30 \cdot 3 - 0;30 = 1;30 - 0;30 = 1.$ (lines 10-12)

[21] For the technical term *manâtum*, see my paper: K. Muroi, Reexamination of the Susa Mathematical Text No.12: A System of Quartic Equations, *SCIAMVS*, Vol.2 (2001), pp. 3–8.

The number 0;30 is called “the one which is subtracted from the width” in line 10.

6. $1 \cdot 9; 22, 30 = 9; 22, 30.$ (line 12)
7. $1, 0 + 9; 22, 30 = 1, 9; 22, 30.$ (lines 13 and 14)

In spite of these obscurities it is certain that our text solves the next quadratic equation in order to obtain the upper breadth u:

$$14; 3, 45u^2 - 1, 9; 22, 30u = 4; 41, 15 \tag{1}$$

Multiplying both sides of (1) by 14;3,45, the text obtains

$$X^2 - 1, 9; 22, 30X = 1, 5; 55, 4, 41, 15, \tag{2}$$

where $X = 14; 3, 45$ (lines 8 and 9).

The equation (2) is solved by the usual method, completing the square. That is, after the calculations of

$$1/2 \cdot 1, 9; 22, 30 = 34; 41, 15, \quad \text{(line 14)}$$
$$(34; 41, 15)^2 = 20, 3; 13, 21, 33, 45, \quad \text{(line 15)}$$
$$1, 5; 55, 4, 41, 15 + 20, 3; 13, 21, 33, 45 = 21, 9; 8, 26, 15, \quad \text{(lines 16 and 17)}$$

the text obtains from (2)

$$(X - 34; 41, 15)^2 = 21, 9; 8, 26, 15$$

which is followed by

$$X - 34; 41, 15 = \sqrt{21, 9; 8, 26, 15} = 35; 37, 30, \quad \text{(line 17)}$$
$$X = 35; 37, 30 + 34; 41, 15 = 1, 10; 18, 45, \quad \text{(lines 18 and 19)}$$

and finally the upper breadth

$$u = 1, 10; 18, 45 \div 14; 3, 45 = 5. \quad \text{(nindan, lines 19–21).}$$

Further the lower breadth v and the depth z are calculated according to the following equations, which must have been given in the statement:

$$v = (1/2) \cdot u + 0; 30 = 2; 30 + 0; 30 = 3, \quad \text{(nindan, lines 21 and 22)}$$
$$z = 12\{0; 30 + \overline{12}(u - v)\} = 12(0; 30 + 0; 10)$$
$$= 12 \cdot 0; 40 = 8. \quad \text{(kùš, lines 22–24)}$$

Verification begins at line 24. First the trapezoidal cross-section S of a canal or a hole is calculated and then its volume V is divided by S in order to obtain the length x:

$$S = (1/2) \cdot (5+3) \cdot 8 = 32, \qquad \text{(lines 24–27)}$$
$$\therefore x = V/S = 24,0 \cdot \overline{32} = 24,0 \cdot 0;1,52,30 = 45.$$
(nindan, lines 27 and 28)

Second Problem

As to the second problem we can recognize several typical expressions of excavation problems.

1. *ú-šà-pí-il* "I excavated" in line 31.
2. a-šà *ka-la-ak-ki* gal "the area of the large hole" in line 33.
3. tùn *šà a-ta-ap pa-na-nim* "the depth of the former canal" in line 34.
4. *a-na* tur *aš-lu-uṭ* "I cut off $(\cdots)$ for the small" in line 35.
5. *šà ka-la-ak-ku* ugu *ka-la-ak-ki i-te-ru* "the amount by which (the length of) the (large) hole exceeded (the length of) the (small) hole" in line 37.
6. igi 12 *šu-up-li* "the reciprocal of 12 of the depth" in line 39.

Note further the words in line 32:

2-kam *ta-ad-di-in* 2-*tu* (*šanītu*) "You gave the second [depth (?)]. A second time $(\cdots)$".

