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A Sanskrit Mathematical Anthology

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I Introduction

I.1 Text

A small mathematical work in Sanskrit is appended to Śrīdhara's *Triśatī* in a manuscript preserved in Hemacandrācārya Jaina Jñāna Mandira, Pāṭaṇa, North Gujarat. The former begins in the same line of writing in which the latter ends and both have been copied by the same person named Harṣatilaka, who calls himself *muni* (abbreviated mu°) or an ascetic.

The work contains neither the title of the work nor the name of the author. It does not have a *maṅgala śloka* (an introductory auspicious verse in which the author expresses his homage to god) that every genuine Sanskrit work should have. It has a very simple colophon by the scribe: '(This anthology was) written down by Muni Harṣatilaka. Let there be good fortune'; but it does not have a colophon of the work itself. None, except one (verse 30), of the mathematical rules and examples treated in it occurs in the *Triśatī*, which Harṣatilaka copied immediately before he began to work on it. But the first eight verses are found also in the *Pāṭīgaṇita*, another, larger work of Śrīdhara: they are most probably quotations either from that work or from his lost work, *Navaśatī*.[1] The last verse (31) mentions a Jayaśekhara: '(This was) told by the venerable Jayaśekhara'; this Jayaśekhara seems to be the author either of verses 17–31 (section 3) or of verse 31 only.

These circumstances indicate that the work was not a genuine, original work but a loose compilation or anthology of earlier rules and examples accompanied by brief prose commentaries on the examples. In that sense, it resembles the *Bakhshālī Manuscript*[2] and another Pāṭaṇa manuscript, HJJM 8894.[3] Presumably, the compilation was meant to be an appendix to the *Triśatī*.

I have not so far been able to find other manuscripts of the same work but that the present manuscript is not the autograph but a copy is proved by a number of

[1]For the *Navaśatī* see Hayashi 1995b, 235.

[2]Hayashi 1995a.

[3]Hayashi 1995a, 464–484.

corruptions of the text (see the footnotes in § II).

I.2 Contents and sectioning

All the topics treated in the anthology (see Table 1) belong to the field called *pāṭī-gaṇita* or 'mathematics of algorithm' in contrast to *bīja-gaṇita* or 'mathematics of seeds' (i.e., algebra) and are divided in three sections:

section 1: verses 1–8, section 2: verses 9–16, section 3: verses 17–31.

Section 1, all verses of which are found also in the *Pāṭīgaṇita*, ends with the numerical figure '1' and section 2 with 'cha // śrī //'; both are end marks and commonly found in Indian manuscripts.

Section 1 treats algebraic problems on fee and price: they belong to the category called 'the procedure for mixture' (*miśraka-vyavahāra*) in *pāṭī*; section 2 deals with problems on buying and selling: they belong to 'the procedure for mixture' in some texts and to 'the proportion' (*anupāta*) in others; and section 3 treats problems on geometry and mathematical series: they belong to 'the procedure for field (or plane figures)' (*kṣetra-vyavahāra*), 'the procedure for excavation (or solid figures)' (*khāta-vyavahāra*), and 'the procedure for series' (*śreḍhī-vyavahāra*).

I.3 Characteristic features

As told above, eight verses are commonly found in the anthology and in the *Pāṭī-gaṇita* of Śrīdhara (8th century) and they are not found in the other extant works of his, *Triśatī* and *Gaṇitapañcaviṃśī*. The *Pāṭīgaṇita* is preserved in a uniquely extant, incomplete manuscript and a larger work called *Navaśatī* of the same author is not available now. The anthology is therefore an important document for the study of the transmission of Śrīdhara's works.

One of the topics common to the anthology and the *Pāṭīgaṇita* is the Indianized hundred fowls problem. The anonymous commentator of the *Pāṭīgaṇita* had given six solutions to it. The prose commentary of the anthology gives nine solutions. See under verses 7–8 in § IV.1.2 below.

The anthology elaborately treats the buying-selling problem. The only other texts comparable to it in this regard are the *Bakhshālī Manuscript* and the *Mahā-siddhānta*. See § IV.2 below.

As mentioned above, the anthology (verse 31) refers to a mathematician called Jayaśekhara, who is new to us. A Jayaśekhara Sūri wrote a *Kṣetrasamāsa* (Jaina geographical/geometrical work), several manuscripts of which (the oldest one is dated 1579 C. E.) are preserved in Rajasthan and Gujarat;[4] an investigation of those manuscripts may throw new light on the problem of the identification of our Jayaśekhara.

[4]CESS, vol. 3, p. 63a, and vol. 5, p. 117a.

Table 1: Contents of the anthology

Contents	Verses
1. Fee and price	
1.1. Fee to a dancer (*nartaka-dāna*)	
Rule 1: fee to a dancer	1
Ex. 1: fee to a dancer	2–3
Ex. 2: wages for carriers of a palanquin	4
1.2. Prices of living things (*jīva-mūlya*)	
Rule 1: prices of living things	5–6
Ex. 1: pigeons, cranes, swans, peacocks	7–8
2. Buying and selling (*kraya-vikraya*)	
Rule 1: capital	9
Ex. 1: capital	10
Ex. 2: capital	11
Rule 2: profit	12–13
Rule 3: buying rate	14
Rule 4: selling rate	15
Rule 5: capital and profit	16
3. Circular figures and distribution of fruit	
3.1. Circle segments (*cāpa-kṣetra*)	
Rule 1: chord, bow, arrow, diameter	17–19
Ex. 1: chord, bow, arrow, diameter	20–21
Rule 2: intersection of two circles	22
Ex. 2: Moon and Rāhu	23
Rule 3: a broken bamboo	24
Ex. 3: a broken bamboo	25
Rule 4: a peacock's attack on a snake	26–27
Ex. 4: a peacock's attack on a snake	28–29
3.2. Circular wells (*kūpa*)	
Rule 1: capacities of tapered wells	30
3.3. Distribution of fruit (*phala-bhajana*)	
Rule 1: number of all the fruit plucked	31

I.4 Weights and measures

The units for weights and measures used in the anthology are quite common except for the first three (two if *ya*° and *yaṃ* stand for the same denomination) of the four monetary denominations.

Monetary units.

Two names and two abbreviations occur: *tārā* (or *tāra*) (comm. on verses 12–13), *ya*° (comm. on verse 4), *yaṃ* (comm. on verse 14), *rūpa* (verses 3, 8, 10, 11). All these are presumably equivalent.

1 *tārā* (or *tāra*) = 1 *ya*° = 1 *yaṃ* = 1 *rūpa*.

Linear measures.

Four names occur: *aṃgula* (comm. on verses 20–21, verse 23), *kara* (verses 21, 25, 27), *krośa* (verse 4), *hasta* (comm. on verses 20–21, verse 25, comm. on verses 28–29). Out of these, *kara* and *hasta* are equivalent.

24 *aṃgulas* = 1 *kara* or *hasta* (cubit),
8000 *hastas* = 1 *krośa*.

Capacity.

Only one unit for capacity occurs: *kuḍaba* (verse 10).

Time.

Two time units occur: *dina* (verses 1, 2), *divasa* (verse 3, in a doubtful reading), *yāma* (verse 2).

4 *yāmas* = 1 *dina* or *divasa* (length of daylight).

I.5 Word numerals

Three word numerals occur:

3: *anila* or the fires (verse 25).
16: *nṛpa* or the kings (verse 25).
24: *jina* or the winners (verse 30).

I.6 Linguistic peculiarities

I.6.1 Phonetic and orthographic peculiarities

ā for *a*: *lābheta* (verse 10). *i* for *ī*: *kriyaṃte* (verse 10). *rgr* for *gr*: *argra* (verse 25). *rgr* for *rg*: *vargra* (verses 17, 18, 23, 30). *ṣ* for *kh*: *muṣa* (verse 27), *liṣita* (colophon), *śiṣaṃḍī* (verse 29), *śeṣara* (colophon), *ṣaṃḍa* (verse 24). Repetition of consonants after *r*: *arddha* (verses 10, 11, 26), *gartta* (verses 26, 27), *narttaka* (intro. to verse 1, verse 3, comm. on verse 4), *nirddiṣṭa* (verse 13), *sarvva* (verse 31). Irregular *sandhi*: the regular euphonic changes do not occur in the ablative ending -*āt* (verses

17, 18, 19, 26, comm. on verse 23) and in the word, *pṛthak* (verses 1, 22, 24). Cf. also *syāt* in verse 31 (but *syād* in verse 27) and *ṣaṭ* in verse 17 (but *ṣaḍ* in verse 3). The *avagraha* is used several times but once it is used in a strange manner: *hasta 9 aṃgula 9 'ṃgulabhāgā ...* (comm. on verses 20–21). Hiatus: *tasya udāhṛtaṃ* (verse 14). Hiatus-bridger *m*: *śatamarddhaṃ* (verse 10) means fifty according to the prose comment. This implies that the commentator regarded the passage as equivalent to *śatārdhaṃ* or *śatasyārdhaṃ*, that is, 'a half of a hundred'. But, of course, the passage allows another, grammatically better interpretation. See under verse 10 in § IV.2 below.

I.6.2 Grammatical and semantic peculiarities

Irregular number. Sg. for du.: *dhanalābho hi jāyate* (verse 16).

Irregular gender. *ānayanaḥ* (comm. on verses 12–13, 14, 16) for *ānayanaṃ*. *krayaṃ* (verses 12, 14) for *krayaḥ*. *vikrayaṃ* (verses 12, 15) for *vikrayaḥ*. *lābhaṃ* (verse 15) and *lābhā* (verse 11) for *lābhaḥ*.

Unusual (Apabhraṃśa) case endings. Nom. sg. neut. *-iṃ*: *yutiṃ* (verses 14, 16). Nom. sg. neut. pr. *taṃ* (verses 14, 15).

Omission of case endings. This occurs very often in tabular presentation of numerical data called *nyāsa*. The nominative case of *ānayana* occurs without a case ending but with the predicate *samāpta* having the male ending twice (comm. on verses 4 and 15) and having the female ending once (comm. on verse 11). The nominative case of *dāna*, too, occurs once without a case ending but with the predicate *samāpta* having the male ending (comm. on verse 4). In verses, too, omission of case endings occurs. *vikraya lābhadravya⟨ṃ⟩ guṇayet* (verse 9, *vikraya* in the sense of *vikrayena*). *tadagramargre nṛpahasta lagnaṃ* (verse 25, *-hasta* in the sense of *-haste*).

