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The Turyayantraprakāśa of Bhūdhara: Chapters One to Ten

SaKHYa*

I Introduction

I.0.1 The quadrant (Sanskrit: *turīya-yantra*, *turya-yantra*, *turyagola-yantra*) is a graduated quarter circle with which the altitude of a heavenly body can be measured. The sine quadrant carries, in addition, a series of lines running parallel to one or both the radii. These parallel lines allow us to convert the angle of altitude into the corresponding sine and cosine and to solve trigonometric problems graphically. It is difficult to say when and where the simple quadrant was invented. In his *Almagest*, Ptolemy describes a simple quadrant in connection with what came to be known in later times as the 'Mural Quadrant' which is set up on the north-south line and is used to measure the latitude of the locality and the obliquity of the ecliptic.[1]

The quadrant is mentioned for the first time in India by Brahmagupta in his *Brāhmasphuṭasiddhānta* (AD 628) where he describes the construction and use of several astronomical instruments.[2] Of these, the *cakra* (circle), *dhanus* (semi-circle) and *turyagola* (quadrant) are closely related in shape and function. *Cakra* is a circular wooden plate with its circumference graduated into 360 degrees, *dhanus* is its half, and *turyagola* the quarter. In all the three, a perforation is made at the center into which a peg is inserted like an axis, and also a plumb-line is suspended from the center. These instruments are so held towards the sun that the axis throws a shadow on the circumference. Then the arc intercepted between the nadir (indicated

*SaKHYa, which means 'friendship' in Sanskrit, is an acronym that refers to the study group consisting of Sreeramula Rajeswara Sarma, Takanori Kusuba, Takao Hayashi, and Michio Yano. The present paper was prepared in the weekly meetings held at Kyoto University from October 2013 to March 2014, while Sarma was staying in Kyoto as a visiting professor of the university. Setsuro Ikeyama and Lv Peng also actively attended the meetings.

[1] Cf. G. J. Toomer, *Ptolemy's Almagest* (translated and annotated), New York 1984, Book I, Chapter 12, pp. 62-63; see also Fig. D on p. 62.

[2] Cf. Sreeramula Rajeswara Sarma, 'Astronomical Instruments in Brahmagupta's Brāhmasphuṭasiddhānta,' *Indian Historical Review*, 13 (1986-87), 63-74; reprinted in pp. 47-63 of Sreeramula Rajeswara Sarma, *The Archaic and The Exotic: Studies in the History of Indian Astronomical Instruments*, New Delhi: Manohar, 2008. See also Yukio Ôhashi, 'Astronomical Instruments in Classical Siddhāntas,' *Indian Journal of History of Science* 29(2), 1994, 155-313.

by the plumb-line) and the shadow is the zenith-distance. Brahmagupta prefers the semi-circular variety, for he explains all functions in connection with *dhanus* and adds that the same can be done with *turyagola*:

> The *Turya-golaka-yantra* is [constituted by] a half of the *Dhanur*[-*yantra*]. It is marked with ninety degrees; as in the *Dhanur*[-*yantra*], here also [can be determined the time in] *ghaṭikās*, the degrees of the zenith-distance (*natāṃśa*), the degrees of altitude (*un-natāṃśa*), the distance between two planets (*grahāntara*) and other [parameters].[1]

Brahmagupta's successors show a marked preference for the *cakra*, presumably because it has the ideal shape; the quadrant is not mentioned by Lalla (8th century) in his *Śiṣyadhīvṛddhidatantra* or by Śrīpati in his *Siddhāntaśekhara* of 1039, while the *dhanus* is ignored by Bhāskara II in his *Siddhāntaśiromaṇi* of 1150.

I.0.2 The sine quadrant (Arabic: *rub^c al-mujayyab*) was developed in Baghdad in the ninth century. François Charette and Petra G. Schmidl convincingly argue that the earliest Arabic text bearing the title 'The construction of a quadrant with which the sine, the declination and the hours of daylight that elapsed can be determined' was authored by the famous al-Khwārizmī himself.[2] In view of its advantages, the sine quadrant began to be incorporated on the back of the astrolabes.

Along with the astrolabe, the sine quadrant appears to have been transmitted to India from the Islamic world sometime in the early medieval period. While the astrolabe, the Islamic instrument par excellence, was received as the *yantrarāja*, 'king of astronomical instruments,' the sine quadrant, because of its usefulness in solving trigonometric problems graphically, was also absorbed into the repertoire of Indian astronomical instruments.[3]

I.0.3 In India Padmanābha is the first to describe the sine quadrant in his *Dhruva-bhramaṇādhikāra* of 1423. On the reverse side of the *Dhurvabhrama-yantra*, which he designed for night time measurements, he incorporated the sine quadrant for

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[1] *Brāhmasphuṭasiddhānta* 22.17:

अङ्कितमंशनवत्या धनुषोऽर्धं तुर्यगोलकं यन्त्रम् ।
घटिकानतोन्नतांशग्रहान्तराद्यं धनुर्वदिह ॥17॥

[2] Cf. François Charette & Petra G. Schmidl, 'al-Khwārizmī and Practical Astronomy in Ninth-Century Baghdad. The Earliest Extant Corpus of Texts in Arabic on the Astrolabe and other Portable Instruments,' *SCIAMVS* 5 (2004), 101-98, esp. 154-55 and 179-81.