Although we cannot restore the statement of this problem either, it seems that a canal has been enlarged, that is, the bottom of a canal has been deepened. Judging from the calculations performed in the text, the equations dealt with in this problem are:

$$x - y = 0;10 \tag{3}$$
$$z = 12(x-y) \tag{4}$$
$$(x^2+y^2)z + xy(z+1) + (1/13) \cdot (x^2+y^2) = 1;15 \tag{5}$$

where x is the length of the hole, y the width, and z the depth. Before discussing the geometrical significance of equation (5) and what the hole (=*kalakkum*) is, we had better examine the details of the solution which begins at line 36. Multiplying both sides of (5) by 13, the text obtains:

$$13(x^2+y^2)z + 13xy(z+1) + x^2 + y^2 = 1;15 \cdot 13 = 16;15, \qquad \text{(line 36)}$$

and after the application of the identity,

$$x^2 + y^2 = (x-y)^2 + 2xy,$$

subtracts $(x-y)^2$, which is $0;1,40$ according to equation (3), from both sides,

$$\begin{aligned}
&13(x-y)^2z + 13\cdot 3xyz + 13xy + 2xy \\
&= 16;15 - 0;1,40 \\
&= 16;13,20. \qquad\qquad \text{(lines 37 and 38)}
\end{aligned} \tag{6}$$

Next the text calculates the reciprocal of z according to equations (3) and (4):

$$\bar{z} = \overline{12}\cdot\overline{0;10} = 0;5\cdot 6 = 0;30, \qquad\qquad \text{(lines 38–40)}$$

where $0;30$ is called *tāluku* "product".[22] Multiplying both sides of (6) by $0;30$, the text obtains:

$$\begin{aligned}
&13(x-y)^2 + (13\cdot 3 + 13\cdot 0;30 + 2\cdot 0;30)xy \\
&= 16;13,20\cdot 0;30 \\
&= 8;6,40, \qquad\qquad \text{(lines 40 and 41)}
\end{aligned}$$

and subtracting $13(x-y)^2 (= 13\cdot 0;1,40 = 0;21,40)$ from the results,

$$(13\cdot 3 + 13\cdot 0;30 + 2\cdot 0;30)xy = 8;6,40 - 0;21,40 = 7;45. \qquad \text{(lines 41–43)}$$

Calculating the coefficient of xy, the text obtains:

$$\begin{aligned}
(39 + 6;30 + 1)xy = (39 + 7;30)xy &= 46;30xy = 7;45 \\
\therefore xy &= 0;10. \qquad\qquad \text{(lines 43–49)}
\end{aligned}$$

The last division of $7;45$ by $46;30$ is carried out by trial and error as in BM 85196, No.15. Next begins a series of computations according to the usual method of Babylonian mathematics to obtain x and y from $x-y=a$, $xy=b$:

$$\{(x-y)/2\}^2 = 0;5^2 = 0;0,25 \qquad\qquad \text{(lines 49 and 50)}$$

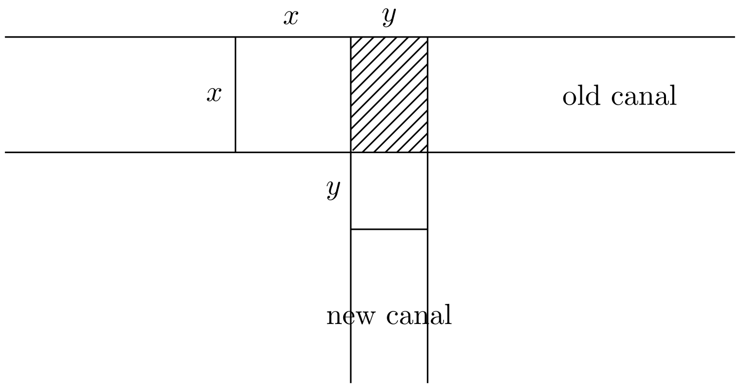
$$xy + \{(x-y)/2\}^2 = \{(x+y)/2\}^2 = 0;10 + 0;0,25 = 0;10,25, \qquad \text{(lines 50 and 51)}$$