Unusual use of relative pronouns. *kṛtir yā ... tena ...* (verse 24). *kṛtiḥ ... yā ... phalaṃ tad* (verse 26).

Unusual meanings. The word *āryā* is employed not only for Āryā meter but also for Anuṣṭubh meter (see the introductions to verses 12–18) and for Upajāti meter (see the introduction to verse 30). The word *taṣṭa* (lit. 'chiseled'), which usually refers to a modulo operation, is used in the sense of *bhakta* ('divided') in verse 22: the former operation is meant for the remainder while the latter for the quotient. The word *bhāga* ('part') is used in the sense of *tri-bhāga* ('one-third') in verse 11, but probably the last *pāda* of verse 11 in which the word occurs is corrupt. The word *vṛtta* ('happened'), which usually means 'a circle' in mathematics, is used in the sense of 'the diameter of a circle' in the introduction to verses 17–19. The word *vedha* ('penetrating'), which usually means 'a depth' in mathematics, is also used to denote the diameter of a circle in verse 20. The compound *kriyā-sūtra* is used in the sense of *karaṇa-sūtra* ('a procedural rule') in the introduction to verse 22. The word *kriyā* ('action') can naturally mean mathematical procedures,[5] but the compound *kriyā-sūtra* is hardly seen in mathematical works.

I.7 Abbreviations

In this manuscript, as in other mathematical manuscripts, *udāharaṇa* is very often abbreviated *u*° (comm. on verses 15, 16), *udā*° (comm. on verses 2–3, 7–8, 10, 23, 25, 28–29), and *udāha*° (comm. on verses 12–13, 20–21). Other abbreaviations are as follows.

gu° for *guṇaka* (comm. on verses 7–8).
dvi° for *dvitīya* (comm. on verses 15, 16).
nyā° for *nyāsa* (comm. on verses 20–21).
mu° for *muni* (colophon).
ya° for a monetary unit? (comm. on verse 4).
yaṃ for a monetary unit? (comm. on verse 14).

I.8 Manuscript

Hemacandrācārya Jaina Jñāna Mandira (Pāṭaṇa), No. 10727. Title in the catalogue: Gaṇitasāra. Author in the catalogue: Śrīdharācārya. Scribe: Harṣatilaka. Date: unknown. Language: Sanskrit. Script: Devanāgarī. Material: paper. Size: 24.5×10 cm. Extent: fol. 22a, line 4 to fol. 23b, line 13. 12 lines to a page on fol. 22 and 13 lines to a page on fol. 23. About 47 letters to a line on fol. 22 and about 53 letters to a line on fol. 23.

II Text

[The irregular *sandhis* and the irregular forms of words mentioned in § I.6 above are left unchanged in this edition. The verse numbers are mine. In the Ms. the numerical figure for one (1) is put at the end of verses 2, 8 (first line), 13, 16, and 30. Notation for this edition — Ms.: the manuscript used for this edition. PG: Shukla's edition of the *Pāṭīgaṇita*. PG/Ms.: the uniquely extant manuscript on which the PG is based. (x): delete x. ⟨x⟩: add x. x] y T: T reads y for x.]

// siddhaṃ[6] // narttakadānānayane karaṇasūtramāryā(ḥ) //

pūrvonaparadināṃśān phal⟨ā⟩hatānprekṣakai⟨ḥ⟩ pṛthak hṛtvā ⟨/⟩[7]
pūrvā⟨n⟩ pare nidadhyād dānaṃ vyāvṛtt⟨i⟩guṇitāste //1//[8]

[5]Cf. the title of the third chapter, Kālakriyā ('Procedures on Time'), of the *Āryabhaṭīya*.

[6]Expressed by a symbol like '80'. Cf. Sircar 1965, 92–97.

[7]pūrvona] pūrvāna Ms.; pūrvonaparadināṃśān] pūrvonāparadīnāṃśān PG/Ms.; pṛthak hṛtvā] pṛthagghṛtvā PG.

[8]= PG 68. Āryā meter.

udā° //

ekena yāmamekaṃ dvau yāmau nṛtya(ḥ)manyapuruṣeṇa ⟨/⟩[9]
dṛṣṭaṃ yāmatritayaṃ apareṇa (ne) dināṃtamanyena //2//[10]
narttakapuṃso deyā tai rūpāṇāṃ (tu) ṣaḍanvitā navatiḥ /[11]
tasyāḥ kiṃ kena bhaved deyaṃ dṛṣṭānusāreṇa //3//[12]

nyāsaḥ //

1	1	1	1
1	2	3	4
4	4	4	4
4	3	2	1

datta 96 ⟨/⟩ labdhaṃ yathākramaṃ / 6 / 14 / 26 / 50 //

dvitīyodāharaṇaṃ //

yugyaṃ krośatritayaṃ daśabhi⟨ḥ⟩ puruṣaiḥ śatena netavyaṃ /[13]
krośe krośe tebhyo vyāvṛttā dvitripaṃca(ja)janā⟨ḥ⟩ //4//[14]

nyāsaḥ //

2	3	5
1	1	1
1	2	3
3	3	3
10	8	5

[15] datta śataṃ 100 ⟨/⟩ labdhaṃ

6	22	70
2	1	5
3	2	6

ya° //

evaṃ narttakadāna samāptaḥ //

// jīvamūlyānayane karaṇasūtramāryādvayaṃ //

[9] nṛtya(ḥ)m] dṛṣṭam PG.

[10] = PG (86). Āryā meter. dṛṣṭaṃ] saṃdṛṣṭaṃ PG, sandṛṣṭaṃ PG/Ms.; tritayaṃ] tṛtayaṃ Ms., trayaṃ PG.; anyena] apareṇa PG.

[11] puṃso] pakṣe PG.; deyā tai] divasaiḥ Ms.; navatiḥ] na bhavati Ms.

[12] = PG (87). Āryā meter. tasyāḥ] kasmāt Ms.; bhaved] pṛthak PG.; deyaṃ dṛṣṭānu] deya dṛṣṭavānu PG/Ms.

[13] yugyaṃ] yugmaṃ Ms.

[14] = PG (88). Āryā meter. krośe krośe] krośadvau Ms., which expression presumably originates from 'krośe 2' in the parent manuscript; vyāvṛttā] vyāvṛtta Ms., vyāvṛttāḥ PG; janā⟨ḥ⟩] narāḥ PG.

[15] To the right of the second row of this table the following comment (including the '=') is written: = idaṃ narttakadānalabdhacakra. See the fn. in the translation below.

kramaśo yathoktajīvān guṇayediṣṭaika⟨jīva⟩mūlyena ⟨/⟩[16]
tebhyaḥ pṛthak ⟨sva⟩mūlyaṃ viśodhya śeṣaṃ tathā hanyāt //5//[17]
icchāṃ vinā svabuddhyā yathā tadaikyena tulyatecchā[22b]yā⟨ḥ⟩ /[18]
utpannaistatroktān guṇayet sarvaṃ ca jīvaṃ syāt //6//[19]

udā° // dvayaṃ //

tribhiḥ pārāpatāḥ paṃca(ḥ /) paṃcabhiḥ sapta sārasā⟨ḥ⟩ /[20]
saptabhirnava haṃsāśca(/) navabhistrīṇi barhiṇaḥ //7//[21]
rājaputravinodārthaṃ(/) jñātamūlyaṃ yathoditaṃ /(/ 1)[22]
śatenaikena rūpāṇāṃ jīvānāṃ śatamānaya(ḥ) //8//[23]

nyāsaḥ //

5	7	9	3	⟨100⟩
3	5	7	9	⟨100⟩

gu° 3 / 4 / 5 / atha 2 / 6 / 4 / atha 1 / 8 / 3 // atha 12 / 1 / 2 / atha 7 / 1 / 5 / atha 3 / 9 / 1 / atha 4 / 7 / 2 / atha 8 / 4 / 2 / atha 9 / 2 / 3 //[24] ebhiḥ pārāpatādīn jīvān trīn guṇayitvā tathā mūlyaṃ (/) tadaikyamicchātaḥ śodhayitvā //[25] 1

krayavikraye lābhamūlānayane karaṇasūtramāryā //

vikraya lābhadravya⟨ṃ⟩ guṇayet krayavikrayāṃtareṇa bhajet ⟨/⟩[26]
yallabdhaṃ tanmūlānayane pracakṣate gaṇitaṃ //9//[27]

udā° // dvayaṃ //

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[16]yathokta] yadodita Ms.; jīvān] jīvā Ms., jīvād PG/Ms.; iṣṭaika] iṣṭeka Ms.

[17]= PG 63. Āryā meter. viśodhya] visodhya Ms., viśobhya PG/Ms.

[18]tadaikyena] taddhaikyena Ms.

[19]= PG 64 (2nd half is different). Āryā meter. utpannaistatroktān] utpanneṣu ca kudṛk (metrically inferior) Ms., tadguṇamūlyahatāvapi PG; guṇayet...syāt] na śiṣṭamṛṇaṃ khaṃ yathā hyaguṇe (metrically inferior) PG.

[20]pārāpatāḥ] pārāvatāḥ PG, parāvatāḥ PG/Ms.; sārasā⟨ḥ⟩] sarasāḥ PG/Ms.

[21]= PG (78). Anuṣṭubh meter. navabhistrīṇi barhiṇaḥ] navabhirbarhiṇastrayaḥ PG.

[22]jñāta] jñātvā PG.

[23]= PG (79). Anuṣṭubh meter.

[24]8 / 4 / 2 /] 4 / 8 / 2 / Ms.

[25]trīn] tri Ms.; tadaikyamicchātaḥ śodhayitvā] tadekamicchāguṇitatvā Ms. (the text seems to be corrupt here).

[26]-vikrayāṃtareṇa bhajet] -vikrayaṃ / tāravibhajet Ms.