[3] For beautiful photos of specimens of the *yantrarāja* and the quadrant, see respectively pp. 198-201 and pp. 204-08 in: David Pingree, *Eastern Astrolabes*, Historic scientific instruments of the Adler Planetarium & Astronomy Museum, vol. 2, Chicago: Adler Planetarium & Astronomy Museum, 2009.

daytime observations.[1]

Subsequently the sine quadrant was discussed by several writers, such as Jñānarāja in his *Siddhāntasundara* of 1503, Kamalākara in his *Siddhāntatattvaviveka* of 1568, Cakradhara in his *Yantracintāmaṇi* composed before 1621, Bhūdhara in his *Turyayantraprakāśa* of ca. 1572 and Nandarāma in his *Yantrasāra* of 1771. Of these, Cakradhara's *Yantracintāmaṇi* and Bhūdhara's *Turyayantraprakāśa* are exclusively devoted to the sine quadrant.

Cakradhara, son of Vāmana, composed the *Yantracintāmaṇi* together with a commentary called *Vivaraṇa*. He does not mention the date of his composition, but the work is referred to by name and cited by Nṛsiṃha Daivajña of Kāśī in 1621 in his commentary on Bhāskarācārya's *Siddhāntaśiromaṇi*.[2] Therefore, the *Yantracintāmaṇi* must have been written sometime between 1423 and 1621, probably in the sixteenth century. Cakradhara coined a special name *Yantracintāmaṇi* (lit. 'wishing gem of an instrument') for the sine quadrant, but this name did not catch on as Mahendra Sūri's name *Yantrarāja* for the astrolabe did. Like the ordinary quadrant, the sine quadrant also continued to be referred to as *turīya-yantra* or *turya-yantra*.

The *Yantracintāmaṇi* is a slender text of just 26 verses. Cakradhara asserts that his work, though small, carries much substance:

> I shall teach an instrument which does not require mathematical calculations (*gaṇitānapekṣya*), which can quickly determine the time and other elements (*samayādikānām āśuprabodhaṃ*) and which makes use of principles that have never been used before (*apūrvayukti*). [My work], though small (*alpa*), carries much substance (*analpakārtha*), [is composed in] fine verses and dispels the darkness of ignorance. Those who understand [the use of] this instrument in all the details will understand the whole range of the best mathematical astronomy.[3]

The *Yantracintāmaṇi* enjoyed great popularity. Besides the author's own *Vivaraṇa*, two other commentaries are extant: *Yantradīpikā* by Rāma Daivajña (ca.

[1] Cf. Sreeramula Rajeswara Sarma, 'The Dhruvabhrama-Yantra of Padmanābha,' *Saṃskṛtavimarśaḥ, Journal of Rashtriya Sanskrit Sansthan, World Sanskrit Conference Special*, 6 (2012), 321-43. See also Yukio Ôhashi, 'Early History of the Astrolabe in India,' *Indian Journal of History of Science* 32(3), 1997, 199-295, which contains a critical edition, translation and commentary of Padmanābha's *Yantrarājādhikāra* on the southern astrolabe.

[2] Cf. Bhāskarācārya, *Siddhāntaśiromaṇi*, with his own *Vāsanā* and the *Vārttika* by Nṛsiṃha Daivajña, ed. Murali Dhara Caturveda, Sampurnanand Sanskrit University, Varanasi 1981, p. 456.

[3] *Yantracintāmaṇi* 1cd-2:

यन्त्रं प्रवक्ष्ये गणितानपेक्ष्यमाशुप्रबोधं समयादिकानाम् ॥ 1cd॥
अपूर्वयुक्त्यल्पमनल्पकार्थं सद्वृत्तमज्ञानतमोऽपहारि ।
विदन्ति ये यन्त्रमिदं सभेदं पश्यन्ति तेऽग्र्यं गणितं समस्तम् ॥ 2॥

1625) and *Turyayantropapatti* or *Yantracintāmaṇi-sūtrāṇām upapatti* by Dādābhāī (ca. 1719). Moreover, some 90 manuscript copies of this work are known to exist.[1]

I.1.1 Bhūdhara also composed an exclusive text on the sine quadrant with the title *Turyayantraprakāśa*.[2] Besides this work, three commentaries by him are known, viz., the *Sūryasiddhāntavivaraṇa* on the *Sūryasiddhānta*, *Mañjarī* on the *Svarodaya* of unknown authorship and *Sodāharaṇalaghumānasa* on the *Laghumānasa* of Muñjāla.[3]

At the beginning of the *Turyayantraprakāśa* and also at the beginning of the *Sūryasiddhāntavivaraṇa*, Bhūdhara informs us that he is a resident of Kāmpilya on the banks of the river Ganga. This city, known from the times of the *Mahābhārata*, survives as the modern Kampil (27° 37′ 12″ N; 79° 16′ 48″ E), a small town on the banks of the Ganga in the state of Uttar Pradesh. It is also sacred to the Jainas as the birth place of Vimalanātha, the thirteenth Tīrthaṅkara.