$$(x+y)/2 = \sqrt{0;10,25} = 0;25, \qquad\qquad \text{(lines 51 and 52)}$$

[22] The Akkadian word *tāluku(m)* is a derivative of the verb *alāku(m)* "to go", and its usual meaning is "Gang, Lauf, Bahn" (AHw, p.1312). But it is erroneously used here instead of the mathematical term a-rá "a factor, a multiplicand, a product". The cause of this mistake may lie in the fact that the Sumerian word a-rá has many different meanings in general and one of the Akkadian equivalents of it is *alaktu(m)* "gait, road, way" (CAD, Vol.1, A ⊠, p.297). For another occurrence of *tāluku(m)* in the Susa mathematical texts, see my paper: K. Muroi, Reexamination of Susa Mathematical Text No.8, *Sugakushi Kenkyu* No.140 (1994), pp. 50–56.

$$\therefore x = (x+y)/2 + (x-y)/2 = 0;25 + 0;5 = 0;30$$
$$y = (x+y)/2 - (x-y)/2 = 0;25 - 0;5 = 0;20. \quad \text{(lines 52 and 53)}$$

Thus the mathematical purport of the second problem can be clearly understood. But a difficult question to answer remains, that is, what are x and y? or what is the relation between "a canal (=*atappum*)" and "a hole (=*kalakkum*)"? My answer is as follows:



The shadowed area is deeper by 1 kùš than the bottom of the canal

Fig. 3

There is a canal whose width and depth are x nindan and z kùš, respectively (cf. Fig.3). The side of the canal is cut off in part in order to make a new canal which is at right angles with the old one and whose width and depth are y nindan and z kùš, respectively. Moreover the junction of the two canals is deepened by 1 kùš, probably in order to store up silt.[23] Therefore this part of the canals is x in length, y in width, and $z+1$ in depth. Two holes adjoining to it, whose dimensions are x by x by z and y by y by z, respectively,are considered in the text. The former is called *kalakki* gal "the large hole" in line 33. The latter must be *kalakki* tur "the small hole". The unknown quantities asked for in the problem are the length of the large hole (=x) and the length of the small hole (=y). We might expect here that x is named, for example, uš *kalakki* gal "the length of the large hole" instead of *kalakki* gal "(the length of) the large hole". Since x is originally the width of the old canal, the scribe of Susa must have omitted uš "the length" in order to avoid confusion. In fact neither uš nor sag "the width" occurs in the text at all. Now let us return to equation (5). We can interpret the first and the second terms of the left side of (5) as the sum of the volumes of the large hole, of the small hole and of the junction of

[23]In modern Japan we sometimes see the same device for irrigation canals.

the two canals. As to the third term, $(1/13) \cdot (x^2 + y^2)$, I think that it does not have a geometrical significance. It is presumably added to the volumes in order to make the equation more complicated. We can take an example of the complexity of this kind from the so-called series texts, that is, ancient drill books in mathematics:

$$xy = 10, 0$$
$$x^2 + (1/11) \cdot f(x, y) = 16, 40$$
$$f(x, y) = 2[(1/7)\{2y + 3(x - y)\}]^2 + x^2,$$

whose answer is

$$x = 30 \text{ and } y = 20.$$[24]

In any case, there is no doubt that the second problem of the Susa mathematical text No.24 is a sophisticated system of cubic equations.

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[24]VAT 7537, No.1. For the transliteration and translation of the text, see my paper: K. Muroi, The Expressions of Zero and of Squaring in the Babylonian Mathematical Text VAT 7537, *Historia Scientiarum*, Vol.1 (1991), pp. 59–62.