[27]Āryā meter. The last pāda is irregular. nayane] nayanaṃ/ Ms.

kuḍabā⟨ḥ⟩ sapta kriyaṃte rūpeṇa paṃca vikrayet /(/)
śatamarddhaṃ ca lābheta taṃ(X) lābhamūlamānayet //10//[28]

kraya 7 / vikraya 5 / lābha 50 // labdhaṃ mūlaṃ 125 //

krītvā rūpeṇa sārddhāṣṭau ṣaṭ pādonaḥ pradīyate ⟨/⟩[29]
yatra (//) lābhaikonatriṃśat (/) bhāgayuktena kiṃ mūl(y)aṃ //11//[30]

kraya $\begin{array}{|c|}8\\1\\2\end{array}$ vikraya $\begin{array}{|c|}6\\1\\4\end{array}$ lābha $\begin{array}{|c|}29\\1\\3\end{array}$[31] labdhaṃ mūladhanaṃ $\begin{array}{|c|}61\\1\\3\end{array}$ evaṃ mūlānayana samāptā //[32]

// lābhānayane karaṇasūtramāryā ⟨//⟩

krayaṃ ⟨ca⟩ vikrayaṃ caiva tayoḥ parasparāṃtaraṃ ⟨/⟩[33]
tenaiva tāḍitaṃ mūl(y)aṃ vikrayeṇaiva saṃbhajet //12//[34]
(la) yallabdhaṃ tadbhavellābho (/) nirddiṣṭaṃ pūrvasūribhiḥ //13//[35]

kraya 7 vikraya 5 lābha 0 // mūladhanaṃ 125 labdhaṃ tārā 50 //

udāha° // nyāsaḥ // ⟨kraya⟩ $\begin{array}{|c|}8\\1\\2\end{array}$ vikraya $\begin{array}{|c|}6\\1\\4\end{array}$ lābha ⟨0⟩ mūladhanaṃ $\begin{array}{|c|}61\\1\\3\end{array}$ ⟨labdhaṃ⟩ lābha $\begin{array}{|c|}29\\1\\3\end{array}$ evaṃ lābhānayanaḥ samāptaḥ //

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[28] Anuṣṭubh meter. lābheta] lābhena Ms.

[29] pādonaḥ] pādonau Ms.

[30] Anuṣṭubh meter. Probably the last pāda is corrupted because it is metrically irregular and the word bhāga, which simply means 'part', is used for 'one-third.' A better (or expected) expression is: tryaṃśayugmūlamānayet.

[31] $\begin{array}{|c|}29\\1\\3\end{array}$] $\begin{array}{|c|}29\\1\\2\end{array}$ Ms.

[32] One illegible letter after the double daṇḍa.

[33] tayoḥ parasparāṃtaraṃ] tayo paramaṃparaṃ Ms.

[34] Anuṣṭubh meter.

[35] Anuṣṭubh meter (half stanza).

// krayānayane karaṇasūtra[23a]māryā //

dhanalābhayutiṃ caiva vikrayenaiva tāḍitaṃ /(/)[36]
vibhaktaṃ taṃ ca mūlena krayaṃ tasya udāhṛtaṃ //14//[37]

kraya 0 // vikraya 5 lābha 50 // mūladhanaṃ 125 ⟨//⟩ labdhaṃ kraya (2)7 ⟨//⟩

dvitīyodāharaṇanyāsaḥ // kraya 0 // vikraya $\left\|\begin{array}{c}6\\1\\4\end{array}\right\|$ lābha $\left\|\begin{array}{c}29\\1\\3\end{array}\right\|$[38] yaṃ [39]labdhaṃ krayaḥ $\left\|\begin{array}{c}8\\1\\2\end{array}\right\|$ evaṃ krayānayanaḥ samāptaḥ //

// vikrayānayane karaṇasūtramāryā //

krayena tāḍitaṃ mūlaṃ (/) lābhaṃ mūlayutaṃ ⟨tathā⟩ /
tenaiva bhājitaṃ taṃ ca (/) vikrayaṃ jāyate sphuṭaṃ //15//[40]

kraya 7 vikraya 0 // lābha 50 mūladhanaṃ 125 ⟨//⟩ labdhaṃ vikraya 5 ⟨//⟩

dvi° u° // nyāsaḥ // kraya $\left\|\begin{array}{c}8\\1\\2\end{array}\right\|$ vikraya 0 lābha $\left\|\begin{array}{c}29\\1\\3\end{array}\right\|$ [41]labdhaṃ vikraya (//) $\left\|\begin{array}{c}6\\1\\4\end{array}\right\|$ evaṃ vikrayānayana samāptaḥ //

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[36] vikrayenaiva] vikrayainai Ms.

[37] Anuṣṭubh meter.

[38] $\left\|\begin{array}{c}29\\1\\3\end{array}\right\|$] $\left\|\begin{array}{c}29\\1\\4\end{array}\right\|$ Ms.

[39] The following sentence is missing here: mūladhanaṃ $\left\|\begin{array}{c}61\\1\\3\end{array}\right\|$.

[40] Anuṣṭubh meter.

[41] The following sentence is missing here: mūladhanaṃ $\left\|\begin{array}{c}61\\1\\3\end{array}\right\|$.

// lābhamūlānayane karaṇasūtramāryā //

dhanalābhayutiṃ caiva vikrayāṃtaratāḍitaṃ /
krayeṇa bhājitaṃ caiva dhanalābho hi jāyate //16//[42]

kraya 7 vikraya 5 lābha (/) 0 mūladhanaṃ 0 // mūlalābhāni (/) 175 ⟨//⟩ labdhaṃ mūladhanaṃ 125 śeṣaṃ labdhaṃ lābhadhanaṃ 50 //

dvi° u° // nyāsaḥ // kraya $\left\|\begin{array}{c}8\\1\\3\end{array}\right\|$ vikraya $\left\|\begin{array}{c}6\\1\\4\end{array}\right\|$ lābha 0 mūlalābhamiśraṃ $\left\|\begin{array}{c}90\\2\\3\end{array}\right\|$[43] labdhaṃ mūladhanaṃ $\left\|\begin{array}{c}61\\1\\3\end{array}\right\|$ śeṣa lābhadhanaṃ(//) $\left\|\begin{array}{c}29\\1\\3\end{array}\right\|$ evaṃ lābhā⟨na⟩yanaḥ samāptaḥ // cha // śrī // //

// vṛttakṣetrasya jīvā(//)cāpabāṇavṛttādīnāṃ parijñā(/)⟨nā⟩ya karaṇasūtramāryādvayaṃ //

śaro(/)navṛttaviṣkaṃbhāt(/) śaraghnāt caturāhatāt /
padaṃ jīvā tathā bāṇava(/)rgrāt ṣaṭguṇitāt punaḥ //17//[44]
jyāvargrasahitāt mūlaṃ(/) phalaṃ cāpaṃ prajāyate //18//[45]
jyāv⟨y⟩āsa(X)kṛtiviś(l)eṣamūlonāt vyāsato dalaṃ bāṇaḥ /⟨/⟩
bāṇajyādalakṛtiyutiriṣubhaktā vyāsamānaṃ syāt //19//[46]

udāha° //

vṛtte daśa vyāsa iṣu⟨r⟩dvayaṃ tat jyā kā dvayaṃ yatra śaro 'ṣṭa jīvā ⟨/⟩
kiṃ tatra cāpaṃ daśa yatra vedho jīvāṣṭa kastatra śaraḥ śaro vā //20//[47]
yatra dvayaṃ jyā(')ṣṭakaronmitātra(/)

[42] Anuṣṭubh meter. caiva] ca/ Ms.

[43] $\left\|\begin{array}{c}90\\2\\3\end{array}\right\|$] $\left\|\begin{array}{c}9\\1\\1\\3\end{array}\right\|$ Ms.

[44] Anuṣṭubh meter.

[45] Anuṣṭubh meter (half stanza).

[46] Āryā meter. The first pāda is irregular.

[47] Indravajrā meter.

vyāsaḥ kiyān vṛttaphalāni jalpa //21//[48]

jyārthaṃ[49](//) nyāsa[50] $\begin{matrix}2\\10\end{matrix}$ labdhaṃ jīvā ⟨8 //⟩ cāpārthaṃ nyāsaḥ //[51] $\begin{matrix}2\\8\end{matrix}$ amūladarāśiriti jñānena labdhamāsannamūlaṃ hasta 9 aṃgula 9 'ṃgulabhāgā [23b] $\begin{matrix}3\\25\end{matrix}$ ⟨//⟩[52] bāṇārthaṃ(//) nyā° //[53] $\begin{matrix}8\\10\end{matrix}$ labdhamiṣumānaṃ 2 ⟨//⟩ vyāsārthaṃ nyā° //[54] $\begin{matrix}2\\8\end{matrix}$ labdhaṃ vyāsamānaṃ 10 //

kriyāsūtraṃ /

grāsonau viṣkaṃbhau tadyutitaṣṭau pṛthak viparyastau⟨/⟩[55]
grāsaghnau saṃbhavataḥ(/) śarau pṛthak vṛttayo⟨ḥ⟩ kramaśaḥ //22//[56]

udā° //

iṃdorvyāsaḥ(/) paṃcavargrastu rāhordvāpaṃcāśattena saptāṃgulāni /[57]
grastānīṃdorasya viṣkaṃbhataśca brūhi kṣipraṃ tatra bāṇaṃ ca mitra //23//[58]

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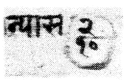
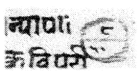
[48]Indravajrā meter (half stanza).

[49]jyārthaṃ] jyārddhaṃ Ms.

[50]Figure with 'nyāsa' in Ms.: 

[51]Figure with 'nyāsaḥ //' in Ms.: 

[52]jñānena] jñāyena Ms.

[53]Figure with 'nyā° //' in Ms.: 

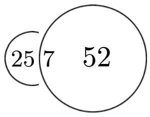
[54]Figure with 'nyā° //' in Ms.: 

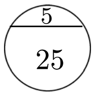
[55]viṣkaṃbhau] viṣkaṃbhoḥ Ms.; tadyuti] tarkkāti Ms.

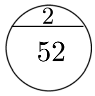
[56]Āryā meter. grāsaghnau] grāsaghno Ms.

[57]rāhor] rāhuḥr Ms.; dvā] dvau Ms.