Bhūdhara belonged to the Bhāradvāja-gotra. His grandfather's name is variously mentioned as Khemaśarman or Somaśarman.[4] His father Devadatta was said to have been honoured by King Jalāl al-Dīn Akbar (r. 1556-1605).

I.1.2 Bhūdhara's commentary on the *Sūryasiddhānta* was composed in 1572. But no date is mentioned in the *Turyayantraprakāśa*. In this work, he mentions that his father Devadatta was honoured by Akbar. But this fact is not mentioned in his commentary on the *Sūryasiddhānta*. Therefore, it is likely that Devadatta was honoured by Akbar sometime after 1572 and that the *Turyayantraprakāśa* was composed thereafter. The *Sūryasiddhāntavivaraṇa* commences as follows:[5]

> काम्पिल्ये सुरसिन्धुबन्धुरतटे ज्योतिर्विदामग्रणीर्
> भारद्वाजकुलेऽमले समभवत् श्रीसोमशर्माह्वयः ।
> तत्पुत्रो नृपवृन्दवन्दितपदः श्रीदेवदत्ताभिधः
> कीर्त्या निर्मलयोज्ज्वलाः समतनोद्यः पङ्क्तिसंख्या दिशः ॥ २॥
> भूधरः तत्सुतः सूर्यसिद्धान्तं विवृणोम्यहम् ।

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[1]David Pingree, *Census of the Exact Sciences in Sanskrit*, Philadelphia 1970ff [henceforth CESS], 3, 36-37; 4, 88; 5, 103-04.

[2]Ibid., 4, 331-32.

[3]Ibid., 5, 265, mentions a *Bhūdharasāriṇī* by Bhūdhara; Śaṃkara Bālakṛṣṇa Dīkṣita, *Bhāratīya Jyotiṣa*, Hindi translation by Śivanātha Jhārakhaṇḍī, second edition, Lucknow 1963, p. 625, refers to a commentary by a Bhūdhara on the *Narapatijayacaryā*. It cannot be ascertained whether these Bhūdharas are identical with the Bhūdhara of Kāmpilya.

[4]In the two manuscript copies of the *Turyayantraprakāśa*, the name is given as Khemaśarman, so also in a manuscript of the commentary on the *Sūryasiddhānta* which was consulted by K. S. Shukla. However, in another manuscript of the same work, the name occurs as Somaśarman; cf. CESS 4, 332. Khemaśarman is a vernacular form of the Sanskrit Kṣemaśarman.

[5]Cited in CESS 4, 332.

गुरूणां पादयुगलकमलभ्रमरायितः ॥ ३ ॥[1]

In Kāmpilya, [situated] on the beautiful/curved banks of the divine river [Gaṅgā], in the pure Bhāradvāja clan [was born] the illustrious Somaśarman who was the foremost among the astronomers/astrologers. His son Devadatta, whose feet were worshipped by hosts of kings, brightened the ten directions with his spotless fame. I, his son Bhūdhara, elucidate the *Sūryasiddhānta*, having become a bee at the pair of lotus feet of the teachers/elders.

I.1.3 The commentary on the *Laghumānasa* of Muñjāla (or Mañjula) is preserved in a unique manuscript in the Sampurnanand Sanskrit University at Varanasi (serial no. 36944 and accession no. 2970).[2] This manuscript is incomplete and breaks off after verse 27 of chapter 3 (Tripraśnādhikāra). The name of the commentator is not available in the manuscript. But from the contents of the commentary, K. S. Shukla concludes that it must be Bhūdhara, because all the calculations in chapter 3 are made for the city of Kāmpilya and because the calculation of *ahargaṇa* in chapter 1 is made for 'Wednesday noon, the 15th *tithi* in the light fortnight of Āṣāḍha in Śaka 1494 (corresponding to June 25, AD 1572)' just as Bhūdhara's commentary on the *Sūryasiddhānta* calculates the *ahargaṇa* 'for the 15th *tithi* of the light fortnight of Āṣāha, Śaka 1494.' Therefore, this commentary on the *Laghumānasa* also must have been composed at about the same time as the commentary on the *Sūryasiddhānta*, i.e., ca. 1572.

I.1.4 Bhūdhara's *Turyayantraprakāśa* (Light on the [Sine] Quadrant) is rather a large work, consisting of 265 verses distributed into 21 short chapters of uneven length. The text is available in the following two manuscripts:

- V: Varanasi, Sampurnanand Sanskrit University, No. 35097.
- B: Baroda, Oriental Institute, Baroda, No. 12828;[3] it is incomplete, breaks off at 13.13.[4]

[1]In the *Mañjarī*, verse 2 is the same as verse 2 in the *Sūryasiddhāntavivaraṇa*, but verse 3 is slightly altered:

गुरूणां पादयुगलकमलभ्रमरायितः ।
भूधरः [sic!] तत्सुतः श्रीमान्यथाज्ञानं तनोत्यदः ॥ ३ ॥

Cf. CESS 4, 332.

[2]Cf. K. S. Shukla, *A Critical Edition of the Laghumānasa of Mañjula*, Indian National Science Academy, New Delhi 1990, pp. 38-40. This commentary is not noticed in the CESS.

[3]In the catalogue it is wrongly attributed to Bhāskara Daivajña, son of Devadatta Daivajña.

[4]In this manuscript, the verses of the first four chapters are numbered continuously from 1 to 69, but in the remaining chapters, consecutive numbers are given to the verses separately in each chapter.

The manuscripts are not dated, but probably belong to the nineteenth century. They appear to have been copied from the same or similar defective source and exhibit the same lacunae. Lacunae are noticeable, in particular, in chapter 1 which ends in a half verse. In 1.4, the word for 'son' is missing. In 1.7-14ab, where the construction of the sine quadrant is explained and the technical terms for the different parts of the instruments are introduced, the definition of the term *mṛgāsya*, which occurs very often later, is missing. Moreover, the titles of chapters are not transmitted completely in the colophons; sometimes only the topic of the chapter is mentioned together with the serial number of the chapter (*ity akṣāṃśavicāraḥ saptamaḥ*), sometimes only the serial number of the chapter followed by '*adhyāya*' (*iti ṣaṣṭho 'dhyāyaḥ*), and some other times just the serial number (*iti caturthaḥ*).

Contents of the *Turyayantraprakāśa*

Chap.	Content in Sanskrit	Vv.	Content in English
1	Yantraracanā	17	Construction of the instrument
2	Unnatāṃśavedhavicāra	2	Measuring the altitude
3	[Dhanurjyāśaravicāra]	35	[Arc, chord and arrow]
4	[Madhyonnatāṃśavicāra]	13	[Meridian altitude of the sun]
5	Krāntivicāra	9	Declination
6	[Arkāṃśavicāra]	9	[Solar longitude]
7	Akṣāṃśavicāra	10	Terrestrial latitude
8	Chāyāvicāra	17	Shadow of the gnomon
9	Sūryonnatāṃśavicāra	5	Altitude of the sun
10	Divasarātrivicāra	6	Length of the day and the night
11	Madhyāhnāvadhyava-śiṣṭadinavṛttavicāra	31	Diurnal circle remaining up to midday
12	Unnatāṃśavicāra	7	Altitude
13	Unnatāṃaśajñāna	9	Knowledge of the altitude
14	[Digaṃśajñāna]	18	[Knowledge of the azimuth]
15	Digjyājñāna	12	Azimuth cosine
16	Horādijñāna	7	*Horā* and other matters
17	Sandhyākālajñāna	5	Time of the twilight
18	Tattaddiksthadeśajñāna	15	Localities situated in different directions
19	Lagnamānajñāna	14	Measure of the ascendant
20	Lagnaprakāra	12	Method of the ascendant
21	Parvatādyunnatijñāna	12	Heights of mountains and others

Besides the lacunae, Bhūdhara's use of some technical terms is rather unusual: he employs *kramajyā* in the sense of horizontal parallel as against the conventional meaning of 'sine', *utkramajyā* in the sense of 'vertical parallel' as against 'versed sine', *ghaṭī* and *pala* in the sense of the degree and minute of arc respectively whereas traditionally these terms denoted the well-known sexagesimal units of time. Attention has been drawn to these peculiar usages at the appropriate places in our commentary.

II Text and Translation

.... Conventions and signs ....

- Following manuscript V, we give a set of consecutive numbers to the verses separately in each chapter but supply the chapter number before each verse number for easy reference.
- The symbol ∀ in the text indicates the beginning of a new page of the manuscripts; the manuscript and the page are specified in the margin.
- The abbreviation 'Em' in the apparatus means our suggestion of emendation without a support of manuscripts.
- A double slash with a subscript, $//_n$, in the translation indicates the end of the translation of verse n. It should however be noted that a sentence sometimes continues in the next verse and, due to the syntactical difference between Sanskrit and English, we were not always successful in strictly separating the translations of the two consecutive verses.
- A pair of brackets, [ ], indicates the word(s) added to the translation to complete the syntax of the sentence; a pair of parentheses, ( ), encloses either the original Sanskrit word(s) or our explanation of the immediately preceding word(s).

....

भूधरविरचितः तुर्ययन्त्रप्रकाशः

∀श्रीगणेशाय नमः ।[1] V1b, B1b

Salutation to the auspicious Gaṇeśa!

II.1 Chapter One: Construction of the instrument[2]

अस्ति शम्भोः पदं भव्यं काम्पिल्यपुटभेदनम् ।
उल्लसज्जाह्नवीलोलकल्लोलैः पावनीकृतम् ॥१.१॥

[1] B: ओं स्वस्ति सिद्धं ॥ ओं श्रीगणेशाय नमः ओं नमो भगवते वासुदेवाय नमः ॥ ओं

[2] Chapters 1-3 are composed in *Anuṣṭubh* meter; chapters 4-10 in *Āryā* and its variants *Gīti*, *Upagīti*, etc.