[58]Śālinī meter. viṣkaṃbhataśca] viṣkaṃbhasya Ms.

nyāsa[59] 

labdhau laghuvṛtte śara 5 vṛhadvṛtte śara 2 ⟨//⟩ atra jyārthaṃ punaḥ(/) nyāsaḥ //[60] 

śaronavṛttaviṣkaṃbhāt ityādinā labdhaṃ jīvā 20 ⟨//⟩ dvitīyavṛttasya jyārthaṃ nyāsaḥ[61] 

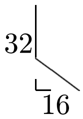
pūrvoktanyāyenaiva labdhaṃ jīvā 20 ⟨/⟩ iyameva rāhumupavistṛ⟨ṇo⟩ti //

⟨kriyāsūtraṃ /⟩

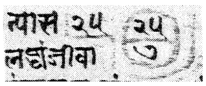
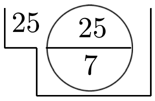
vaṃśāgramūlāṃtarabhūkṛtiryā vaṃśoddhṛtā tena(/) pṛthak yutonaḥ ⟨/⟩[62]
vaṃśo dale staḥ kramaśastadīye vaṃśasya ṣaṃḍe śrutikoṭirūpe //24//[63]

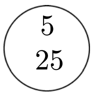
udā° //

samapradeśe dvyanila32pramāṇo vaṃśaḥ sa vegāt pavanasya bhagnaḥ /
tadagramargre nṛpa16hasta lagnaṃ karai⟨ḥ⟩ kiyadbhiḥ sthalataḥ sa bhagnaḥ //25//[64]

⟨nyāsaḥ⟩[65] 

⟨jāte ṣaṃḍe hasta 20 / 12 //⟩

[59]Figure with 'nyāsa' in Ms.:  

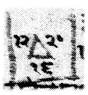
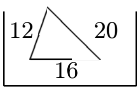
[60]Figure with 'nyāsaḥ //' in Ms.:  

[61]Figure with 'nyāsaḥ' in Ms.: 

[62]yutonaḥ] yutonau Ms.

[63]Indravajrā meter.

[64]Upajāti meter. sthalataḥ] stalataḥ Ms.

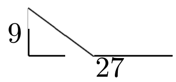
[65]Figure in Ms.:  

⟨kriyāsūtraṃ /⟩

kṛti⟨ḥ⟩ staṃbhasya yā vyālavilāṃtarahṛtā phalaṃ /[66]
tadvyālagarttaviśleṣāt śodhyaṃ tasyārddhasammitaiḥ //26//[67]
karai⟨ḥ⟩ syādgarttamuṣato yogo vyālakalāpinoḥ //27//[68]

udā° //

staṃbhasya mūle bilamasti tasya navocchritasyopari yaḥ śiṣaṃḍī /[69]
trisaṃguṇastaṃbhamite ṃtare 'hiṃ dṛṣṭvā vrajaṃtaṃ bilamatra tiryak //28//[70]
papāta tasyopri tadā kiyadbhiḥ karairbilādāyutiretayo⟨ḥ⟩ syāt //29//[71]

nyāsaḥ //[72] 

jātā vilāyutyormadhye hasta 12 /[73] ⟨agre hasta⟩ 15 //
kūpaphalānayane karaṇasūtramāryāḥ //

talasya vaktrasya ca tadyuteśca vargraikyavargrasya daśāhatasya /
padaṃ vyadhaghnaṃ jinabhakta(ṃ)metat kūpe phalaṃ tattrilavaśca sūcyāṃ //30//[74]

kathitāṃśaphalāṃkahateḥ kṛtirdviguṇitā tayonitā nūnaṃ /[75]
sarvvaphalānāṃ saṃkhyā syāt śrījayaśeṣareṇoktaṃ //31//[76]

mu° harṣatilakena liṣitā // śubhaṃ bhavatu(ḥ) //

[66] vyāla] cyāla Ms.

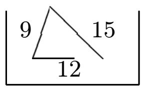
[67] Anuṣṭubh meter. śodhyaṃ] sodhyaṃ Ms.

[68] Anuṣṭubh meter (half stanza).

[69] mūle] mūlau Ms.; asti] asri Ms.

[70] Upajāti meter.

[71] Upendravajrā meter (half stanza). tadā] tat Ms.; bilādāyutir] bilāgrāṣutir Ms.

[72] Figure with 'nyāsaḥ //' in Ms.:  

[73] -yutyor] -yutyār Ms.

[74] Upajāti meter.

[75] tayonitā] tayo–tā Ms. (only a short mātrā line is written between yo and tā).

[76] Āryā meter.

III Translation

[⟨...⟩: additions to the translation. (...): a brief explanation of a word in the translation.]

Let there be success. One Āryā stanza for a procedural rule ⟨to be employed⟩ in calculating the fee to a dancer:

1. Having separately divided by the spectators the day-fractions each decreased by the preceding terms and multiplied by the fruit (total fee), one should add the former to the latter ⟨in the sequence⟩. They (the results) are multiplied by the leaving ⟨people⟩. ⟨The results are⟩ the fee ⟨to be paid by each group of spectators⟩.

Ex.:

2–3. A dance was seen by one man for one *yāma*, by another for two *yāmas*, by another for three *yāmas*, and still by another up to the end of the day.// ⟨A total of⟩ ninety-six *rūpas* should be given to the male dancer by them. What ⟨part⟩ of that ⟨total amount⟩ should be paid by whom, in proportion to the seen ⟨period of time⟩?

Setting-down

1	1	1	1	⟨leaving people⟩
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	⟨day-fraction⟩
4	3	2	1	⟨spectators⟩

Given 96 ⟨*rūpas*⟩. Obtained are, in the same order, 6, 14, 26, and 50 ⟨*rūpas*⟩.

Second example:

4. A palanquin is to be carried three *krośas* for a hundred ⟨monetary units⟩ by ten men, out of whom two, three and five people withdraw at ⟨the completion of⟩ each *krośa*, ⟨respectively⟩.

Setting-down

$\frac{2}{1}$	$\frac{3}{1}$	$\frac{5}{1}$
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
10	8	5

[77] Given a hundred ⟨monetary units⟩ 100. Obtained

$6\frac{2}{3}$	$22\frac{1}{2}$	$70\frac{5}{6}$	*ya.*[78]

[77] To the right of the second row of this table the following comment is written: '= This is a table of the answer for ⟨the problem of⟩ the fee of a dancer.' This seems to refer to the next table.

[78] *ya* seems to be an abbreviation of the monetary unit employed in this example.

Thus ends ⟨the section for⟩ the fee to a dancer.

Two Āryā stanzas for a procedural rule ⟨to be employed⟩ in calculating the prices of living things:

5–6. One should multiply, one by one, the said ⟨numbers of⟩ living things by the ⟨unit⟩ price of an optional living thing. Having subtracted each price from them, one should multiply the remainders// except the requisite term by one's own intelligence in such a way that the requisite term is equal to the sum of them (the products). One should multiply what have been told there (in the problem) by what have been produced ⟨as the multipliers in the preceding step⟩. There will be ⟨the numbers of⟩ all living things.

Ex. ⟨in⟩ two ⟨Anuṣṭubh stanzas⟩:

7–8. Five pigeons are ⟨sold⟩ for three *rūpas*, seven cranes for five, nine swans for seven, and three peacocks for nine.// For the pastime of the prince bring one hundred living things with the prices known as told above for one hundred *rūpas*.

Setting-down

5	7	9	3	100
3	5	7	9	100

Multipliers: 3, 4, 5; 2, 6, 4; 1, 8, 3; 12, 1, 2; 7, 1, 5; 3, 9, 1; 4, 7, 2; 8, 4, 2; and 9, 2, 3. Having multiplied the three kinds of living things beginning with pigeons and also the ⟨corresponding⟩ prices by them, and subtracted their sum from the requisite, ...[79]

⟨End of section⟩ 1.

One Āryā stanza for a procedural rule ⟨to be employed⟩ in calculating the capital for profit in the ⟨problem of⟩ buying and selling:

9. One should multiply the money of profit by the selling rate and divide by the difference of the buying and the selling rates. What is obtained is called the computed one in calculating the capital.

Exs. ⟨in⟩ two ⟨Anuṣṭubh stanzas⟩:

10. Seven *kuḍabas* ⟨of a certain commodity⟩ are bought for one *rūpa*. One would sell ⟨the same thing⟩ at the rate of five ⟨*kuḍabas* per *rūpa*⟩ and obtain

[79] The text is corrupted.

half of one hundred ⟨*rūpas*⟩.[80] One should calculate the capital for the profit.

⟨Setting-down:⟩ buying rate 7, selling rate 5, profit 50. The capital obtained is 125.

11. Eight and a half ⟨of a certain thing⟩ are bought for one *rūpa* and six less a quarter ⟨of the same thing⟩ are sold ⟨for one *rūpa*⟩. If the profit is thirty less one increased by one-third,[81] then what is the capital?

⟨Setting-down:⟩ buying rate $8\frac{1}{2}$, selling rate $6-\frac{1}{4}$,[82] profit $29\frac{1}{3}$. The capital obtained is $61\frac{1}{3}$. Thus ends the calculation of the capital.

One Āryā stanza[83] for a procedural rule ⟨to be employed⟩ in calculating the profit:

12–13. The buying rate and the selling rate: the difference of the two ⟨should be obtained⟩. By it the capital is multiplied. One should divide ⟨the product⟩ by the selling rate.// What is obtained would be the profit. ⟨This has been⟩ told by earlier scholars.

⟨First example. Setting-down:⟩ buying rate 7, selling rate 5, profit 0,[84] capital 125. What is obtained is 50 *tārās*.

⟨Second⟩ example. Setting-down: buying rate $8\frac{1}{2}$, selling rate $6-\frac{1}{4}$, profit 0, capital money $61\frac{1}{3}$. The profit obtained is $29\frac{1}{3}$. Thus ends the calculation of the profit.

One Āryā stanza[85] for a procedural rule ⟨to be employed⟩ in calculating the buying rate:

14. The sum of the ⟨capital⟩ money and the profit is multiplied by the selling rate and it is divided by the capital. It is said to be the buying rate.

⟨First example. Setting-down:⟩ buying rate 0, selling rate 5, profit 50, capital money 125. The buying rate obtained is 7.