तत्रागणेयगुणिनामग्रणीर्गणकोत्तमः ।[1]
देवदत्त इति ख्यातः खेमशर्मात्मजोऽभवत् ॥१.२॥
ज्योतिर्विद्यानुभावेन येन जल्लालदीनृपः ।
वशवर्तीकृतस्तादृक् चक्रवर्ती महीभुजाम् ॥१.३॥[2]
तस्य भूधरनामेति यशोलङ्घितभूधरः ।
आसीदशेषदैवज्ञशेखराकल्पवेषभाक् ॥१.४॥

There is a city [called] Kāmpilya, the auspicious abode of Śiva, which is sanctified by the tossing waves of the river Gaṅgā.//$_1$ There lived the son of Khemaśarman, who was known [by the name] Devadatta, the foremost among those with countless virtues and an excellent astronomer,//$_2$ who, through the power of [his knowledge of] the astral science, captivated [the heart of] such an overlord of the [vassal] kings as the King Jalāl al-Dīn [Akbar].//$_3$ He had [a son [3]] by name Bhūdhara, who by his fame crossed the mountains (whose fame reached far and wide) and who possessed an appearance which is like an ornament on the diadem of all astronomers.//$_4$

परेण भास्करेणेव नत्वा भास्करमीश्वरम् ।
तुर्ययन्त्रप्रकाशोऽयं तेन तावत्प्रकाश्यते ॥१.५॥
योऽयं ज्योतिर्विदाचार्यैरप्राप्तसरणि$^\forall$क्रमः । B2a
सोऽयमुत्कण्ठितैः कण्ठाभरणीक्रियतां बुधैः ॥१.६॥

After paying obeisance to the sun and to Śiva, he (i.e. Bhūdhara) brings to light the *Turya-yantra-prakāśa*, as if he were another sun himself.//$_5$ This procedure (*saraṇi-krama*) [of instrumentation] which was unknown to [previous] masters of astral science may now be welcomed (lit. may be made the neck ornament) by eager scholars.//$_6$

एकेन धनुषा व्यासद्वयेन परिवेष्टितः ।
पूर्णवृत्तचतुर्थांशस्तुर्य$^\forall$यन्त्र इतीर्यते ॥१.७॥ V2a
सूर्योन्नतांशविज्ञानं धनुषस्तत्र जायते ।[4]
स्थापिते धनुषः पृष्ठे स्वसन्मुखमथाटनिः ॥१.८॥[5]
अपसव्यकरे लग्ना कार्मुकस्यादिरुच्यते ।

[1]B: र्गणकौत्तमः

[2]B: वशवर्त्ता B: महि°

[3]Strangely the word for son is missing in this verse. The long attribute in the second half is also very strange.

[4]B: सूर्यौन्नतां°

[5]B: पृष्टे

सव्यहस्ताग्रसंसक्ता तस्यान्तः प्रोच्यते बुधैः ॥१.९॥[1]
कार्मुकस्यादिमारभ्य रेखा पश्चिमपूर्वयोः ।
अन्तमारभ्य रेखा तु दक्षिणोत्तरयोर्मता ॥१.१०॥
केन्द्रे रेखाद्वयस्यादिर्विज्ञेयोऽन्तश्च कार्मुके ।[2]
धनुरन्तमथारभ्य धनुराद्यावधि क्रमात् ॥१.११॥
समा नवतिभागाः स्युः पञ्च षड् वा तदन्तरे ।[3]
या रेखाः सन्ति तासामप्युपयो∀गोऽत्र विद्यते ॥१.१२॥ B2b
पूर्वपश्चिमरेखातः षष्टिरेखाः समाचरेत् ।
एवमेव क्रमज्याश्च विदध्यात्षष्टिसंख्यया ॥१.१३॥
दक्षिणोत्तररेखातो यन्त्रज्ञः कार्मुकावधि ।[4]

The quarter of a full circle which is encompassed by one arc (*dhanus*) and two [half] diameters (*vyāsa*) is called the quadrant (*turya-yantra*). $//_7$ There one obtains the knowledge of the sun's altitude (*unnatāṃśa*) from the arc. When the back (i.e., surface) of the arc is placed [horizontally] in front of oneself, the extremity of the arc$//_8$ which is held in the right hand is called the beginning of the arc; that which is held by the left hand is called its end by the learned.$//_9$ The line from the beginning of the arc [up to the center] is regarded as the west-east line and the line from the end of the arc [up to the center] the south-north line.$//_{10}$ It should be known that both the lines have their starting point at the center (*kendra*) [of the quadrant] and their termination at the arc. Now, starting from the end of the arc up to its beginning,$//_{11}$ there should be ninety equal divisions. Those lines [of division] at intervals of five or six [divisions, i.e., degrees] have their own use.$//_{12}$ From the east-west line, he should draw sixty [vertical parallel] lines. Likewise, he should draw sixty horizontal [parallel] lines (*krama-jyā*, lit. 'regular chords')$//_{13}$ from the south-north line up to the arc, he who knows the instruments (*yantrajña*).

केन्द्रसंनिहिते छिद्रे सूत्रं सूक्ष्मतरं दृढम् ॥१.१४॥
स्थापयित्वाथ त∀स्यान्ते बध्नीयाद्गुरुगोलकम् । V2b
यन्त्राद्बहिः प्रभां वेद्धुं कुर्यात्कूटद्वयं सुधीः ॥१.१५॥
एकं केन्द्रसमीपस्थं धनुःसन्निहितं परम् ।
कूटद्वयमपि च्छिद्रान्वितं कुर्वन्ति केचन ॥१.१६॥[5]
केन्द्रासन्ने तथैकस्मिन्विवरं कुर्वते परे ।
तत्र नक्षत्रवेधं तु केचिच्छिद्रे तु कुर्वते ॥१.१७॥
संस्थापयन्ति नलिकां केचित्कूटसमस्थले ॥१.१८॥

[1]B: तस्यांत

[2]B: केंद्र

[3]B: भागा

[4]B: दक्षिणेत्तर (*ra* in top margin)

[5]B: कुर्वीत

In the hole at the center, a thin strong string$//_{14}$ should be affixed and a heavy round weight (*gurugolaka*) should be tied to its end. The intelligent man should create, outside [perimeter of] the instrument, for observing the light [of a heavenly body], two sighting vanes (*kūṭa*, lit. projection, or projecting bit),$//_{15}$ one close to the center and another close to the arc. Some persons endow both the sighting vanes with holes.$//_{16}$ Some [others] bore a hole only in the [sighting vane] close to the center, and observe the stars through that hole [!].[1]$//_{17}$ Some [others] attach a sighting tube (*nalikā*) at the same place (at the same level) in the [two] sighting vanes (*kūṭa-sama-sthale*).[2]$//_{18}$

इति तुर्ययन्त्रप्रकाशे यन्त्ररचनाविचाराध्यायः ॥[3]

Thus the [first] chapter in the *Turyayantraprakāśa*, the deliberation on the construction of the instrument.

II.2 Chapter Two: Measuring the altitude

सूर्योन्नतांश$^{\forall}$वेधाय धृत्वा यन्त्रं करद्वये । B3a
सूर्याभिमुखमाधाय कूटं केन्द्रसमीपगम् ॥ २.१॥
तथा निजभुजस्याग्रं चालयेद्गणकाग्रणीः ।
यथा छिद्रद्वयस्यान्तः पतेत्प्राभाकरी प्रभा ॥ २.२॥
एवं यन्त्रे पतेत्सूत्रं तत्र चिह्नं समाचरेत् ।[4]
चिह्नावधि धनुःप्रान्तादंशाः सूर्योन्नतांशकाः ॥ २.३॥[5]

For observing the sun's altitude degrees, having held the instrument (*yantra*)[6] in both the hands in such a way that the sighting vane near the center is towards the sun,$//_{1}$ the foremost astronomer should move the tip of his arm in such a way that the sun's ray passes through both the holes.$//_{2}$ [Having done] so, he should make a mark on the point on the arc where the plumb-line falls. The degrees from the end [of the arc] up to that mark are the sun's altitude degrees.$//_{3}$

इति तुर्ययन्त्रप्रकाशे सूर्योन्नतांशवेधविचाराध्यायो द्वितीयः ॥[7]

Thus the second chapter in the *Turyayantraprakāśa*, the deliberation on the measurement of the sun's altitude in degrees.

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[1] It is absurd: something is wrong with the text.

[2] Here also the text is not properly transmitted; the chapter ends in the middle of the verse.

[3] V: रचनाध्यायः

[4] Em: यत्र

[5] B: °दंशा

[6] Throughout the text Bhūdhara refers to the sine quadrant as *yantra*.

[7] V: इति उन्नतांशविधविचारः ॥ २॥ B: विवारा B treats this line as ॥ २२॥

II.3 Chapter Three: Arc, chord and arrow

∀यावन्तः कार्मुकस्यांशा भवेयुर्धनुरादितः । V3a
तावदंशोत्थरेखा या दक्षिणोत्तररेखया ॥ ३.१॥
मिलिता सैव तस्यैव धनुषो ज्या बुधैर्मता ।[1]
एवं धनुषि विज्ञाते ज्याज्ञानं विदुषां भवेत् ॥ ३.२॥

From the beginning of the arc [up to a given point], as many degrees of arc there are, the line that arises from so many degrees and is merged with (i.e., mapped or projected onto) the south-north line//$_1$ is considered by the learned as the chord (i.e., Rsine) of that arc. In this manner, when [the measure of] the arc is known, the scholar will know the corresponding chord [graphically].//$_2$

यद्योजितं यत्साशीतिशतभागमि∀तं भवेत् । B3b
तत्तदा दीर्घधनुषो ज्या स्यात्तल्लघुकार्मुकम् ॥ ३.३॥[2]

When a certain [arc], united with another [arc], measures one hundred and eighty degrees, then the Rsine of the greater arc will be [obtained] from that smaller arc.//$_3$

दक्षिणोत्तररेखाया यावन्तः केन्द्रतोऽंशकाः ।[3]
तावदंशोत्थरेखा ज्या यत्र चापेन संगता ॥ ३.४॥
तत्र चापादितो भागा ये स्युस्तावन्मितं धनुः ।[4]
तस्य ज्या सा भवेदेवं ज्याज्ञाने ज्ञायते धनुः ॥ ३.५॥[5]