Setting-down for the second example: buying rate 0, selling rate $6-\frac{1}{4}$, profit $29\frac{1}{3}$ *yaṃ*. The buying rate obtained is $8\frac{1}{2}$. Thus ends the calculation of the buying rate.

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[80] For another interpretation see the Commentary.

[81] Lit., 'by part.'

[82] The ms. does not use a sign for subtraction and therefore there is no distinction of expression between $6\frac{1}{4}$ and $6-\frac{1}{4}$.

[83] Actually, one and a half Anuṣṭubh stanzas.

[84] This '0' indicates that the profit is unknown.

[85] Actually, one Anuṣṭubh stanza.

One Āryā stanza[86] for a procedural rule ⟨to be employed⟩ in calculating the selling rate:

> **15. The capital is multiplied by the buying rate and the profit is increased by the capital. By this that is divided and the selling rate is produced correctly.**

⟨First example. Setting-down:⟩ buying rate 7, selling rate 0, profit 50, capital money 125. The selling rate obtained is 5.

2nd ex. Setting-down: buying rate $8\frac{1}{2}$, selling rate 0, profit $29\frac{1}{3}$, ⟨capital $61\frac{1}{3}$.⟩ The selling rate obtained is $6 - \frac{1}{4}$. Thus ends the calculation of the selling rate.

One Āryā stanza[87] for a procedural rule ⟨to be employed⟩ in calculating the profit and the capital:

> **16. The sum of the ⟨capital⟩ money and the profit is multiplied by the selling rate and the difference ⟨of the buying and the selling rates, respectively⟩, and divided by the buying rate. The ⟨capital⟩ money and the profit are produced.**

⟨First example. Setting-down:⟩ buying rate 7, selling rate 5, profit 0, capital money 0, ⟨sum of⟩ capital and profit 175. The capital money obtained is 125. The remainder obtained is the profit money, 50.

2nd ex. Setting-down: buying rate $8\frac{1}{2}$, selling rate $6 - \frac{1}{4}$, profit 0, sum of capital and profit $90\frac{2}{3}$. The capital money obtained is $61\frac{1}{3}$. The remainder is the profit money, $29\frac{1}{3}$. Thus ends the calculation of ⟨the capital and⟩ the profit. Let there be purity and auspiciousness.

Two Āryā stanzas[88] for a procedural rule in order to know the chords, the bows (arcs), the arrows, the diameter,[89] etc., of a circular figure:

> **17–19. The diameter of a circle is decreased by the arrow and multiplied by the arrow and by four. Its square root is the chord. The square of the arrow is multiplied by six and// increased by the square of the chord. Its square root ⟨being taken⟩, the bow is produced as the result.// The square root of the difference between the squares of the chord and the diameter is subtracted from the diameter. Its half is the arrow. The sum of the squares of the arrow and half the chord, divided by the arrow, will be the measure of the diameter.**

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[86] Actually, one Anuṣṭubh stanza.

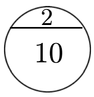
[87] Actually, one Anuṣṭubh stanza.

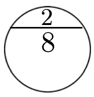
[88] Actually, in the present manuscript, one and a half Anuṣṭubh and one Āryā stanzas.

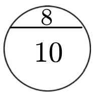
[89] *vṛtta*. See § I.6.2.

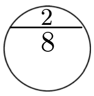
Ex.

20–21. Of a circle, the diameter is ten and an arrow is two ⟨in length⟩. What is the chord? When an arrow is two and the chord is eight, what is the bow? When the diameter[90] is ten and a chord is eight, what is the arrow? Or, when an arrow is// two and the chord is measured by eight *karas*, what is the diameter? Tell the results concerning the circle.

Setting-down for the chord: 

What is obtained is the chord, 8. Setting-down for the bow: 

Knowing that ⟨88 is⟩ a quantity that does not yield a square root, ⟨we calculate, and⟩ an approximate root is obtained, namely, 9 *hastas*, 9 *aṅgulas* and $\frac{3}{25}$ parts of an *aṅgula*, ⟨which is the bow⟩. Setting-down for the arrow: 

The measure of the arrow obtained is 2. Setting-down for the diameter: 

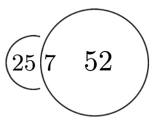
The measure of the diameter obtained is 10.

A procedural rule ⟨for the two arrows in the intersection of two circles⟩:

22. The two diameters decreased by the intersection, interchanged, are severally divided[91] by their sum and multiplied by the intersection. The two arrows of the two circles, respectively, will be produced severally.

Ex.

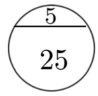
23. The diameter of the moon is the square of five and that of Rāhu fifty-two ⟨*aṅgulas*⟩. By him seven *aṅgulas* of the diameter of the moon were swallowed. Tell quickly, o friend, the arrows in that case.

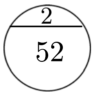
Setting-down: 

Obtained are the arrow of the smaller circle, 5, and

[90] *vedha*. See § I.6.2.

[91] *taṣṭa*. See § I.6.2.

that of the greater circle, 2. Here, another setting-down for the chord: 

Obtained by means of ⟨the rule beginning with the phrase⟩, 'The diameter of a circle is decreased by the arrow' (verse 17), is the chord, 20. Setting-down for the chord of the second circle: 

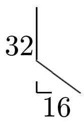
Obtained by means of the same principle told above is the chord, 20. This ⟨chord⟩, indeed, stretches over Rāhu.

A procedural rule ⟨for the problem of a broken bamboo⟩:

> **24. The square of the distance on the ground between the tip and the foot of the bamboo is divided by the bamboo. The bamboo, increased and decreased by that separately and halved, are the two parts of the bamboo, respectively, in the forms of the diagonal and the upright.**

Ex.

> **25. There is a bamboo of height 'two and the fires' (32) on an even ground. It is broken by the power of wind and its tip touches ⟨the ground⟩ in front at a distance of 'the kings' (16) *hastas*. At how many *karas* from the ground is it broken?**

Setting-down: 

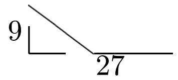
Produced are the two parts, 20 and 12 ⟨*karas*⟩.

A procedural rule ⟨for the problem of a peacock's attack on a snake⟩:

> **26–27. The square of the pole is divided by the distance between the snake and the hole. The quotient is subtracted from the distance between the snake and the hole. At ⟨a distance⟩ measured by the *karas* of half of that ⟨result⟩// from the mouth of the hole the encounter of the snake and the peacock will take place.**

Ex.

> **28–29. There is a hole at the foot of a pole, which is nine ⟨*karas*⟩ in height. A peacock standing at the top of it, seeing a snake at a distance of three times ⟨the height of⟩ the pole rush to the hole, diagonally// attacked upon it. At how many *karas* from the hole will the encounter of the two take place?**

Setting-down: 

Produced are ⟨the distance⟩ between the hole and the ⟨spot of⟩ the encounter 12 *hastas* and, in front, 15 *hastas*.

Āryā stanzas[92] for a procedural rule ⟨to be employed⟩ in calculating the capacity of a circular well:

> **30. The square root of ten times the square of the sum of the squares of the bottom ⟨diameter⟩, the top ⟨diameter⟩ and their sum, multiplied by the depth and divided by 'the winners' (24), is the fruit (capacity) of the well. One third of it is of a pointed one (i.e., of a cone).**

⟨An Āryā stanza for a procedural rule to be employed in calculating the number of fruit:⟩

> **31. The square of the product of the said fraction and the number of the fruit ⟨allotted to each person⟩, multiplied by two and decreased by that ⟨product⟩, will indeed be the number of all the fruit. ⟨This was⟩ told by the venerable Jayaśekhara.**

⟨This manuscript has been⟩ written down by the sage Harṣatilaka. Let there be good fortune.

IV Commentary

IV.1 Fee and price

IV.1.1 Fee to a dancer (nartaka-dāna)

A dancer makes a performance for one day (of daylight) for a total fee p. The i-th group ($i = 1, 2, ..., n$) consisting of a_i spectators see the dance for t_i day ($0 < t_1 < t_2 < ... < t_n = 1$) from the beginning and leave away. The charge for the i-th group, x_i, is determined by means of two proportionate distributions, one by time and the other by person.

First, the total fee is divided in n parts in proportion to the length of each time period $(t_i - t_{i-1})$, where $t_0 = 0$. Since $\sum_{i=1}^{n}(t_i - t_{i-1}) = t_n - t_0 = 1$, the fee for the j-th time period is

$$\frac{p(t_j - t_{j-1})}{\sum_{i=1}^{n}(t_i - t_{i-1})} = p(t_j - t_{j-1}).$$

Next, since the number of spectators in the j-th time period is $\sum_{k=j}^{n} a_k$, the fee

[92] Actually, one Upajāti stanza.

for one person in the j-th time period is

$$\frac{p(t_j - t_{j-1})}{\sum_{k=j}^{n} a_k}.$$

Since the i-th group of spectators see the dance for the first i time periods, the fee for one person of the i-th group is

$$\sum_{j=1}^{i} \frac{p(t_j - t_{j-1})}{\sum_{k=j}^{n} a_k}.$$

Hence follows the fee assigned to the i-th group,

$$x_i = a_i \sum_{j=1}^{i} \frac{p(t_j - t_{j-1})}{\sum_{k=j}^{n} a_k}.$$

Verse 1. Rule 1: fee to a dancer.

The two accompanying examples show that the numerical data, a_i, t_i and p, were tabulated in a table called *nyāsa* (‘setting-down’), although this procedure is not mentioned in verse 1:

a_1	$\cdots$	a_i	$\cdots$	a_n	given p
t_1	$\cdots$	t_i	$\cdots$	t_n	
b_1	$\cdots$	b_i	$\cdots$	b_n	

where b_i is obtained from a_i:

$$b_i = \sum_{k=i}^{n} a_k.$$

The three rows of the table (a_i, t_i, and b_i) are called ‘leaving ⟨people⟩’ (*vyāvṛtti*), ‘day-fraction’ (*dina-aṃśa*), and ‘spectators’ (*prekṣaka*), respectively. The verse gives an algorithm for obtaining every x_i simultaneously as follows.

Diminish each ‘day-fraction’ by its previous term:

$$t_1, \quad \cdots, \quad t_i - t_{i-1}, \quad \cdots, \quad t_n - t_{n-1}.$$

Multiply each term by the ‘fee’:

$$pt_1, \cdots, \; p(t_i - t_{i-1}), \cdots, \; p(t_n - t_{n-1}).$$

Divide each term by the ‘spectators’ below it:

$$\frac{pt_1}{b_1}, \cdots, \frac{p(t_i - t_{i-1})}{b_i}, \cdots, \frac{p(t_n - t_{n-1})}{b_n}.$$

Starting from the first term, add each term successively to the next: the i-th term becomes

$$\sum_{j=1}^{i} \frac{p(t_j - t_{j-1})}{b_j}.$$

Multiply each term by the 'leaving people': the i-th term becomes

$$a_i \sum_{j=1}^{i} \frac{p(t_j - t_{j-1})}{b_j},$$

which is the charge (x_i) for the i-th group.

This verse is identical with PG 68. Cf. GSS 6.230, which gives a similar rule designed for the wages for carriers of a palanquin (*śibikā*). See under verse 4 below.

Verses 2–3. Ex. 1: fee to a dancer.

Given: $n = 4$, $a_i = 1$ man for every i, $t_i = \frac{i}{4}$ day (= i *yāmas*), $p = 96$ *rūpas*. Answer: $b_i = 5 - i$ men, $x_i = 6, 14, 26, 50$ *rūpas*.

These verses are identical with PG (86)–(87), where the fee is to be paid 'to a party of dancers' (*nartaka-pakṣe*) while in the present work it is given 'to a male dancer' (*nartaka-puṃsaḥ*).

Verse 4. Ex. 2: wages for carriers of a palanquin (*yugya*).

In this example t_i stand not for time but for distance. Given: $n = 3$, $a_i = 2, 3, 5$ men, $t_i = \frac{i}{3}$ of the whole distance (= i *krośas*), $p = 100$ monetary units. Answer: $b_i = 10, 8, 5$ men, $x_i = 6\frac{2}{3}, 22\frac{1}{2}, 70\frac{5}{6}$ *ya°*, where '*ya°*' seems to be an abbreviation of the monetary unit used here. See under verse 14 below.

This verse is identical with PG (88). For similar problems see PG (89)–(90) (fee to five brāhmaṇas worshipping the five faces of Śiva) and GSS 6.231–232 (wages for carriers of a palanquin).

IV.1.2 Prices of living things (jīva-mūlya)

It is required to buy, from among n (≥ 3) kinds of living things, p living things in total for q monetary units when a_i living things of the i-th kind cost b_i monetary units. How many (x_i) of each kind should one buy and how much (y_i) does each kind cost? That is to say,

$$\sum_{i=1}^{n} x_i = p, \quad \sum_{i=1}^{n} y_i = q, \quad x_i : y_i = a_i : b_i,$$

where x_i and a_i are positive integers and y_i and b_i are positive numbers. This type of problem with $p = 100$ and $q = 100$ monetary units, just as in verses 5–6, is usually called 'the hundred fowls problem' and appears to have originated in China.[93]

[93] For the Chinese version see Martzloff 1997, 308–310, Yabuuti 2000, 52.

By subtracting the second equation from the first multiplied by b_j/a_j, we have the indeterminate equation,

$$\sum_{i\neq j}\left(x_i\cdot\frac{b_j}{a_j}-y_i\right)=p\cdot\frac{b_j}{a_j}-q,$$

where $y_i = b_i x_i/a_i$, or

$$\sum_{i\neq j}\left\{\left(a_i\cdot\frac{b_j}{a_j}-b_i\right)\cdot z_i\right\}=p\cdot\frac{b_j}{a_j}-q,$$

where $z_i = x_i/a_i$. The answer to the original equations can be obtained from among the solutions to this indeterminate equation by means of

$$x_i = a_i z_i \ \ (i\neq j),\quad x_j = p-\sum_{i\neq j} x_i,$$

$$y_i = b_i z_i \ \ (i\neq j),\quad y_j = q-\sum_{i\neq j} y_i,$$

on the condition that all the x_i's are positive integers. Or, y_i may be obtained by using $y_i = x_i \times (b_i/a_i)$.

Verses 5–6. Rule 1: prices of living things.

Arrange the given numbers in two rows:

a_1	a_2	$\cdots$	a_i	$\cdots$	a_n	p
b_1	b_2	$\cdots$	b_i	$\cdots$	b_n	q

where p is called *icchā* or 'requisite'. Multiply the first row by the unit price of an optional (*iṣṭa*) living thing, say b_j/a_j, and from it subtract the second row; consequently, the j-th term of the first row is reduced to zero and the i-th $(i\neq j)$ and the last (requisite) terms become

$$a_i\cdot\frac{b_j}{a_j}-b_i \quad\text{and}\quad p\cdot\frac{b_j}{a_j}-q,$$

respectively. Find 'multipliers' (*guṇaka*), z_i, such that

$$\sum_{i\neq j}\left\{\left(a_i\cdot\frac{b_j}{a_j}-b_i\right)\cdot z_i\right\}=p\cdot\frac{b_j}{a_j}-q$$

by one's own intelligence (*svabuddhyā*) on the condition that every $a_i z_i$ is a positive integer (although this condition is not explicitly stated in the verses). Then

$$x_i = a_i z_i,\quad y_i = b_i z_i.$$

Verses 5–6 are identical with PG 63–64 excepting the last line (64b), in which restrictions on the sign of z_j are given and instead the last step of the solution, $x_i = a_i z_i$ and $y_i = b_i z_i$, is only implicitly mentioned. PG 64b in Shukla's translation reads as follows.

> ..., and on taking the products of those multipliers (z_i) and the respective rate-prices (b_i), a negative number or zero may not be obtained for the multiplier of the creature which is without a multiplier (z_j). (Shukla 1959, 50; z_i, b_i and z_j supplied by me.)

The same problem is also given two solutions (algorithms) in the GSS and one in the GK. The first solution of the GSS (6.146b–147a) is as follows. Let the least of all the a_i/b_i be a_j/b_j and determine $(n-1)$ positive numbers, u_i, such that

$$\sum_{i \neq j} u_i = p - \frac{a_j}{b_j} \cdot q.$$

Then,[94]

$$y_i = \frac{u_i}{\frac{a_i}{b_i} - \frac{a_j}{b_j}} \ (i \neq j), \quad y_j = q - \sum_{i \neq j} y_i.$$

x_i must have been obtained from $x_i = y_i \times (a_i/b_i)$, although the GSS does not refer to it.

The second solution (GSS 6.151) is as follows. Subtract $b_i p$ from $a_i q$,

$$c_i = a_i q - b_i p.$$

Let us assume, for convenience, that c_i is positive for $1 \leq i \leq j$ and negative for the rest. Multiply their absolute values separately by optional numbers (*iṣṭa*), d_i $(i = 1, 2, ..., n)$, and calculate two sums, e^+ and e^-, such that

$$e^+ = \sum_{i=1}^{j} c_i d_i, \quad e^- = \sum_{i=j+1}^{n} |c_i| d_i.$$

Multiply the above optional numbers by these sums with their positions 'interchanged' (*viparyasta*), that is, d_1 to d_j by e^- and the rest by e^+.[95]

$$v_i = \begin{cases} e^- \cdot d_i \text{ for } 1 \leq i \leq j, \\ e^+ \cdot d_i \text{ for } (j+1) \leq i \leq n. \end{cases}$$

And further multiply these by the corresponding rate quantities, a_i, and by the rate prices, b_i, separately,

$$x'_i = a_i v_i, \qquad y'_i = b_i v_i.$$

[94]Note that the given relationships can be reduced to the equation, $\sum_{i \neq j} \left(\frac{a_i}{b_i} - \frac{a_j}{b_j} \right) y_i = p - \frac{a_j}{b_j} \cdot q$.

[95]Note that these v_i's satisfy the equation, $\sum_{i=1}^{n} c_i v_i = 0$.

Table 2: Solutions of the hundred birds problem in Indian works.

SMA	PGT	BG	GK	PM	z_1	z_2	z_3	z_4	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4
			2		1	3	7	11/3	5	3	21	15	63	49	11	33
3	3	1	1		1	8	3[a]	4[h]	5	3	56	40	27	21	12	36
			3		2	1	8[b]	11/3	10	6	7	5	72	56	11	33
2	5	2	4	2	2	6	4[c]	4	10	6	42	30	36	28	12	36
1	1	3	5		3	4	5[d]	4	15	9	28	20	45	35	12	36
6			6		3	9	1	13/3[i]	15	9	63	45	9	7	13	39
	6		7		4	2	6	4	20	12	14	10	54	42	12	36
7			8		4	7	2	13/3	20	12	49	35	18	14	13	39
			9	1	5	5	3	13/3	25	15	35	25	27	21	13	39
	4		10		6	3	4	13/3[j]	30	18	21	15	36	28	13	39
5			11		7	1	5[e]	13/3[k]	35	21	7	5	45	35	13	39
			12		7	6	1	14/3	35	21	42	30	9	7	14	42
8			13		8	4	2[f]	14/3[l]	40	24	28	20	18	14	14	42
9			14		9	2	3	14/3[m]	45	27	14	10	27	21	14	42
	2		15		11	3	1	5	55	33	21	15	9	7	15	45
4			16		12	1	2[g]	5[n]	60	36	7	5	18	14	15	45

Variants in the three mss. and one ed. of the GK (for the abbreviations see fn. 95):
[a]1 8 3] 18 3 BNR; [b]2 1 8] ∅ BDNR; [c]6 4] 4 6 BDNR; [d]3 4 5] ∅ D; [e]5] 15 BDNR;
[f]2] 1 R; [g]1 2] 11 1 BDNR; [h]4] 4/2 DN; [i]13/3] 13/2 NR; [j]13/3] ∅ N;
[k]13/3] ∅ BDNR; [l]14/3] 14/1 BDNR; [m]14/3] 6/1 BNR, 6/5 D; [n]5] 4/1 BDNR.