Where the Rsine, which is the line that arises from so many degrees as those of [a certain portion of] the south-north line from the center, meets the arc,//$_4$ from the beginning of the arc, as many degrees [of arc] there are [up to that point], by so many [degrees] the arc is measured; the Rsine of that [arc] shall be that [Rsine given]. Thus when the Rsine is known, the [corresponding] arc can be known [graphically].//$_5$

इत्थं नवत्यंशधनुर्ज्या षष्ठ्यंशमिता भवेत् ।[6]
न्यूनाधिकत्वे जानीयात्कृत्वा त्रैराशिकं बुधः ॥ ३.६॥[7]

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[1] V: ज्यो

[2] Em: तल्लघुकार्मुकात्

[3] B: यावंत

[4] B: वापा°

[5] B: भवेद्देवं

[6] B: इस्यं

[7] B: °त्कृत्पा

In this manner, the Rsine of the arc of ninety degrees will be measured by sixty degrees (or parts) [on the south-north line]. When [the arc is] less or more [than any one of the graduations on the rim], the wise should know [the Rsine not graphically but] by calculating with the Rule of Three (i.e., by linear interpolation).//$_6$

वृत्ते तुर्यांश एव ज्या विज्ञाता यन्त्रकोविदैः ।
वृत्तार्धे पूर्णवृत्ते च स्फुटं जीवा न विद्यते ॥ ३.७॥[1]
साशीतिशतभागोनं नवत्यंशाधिकं धनुः ।[2]
∀पात्यं साशीतिशतके यच्छिष्टं कार्मुकं भवेत् ॥ ३.८॥[3] V3b
खाष्टैकांशा १८० ∀धिकं चापं खाद्रिनेत्रेषु २७० पातयेत् ।[4] B4a
ततोऽधिकं खषड्वह्नि ३६० मध्ये शोध्यं धनुर्भवेत् ॥ ३.९॥[5]
उपयोगोऽत्र तस्यापि तुर्ययन्त्रे निरुच्यते ।

The Rsine (*jīvā*) of only the quarter of a circle is known to the experts on the instruments. For a semi-circle or full circle, there is no apparent (*sphuṭā*) Rsine (*jīvā*).//$_7$ The arc less than one hundred and eighty degrees and greater than ninety degrees should be subtracted (*pātyaṃ*) from one hundred and eighty.[6] The remainder is [treated as the degrees of the actual] arc.//$_8$ The arc greater than 'sky-eight-one', i.e., 180 degrees should be subtracted from 'sky-mountains-eyes', i.e., 270. The [arc] still greater than that (270) should be subtracted from 'sky-six-fires', i.e., 360. Let [the remainder] be [treated as the actual] arc.//$_9$ Its application here in the sine quadrant will be explained [below].

आदिमारभ्य धनुषो ये भागाः स्युस्तदंशजा ॥ ३.१०॥[7]
रेखा यन्त्रगता पूर्वापररेखां समाश्लिषन् ।[8]
तत्र रेखान्ततो भागा ये स्युस्तावन्मितः शरः ॥ ३.११॥[9]
भवेत्तस्यैव धनुषो विद्यादेवं सुधीः शरम् ।

Starting from the beginning of the arc, as many degrees as there are [up to a point],//$_{10}$ the [vertical] line (*rekhā*) arising from there on the instrument (*yantra-*

[1]Em: स्फुटा जीवा

[2]V: ०भागोन B: ०शादिकं

[3]B: पात्यांशाशीति B: भवैत्

[4]V: धिकं सापं षद्रि (original *khā* is changed to *ṣa*) B: धिकं वा खाद्रि० B: नेत्रे १७० षु

[5]B: वह्निमध्ये

[6]The author uses the locative case for the minuend throughout this work.

[7]B: धनुष्पो B: भागा B: ०शजाः

[8]B: यत्र B: ०गताः B: ०खाः Em: समाश्लिषेत्

[9]VB: भागाः B: स्युःस्ता०

gatā) touches the east-west line (*pūrvāpara-rekhā*); the units from the end of the [east-west] line up to there, will be the measure of the versed sine (*śara*, lit. 'arrow')//$_{11}$ of the very same arc. Thus the intelligent man should know the versed sine.

धनुर्नवतिभागेभ्योऽधिकं यदि भवेत्तदा ॥ ३.१२ ॥[1]
नवत्यधिकसंख्यायां नवतिं पातयेदथ ।[2]
शेषस्य धनुषो जीवा कर्तव्या सा च षष्टियुक् ॥ ३.१३ ॥[3]
नवत्यंशाधिकस्यैव कार्मुकस्य शरो भवेत् ।

If the arc is greater than ninety degrees, then//$_{12}$ ninety should be subtracted from that number which is greater than ninety; for the remaining [part of the] arc, find out the [half-]chord (*jīvā*). That increased by sixty//$_{13}$ will be the arrow (*śara*) of the arc which is greater than ninety degrees.