To these apply 'the investment procedure' (*prakṣepa-karaṇa*) or the rule for proportionate distribution,

$$x_i = \frac{px'_i}{\sum_{k=1}^{n} x'_k}, \quad y_i = \frac{qy'_i}{\sum_{k=1}^{n} y'_k}.$$

In order to obtain integer x_i's by means of these methods, one has to choose appropriate u_i's in the first method and d_i's in the second, although the GSS is silent about this point.

The algorithm of the GK (miśra 34–35; part 1, p. 92) is the same as that of the present work except that the greatest of all the b_i/a_i is taken to be b_j/a_j.

Verses 7–8. Ex.1: pigeons, cranes, swans, and peacocks.

Given: $n = 4$, $a_i = 5, 7, 9, 3$, $b_i = 3, 5, 7, 9$ *rūpas*, $p = 100$, $q = 100$ *rūpas*. Answer: The prose commentary does not give a complete answer but only lists nine sets of 'multipliers' (z_i). The list does not contain z_4. This means that the commentator canceled the last term (x_4) of the first equation (taking $j = 4$ in the above procedure) and tried to solve the indeterminate equation,

$$3z_1 + 4z_2 + 5z_3 = 50.$$

This has sixteen sets of positive integer solutions, which are necessary and sufficient for the complete solutions of the problem.[96] Table 2 lists the sixteen solutions and the serial numbers, 1 to 9, in the first column (SMA) indicate the order of the nine solutions in the manuscript, where the eighth set of (z_1, z_2, z_3) is corrupted to $(4, 8, 2)$.

Verses 7–8 are identical with PG (78)–(79). The anonymous commentator of the PG canceled x_1 and tried to solve the indeterminate equation,[97]

$$z_2 + 2z_3 + 9z_4 = 50.$$

He refers to the six solutions indicated in the second column (PGT) of Table 2. While commenting on this example, he cites four anonymous Anuṣṭubh verses, which give the fifty-two positive integer solutions to the last equation,

$$(z_2, z_3, z_4) = (50 - 9k - 2i, i, k),$$

where $i = 1, 2, ..., 20$ when $k = 1$; $i = 1, 2, ..., 15$ when $k = 2$; $i = 1, 2, ..., 11$ when $k = 3$; and $i = 1, 2, ..., 6$ when $k = 4$. The author of the four verses points out that all but the last four sets ($k = 4$, $i = 3, 4, 5, 6$) are not appropriate for the original equations because they make x_1 either zero or negative, but misses the two integer solutions, $(3, 1, 5)$ and $(1, 2, 5)$, both of which are appropriate for the original equations. He mentions none of the solutions with a fractional z_4. The six solutions of the commentator of the PG, on the other hand, include the $(3, 1, 5)$ and one solution with a fractional z_4, namely, $(3, 4, 13/3)$, but not the $(1, 2, 5)$ and the other solutions with a fractional z_4.

Exactly the same problem with slight changes in wording and with different monetary units occurs in other works. The GSS (6.152–153) simply states the problem without answer. The BG (138), in its prose part,[98] obtains the three solutions indicated in the third column (BG) of Table 2 by using algebraic symbols (*yā*, *kā*, etc.) for the unknown numbers and remarks, 'In this way, there are many ⟨solutions⟩ due to the optional ⟨element⟩' (*evamiṣṭavaśādanekadhā*). The GK (p. 93 of part 1), in its prose part, reduces the problem to the same indeterminate equation (with z_1, z_2 and z_3) as the present work and obtains all the sixteen solutions in the order indicated in the fourth column (GK) of Table 2.[99] The PM (A21) gives, without any working, the two solutions indicated in the fifth column.

[96] Cf. Shukla 1959, 51 and 95.

[97] The form of the equation the commentator actually dealt with is $-\frac{4z_2}{5} - \frac{8z_3}{5} - \frac{36z_4}{5} = -\frac{200}{5}$, which is obtained by means of the rule of PG 63–64, but he refers to how to rewrite it by means of the 'cancellation' (*apavartana*), 'reduction to the same denominator' (*cheda-sādṛśya*), and 'ceasing of the negative sign' (*ṛṇa-nivartana*).

[98] Jhā 1949, 436–437.

[99] This order has been confirmed in the three manuscripts, Benares 98703 (abbreviated B), Nepal

For similar problems see ex. 1 (illegible except for $q = 20$), ex. 2 (20 of monkeys, horses, and deer for 20 *paṇas*), and ex. 3 (20 of men, women, and śūdras for 20 *maṇḍas*) for BM C7, PG (80) (100 of pomegranates, mangos and wood-apples for 80 monetary units), and GSS 6.147b–149 (72 of peacocks, pigeons, swans, and cranes for 56 *paṇas*) and 150 (60 *palas* of ginger, long pepper, and pepper for 60 *paṇas*).

IV.2 Buying and selling (kraya-vikraya)

One buys a certain amount of a certain commodity and sells the same to obtain a profit. Let m be the capital (*mūla*), k the buying rate (*kraya*) (quantity/price), v the selling rate (*vikraya*) (quantity/price), ℓ the profit (*lābha*), and M the total gain or the sum (*miśra*) of the capital and the net profit.

All the rules given in verses 9 and 12–16 are obtained from the two basic relationships,

$$m + \ell = M, \quad km = vM.$$

The BM (54–57) gives five formulas, three of which are equivalent to those of rules 1, 2, and 5 of the present work. The GSS (6.167) gives a formula equivalent to rule 1. The MS (15.28b–29, 34–35, and 44–45a) gives eight formulas: four of them are exactly the same as rules 1, 3, 4, and 5 and one is equivalent to rule 1. The GK (miśra 1b–2) refers to proportionate relationships among k, v, $(k - v)$, m, ℓ, and M, from which all the buying-selling formulas of the present work can be derived. The GS (1.86, 89) gives three formulas: one of them is the same as rule 1 and one is equivalent to it.

Verse 9. Rule 1: capital.

$$m = \frac{\ell v}{k - v}.$$

MS 15.28b and GS 1.86 give the same formula. GSS 6.167, MS 15.29, and GS 1.89 give formulas for m which can be obtained by substituting 'quantity/price' for k and v in this formula. BM 54 gives $m = \ell / \left(\frac{k}{v} - 1\right)$.

Verse 10. Ex.1: capital.

Given: $k = 7$ *kuḍabas/rūpa*, $v = 5$ *kuḍabas/rūpa*, $\ell = 50$ *rūpas*. The name of the commodity not mentioned. Answer: $m = 125$ *rūpas*.

4-1689 (jyotiṣa 125) (N), and Rājasthāna Cat. II 4720 (R), as well as in Dvivedī's edition (D). Some of the numerical figures for z_i's in them are corrupt and others are missing but it is deducible that their ancestors had all the sixteen solutions. For example, none of them has $(2, 1, 8)$ for (z_1, z_2, z_3) but the corresponding 11/3 for z_4 exists in all of them; one of the four consecutive 13/3's for z_4 is missing in B, D, and R and two of them are missing in N but all the corresponding (z_1, z_2, z_3)'s are extant in all of them. For the details see the notes for Table 2.

The above is the problem of verse 10 according to the commentator but '$\ell = 50$' is based on a rather peculiar interpretation of the expression '*śatamarddhaṃ ca*' as I mentioned earlier (see under 'Hiatus-bridger *m*' in § I.6.1). A grammatically more natural interpretation of the passage would be 'a hundred and a half', which can mean both '$100 + 1/2$' and '$100 + 100/2$'; since fractions are the theme of the next example (in verse 11), the latter value, i.e., 150, best fits in the present example. In that case, the problem is as follows.

Given: $k = 7$ *kuḍabas/rūpa*, $v = 5$ *kuḍabas/rūpa*, $\ell = 150$ *rūpas*. Answer: $m = 375$ *rūpas*.

For similar examples see GSS 6.168, GK miśra (1st example on p. 56 of part 1), and GS 1.87.

Verse 11. Ex. 2: capital.

Given: $k = 8\frac{1}{2}$ /*rūpa*, $v = (6 - \frac{1}{4})$ /*rūpa*, $\ell = 29\frac{1}{3}$ *rūpas*. The name of the commodity not mentioned. Answer: $m = 61\frac{1}{3}$ *rūpas*.

For similar examples see exs. for BM 54–57, GS 1.88.

These two examples (in verses 10 and 11) are repeatedly employed in the prose comments on verses 12–16, too, for explaining the rules given in them.

Verses 12–13. Rule 2: profit.

$$\ell = \frac{(k-v)m}{v}.$$

The word *tārā* occurs immediately before 50 ($= \ell$) in the commentary. As the denomination *rūpa* is used for that value in the example of verse 10, *tārā* may be a denomination of money equivalent to the *rūpa*.

BM 55 gives $\ell = m\left(\frac{k}{v} - 1\right)$.

Verse 14. Rule 3: buying rate.

$$k = \frac{Mv}{m}.$$

In the prose comment on this verse the syllable '*yaṃ*' occurs immediately after $29\frac{1}{3}$ ($= \ell$). As the monetary unit *rūpa* is used for the same 'profit' in the example of verse 11, the syllable may be an abbreviation of a monetary denomination equivalent to the *rūpa*. See under verse 4 above.

The same formula occurs in MS 15.35.

Verse 15. Rule 4: selling rate.

$$v = \frac{mk}{M}.$$

The same formula occurs in MS 15.35.

Verse 16. Rule 5: capital and profit.

$$m = \frac{Mv}{k}, \qquad \ell = \frac{M(k-v)}{k}.$$

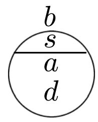
In the solutions of the two examples the commentator gives ℓ without a working process, calling it 'the remainder' (*śeṣa*). This probably implies that he calculated ℓ not by the above formula but by $\ell = M - m$.

MS 15.34 gives the same formulas. BM 56 gives $m = M/\frac{k}{v}$.

IV.3 Circular figures and distribution of fruit

IV.3.1 Circle segments (cāpa-kṣetra)

Let d be the diameter of a circle and a, b, and s the chord, the bow (or arc), and the arrow (or height), respectively, of a segment of the circle.



The formula for b given in verses 17–18 is based on the relationship,

$$a^2 + (\pi^2 - 4)s^2 = b^2, \quad \text{with} \quad \pi = \sqrt{10}.$$

The same relationship with the same value of π was already known to the Jaina philosopher, Umāsvāti (fifth century C. E. or before) and employed mainly by the Jaina people. See UTA 3.11, GSS 7.63 and 73b–75a, MS 15.90–91a, SS 13.39–40, and GS 3.48–49. The same relationship with $\pi = 3$ is used in GSS 7.43b and 45, and with $\pi = 22/7$ in MS 15.94b–97a. Cf. Datta 1929, 124–125 and Datta & Singh 1980, 159–162.

The formulas in verses 17–19, 22, 24, and 26–27, except the above mentioned one, are obtained from the relationship,

$$s(d - s) = \left(\frac{a}{2}\right)^2,$$

which too was known in India by the time of Umāsvāti. See UTA 3.11, AB 2.17b, the Prakrit verse cited in BAB 2.10 (p. 73), BSS 12.41–42a, GSS 7.225b–226a, 227b–228a, and 229b–230a, MS 15.98b–99, SS 13.37–38a, L 204 = GP 26, GK kṣetra 65–66, GM 259, and CCM 66–67. Cf. Datta 1929, 124–125 and Datta & Singh 1980, 159–162.

Verses 17–19. Rule 1: chord, bow, arrow, and diameter.

$$a = \sqrt{4s(d - s)}, \quad b = \sqrt{a^2 + 6s^2}, \quad s = \frac{d - \sqrt{d^2 - a^2}}{2}, \quad d = \frac{(a/2)^2 + s^2}{s}.$$

Verses 20–21. Ex. 1: chord, bow, arrow, and diameter.

(1) Given: $d = 10$, $s = 2$. Answer: $a = 8$.

(2) Given: $a = 8$, $s = 2$. Answer: $b \approx 9$ *hastas* $9\frac{3}{25}$ *aṅgulas*.

(3) Given: $d = 10$, $a = 8$. Answer: $s = 2$.

(4) Given: $a = 8$, $s = 2$. Answer: $d = 10$.

All lengths are in *hasta* unless otherwise indicated.

For similar examples see the verse cited in BAB 2.10 (p. 72), ex. 1 in BAB 2.17b (p. 97), GSS 7.226b–227a, 228b–229a, and 230b–231a, L 205, GK kṣetra (55), and CCM 69.

In the calculation of (2), since 88 ($= a^2 + 6s^2$) is a non-square number (*amūlada-rāśi*, lit. 'a quantity that gives no square root'), the commentator obtains 'an approximate root' (*āsanna-mūla*) for it, that is,

$$\sqrt{88}\ \textit{hastas} \approx 9\ \textit{hastas} + 9\frac{3}{25}\ \textit{aṅgulas}.$$

This approximation was presumably obtained by means of the formula,

$$\sqrt{K} \approx \frac{\left[\sqrt{Ka^2}\right]}{a},$$

where K and a are positive integers. This method has been prescribed by Śrīdhara and others. See Tr 46 = PG 118, MS 15.55, SS 13.36, L 140, and GK kṣetra 30b–31a. $[\sqrt{Ka^2}]$ or the integer part of $\sqrt{Ka^2}$ could be calculated by means of the popular algorithms based on the place-value notation, which were taught by Āryabhaṭa I and others. See AB 2.4, Tr 12–13 = PG 25–26, GP 12, GSS 2.36, GAR 1.3, MS 15.6b–7, GT 23 = SS 13.5, L 22, GK prakīrṇaka 19–20, GS 1.37–38, PV X12–13, and GM 25.

In fact,

$$\sqrt{88}\ \textit{hastas} = \frac{\sqrt{880000}}{100}\ \textit{hastas} \approx \frac{938}{100}\ \textit{hastas} = 9\ \textit{hastas} + 9\frac{3}{25}\ \textit{aṅgulas}.$$

Śrīdhara, too, employs 100 as a for $K = 1000$ and 6250 in his prose comment to the above method (Tr 46) and for $K = 640$ and 4515840 in his solutions to examples for the area of a circle segment (Tr (86)) and for the capacity of a circular well (Tr (91)), respectively.

Verse 22. Rule 2: intersection of two circles.

Let d_1 and d_2 be the diameters of two circles intersecting each other and s the width of the intersection (*grāsa*, lit. 'eating, swallowing'), which is the sum of the two arrows of the segments belonging to each circle ($s_1 + s_2 = s$). Then,

$$s_1 = \frac{s(d_2 - s)}{(d_1 - s) + (d_2 - s)}, \quad s_2 = \frac{s(d_1 - s)}{(d_1 - s) + (d_2 - s)}.$$

The same formula is given in AB 2.18, in BSS 12.42b, in GSS 7.231b–232a, and in GK kṣetra 68.

Verse 23. Ex. 2: Moon and Rāhu.

Given: $d_1 = 25$, $d_2 = 52$, $s = 7$. Answer: $s_1 = 5$, $s_2 = 2$, $a = 20$ (chord). All lengths are in *aṅgula*.

Rāhu is, in Indian mythology, a demon who 'eats' the sun and the moon periodically and, in Indian astronomy, the shadow of the earth that causes lunar eclipse as well as the moon's ascending node.

In graphical representation of eclipses, etc., two or three or four minutes of arc on the sphere are equated to one *aṅgula* on the ground. See PS 11.6, MB 5.53 and 6.55, and SuSi 4.26. According to these conversion ratios, the diameter of the moon would be about 16 *aṅgulas*, at most. The origin of the 25 *aṅgulas* is unknown.

For similar examples see ex. 1 in BAB 2.18 (the moon and Rāhu: $d_1 = 32$, $d_2 = 80$ and $s = 8$), GSS 7.232b–233a (two circles: $d_1 = 32$ *hastas*, $d_2 = 80$ *hastas* and $s = 8$ *hastas*), and GK kṣetra (57) (the moon and the darkness: $d_1 = 13$, $d_2 = 20$ and $s = 6$).

Verse 24. Rule 3: a broken bamboo.

A bamboo of height f is broken by wind at a height x and its tip touches the ground at a distance e from the foot of the bamboo. Let y be the upper part of the bamboo ($x + y = f$). Then,

$$y = \left(f + \frac{e^2}{f}\right) \div 2, \quad x = \left(f - \frac{e^2}{f}\right) \div 2.$$

A formula equivalent to the latter (for a broken pillar) is given in GSS 7.190b–191a. The same formulas are given in MS 15.56–57, in L 149, and in GM 191 (for a monkey's motion). The GK (kṣetra 31b–32a) recommends to use the rule of concurrence (*saṃkramaṇa*) for this type of problems. For the rule of concurrence see Hayashi & Kusuba 1998, 18–19.

Verse 25. Ex. 3: a broken bamboo.

Given: $e = 16$, $f = 32$. Answer: $y = 20$, $x = 12$. All lengths are in *hasta*.

For similar examples see exs. 4 and 5 in BAB 2.17b (pp. 99–100), GSS 7.191b–196a, L 150 = BG 111, GK kṣetra (26), and GM 192 (a monkey's motion). Cf. Shukla 1976, 297–298.

Verses 26–27. Rule 4: a peacock's attack on a snake.

A peacock on the top of a pole of height e, seeing a snake at a distance f from the pole begin rushing to a hole at the foot of the pole, diagonally attacks on and reaches it at a distance x from the pole. Then,

$$x = \left(f - \frac{e^2}{f}\right) \div 2.$$

Here, the two distances covered by the peacock and the snake are assumed to be the same (let it be y), which means that their speeds are the same. The formula for y is not given here.

The same formula is given in MS 15.56–57, in L 151, and in GM 194 (for two monkeys called by Rāma). The BAB (2.17b, p. 98) solves the same problem (for a hawk's attack on a mouse) in general terms (without using a formula) by means of the relationship, $s(d-s) = (a/2)^2$, given in AB 2.17b.

Verses 28–29. Ex. 4: a peacock's attack on a snake.

Given: $e = 9$, $f = 27$. Answer: $x = 12$, $y = 15$. All lengths are in *hasta*. The latter length, $y = 15$, can be calculated either by the same formula as in verse 24 or by $y = f - x$.

For similar examples see exs. 2 and 3 (a hawk's attack on a mouse) in BAB 2.17b (pp. 98–99), L 152 (a peacock's attack on a snake), GK kṣetra (28) (an arrow shot against another arrow), and GM 195 (two monkeys called by Rāma). Cf. Shukla 1976, 296–297.

IV.3.2 Circular wells (kūpa)

Verses 30. Rule 1: capacities of tapered wells.

Let d_1, d_2, and h be the bottom diameter, the top diameter, and the depth, respectively, of a circular well which has a shape of a truncated cone. Then, its capacity is

$$V = \sqrt{\{d_1^2 + d_2^2 + (d_1 + d_2)^2\}^2 \times 10} \times h \div 24,$$

and that of the 'pointed one' (*sūcī*) is

$$V_{\text{sūcī}} = V \div 3.$$

V of the second formula must be the capacity of a columnar well, $V = \sqrt{d^4 \times 10} \times h \div 4$, which is obtained by regarding $d_1 = d_2 = d$ in the first formula.

The first formula is given in Tr 54, too. The second is also given in BSS 12.44, in MS 15.105, in SS 14.43, and in L 217. For a brief history of the treatment of pyramids and cones in India see Hayashi 2004, 734–737.

*IV.3.3 Distribution of fruit (*phala-bhajana*)*

One m-th part of a group of people go to a forest and the i-th person among them plucks i pieces of fruit (mangos, for example). On comming back from the forest, they equally divide all the fruit (x) they plucked among all the people of that group and every person obtains the same number (p) of the fruit.

Let y and z be the numbers of the people of the entire group and of one m-th part of it, respectively. Then,

$$z = \frac{y}{m}, \qquad x = \frac{z(z+1)}{2}, \qquad p = \frac{x}{y}.$$

Verse 31. Rule 1: number of all the fruit plucked.

$$x = 2(mp)^2 - mp.$$

GS 4.48 prescribes a rule for a more general case, where arithmetical progression is used in place of the natural series, that is, the i-th person of one m-th part of the group plucks $(a + (i - 1)d)$ mangos. GS 4.48 is cited in PM A9 and a similar rule for a troop of monkeys occurs in PM A31. GS 4.49 gives an example, where $m = 8$, $p = 20$, $a = 4$, and $d = 6$.

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