यावदंशैः परिमितः शरः स्यात्तावतोऽंशकान् ॥ ३.१४ ॥
पूर्वपश्चिमरेखान्ताद्गणयेत्तु तदंशजा ।
उत्क्र∀मज्या धनुर्यत्र यावदंशमितं स्पृशेत् ॥ ३.१५ ॥ B4b
∀तत्रादिमधनुष्कोटेर्गणितैस्तावदंशकैः । V4a
तस्य बाणस्य तच्चापं भवेदिति विनिश्चयः ॥ ३.१६ ॥

As many units is the measure of the [given] arrow (*śara*), so many units//$_{14}$ should be counted off from the end of the east-west line [on the quadrant]. The vertical line (*utkrama-jyā*) that arises from those units touches an arc measured by a certain number of degrees,//$_{15}$ by so many degrees counted from the extremity (beginning) of the original arc (i.e., arc of the sine quadrant) up to that point, the arc of that arrow will be obtained. This is the rule.//$_{16}$

यदा भवेत्षष्टिभागाधिको बाणस्तदा त्यजेत् ।[4]
षष्टिं चाधिकसंख्यायां यच्छेषं तु बभूव तत् ॥ ३.१७ ॥[5]
क्रमज्यां कल्पयेन्नूनं सा क्रमज्या परामृशेत् ।
दक्षिणोत्तररेखातो यावदंशमितं धनुः ॥ ३.१८ ॥[6]

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[1] B: धनुर्तड

[2] B: नवत्याधिक

[3] B: सा व

[4] B: त्यज्येत्

[5] B: वाधिक◦ V: यच्छेषं

[6] B: यावंदं◦

तावद्भिरंशकैरेव नवत्या संयुतैर्मितम् ।[1]
षष्टिभागाधिकस्येषोः कार्मुकं निश्चितं भवेत् ॥ ३.१९ ॥

When the [given] arrow is higher than sixty units, then sixty should be subtracted from the higher number; what has remained//$_{17}$ should be regarded as a horizontal line (*kramajyā*). Now, that horizontal line will touch an arc of as many degrees measured from the south-north line,//$_{18}$ by so many degrees, increased by ninety, will be determined the measure of the arc for the arrow greater than sixty units.//$_{19}$

यावद्भागं धनुस्तावदर्धज्या यावती भवेत् ।
तावती द्विगुणा तस्य धनुषो ज्यार्द्धमुच्यते ॥ ३.२० ॥[2]

The arc measures a certain number of degrees; the [corresponding] half-chord (*ardha-jyā*) has a certain measure; that much is said to be half of the chord of the doubled arc.//$_{20}$

यावन्तः कार्मुकस्यांशास्तावन्तो धनुरादितः ।[3]
गणयित्वाथ तत्रैव सू∀त्रं संस्थापयेदथ ॥ ३.२१ ॥[4] B5a
दक्षिणोत्तररेखायामाद्यन्ते यच्च संगतम् ।
वृत्तार्धं तस्य रेखायां योजयेच्चिह्नसूत्रकम् ॥ ३.२२ ॥
ततः सूत्रे धनुःप्रान्तस्थापिते चिह्नसूत्रकम् ।
दक्षिणोत्तररेखायां यावदंशोपरिस्थितम् ॥ ३.२३ ॥
तावदंशमिता मौर्वी धनुषस्तस्य जायते ।

As many degrees there are in a [given] arc, having counted so many degrees from the beginning of the arc [on the sine quadrant], hold the string at this point. Now,//$_{21}$ set the string with the mark, i.e., the cursor (*cihna-sūtra*) on the line of the semi-circle[5] that meets the south-north line at its beginning and end.//$_{22}$ Then, when the string is [rotated and] placed at the end of the arc, the string with the cursor is situated on as many units on the south-north line,//$_{23}$ a chord having so many units is obtained for that arc.

[1] V: नवत्यां

[2] Em: द्विगुणितस्य धनुषो

[3] B: ∘स्तावतो

[4] B: गणयित्वाप्य

[5] In other words, put the cursor (called *mṛgāsya*) at the intersection of the string and the semicircle.

ज्यासकाशादथ धनुर्ज्ञातुमिच्छेद्यदा तदा ॥ ३.२४ ॥
∀पूर्वोत्तरे तु रेखायां पूर्वं सूत्रं न्यसेदथ ।[1] V4b
यावदंशमिता मौर्वी तावदंशोपरि न्यसेत् ॥ ३.२५ ॥
चिह्नसूत्रं पुनर्वृत्तार्धरेखायां चिह्नसूत्रकम् ।
संयोजयेत्ततः सूत्रं यत्र चापं समाश्लिषेत् ॥ ३.२६ ॥[2]
तत्रादिमधनुःकोटेर्यावन्तो धनुरंशकाः ।[3]
तावदंशमितं तस्या जीवायाः कार्मुकं भवेत् ॥ ३.२७ ॥

Now, if one wishes to know the arc from the chord,//$_{24}$ he should first place the string upon the south-north line,[4] and as many degrees as the chord (*maurvī*) has, on so many degrees one should place//$_{25}$ the string with the cursor.[5] Again he should [rotate and] put the string with the cursor on the line of the semicircle. Then, the string will touch the arc at a point,//$_{26}$ to which place from the extremity of the original arc there are as many degrees of the arc, the arc for that [given] chord will be measured by so many degrees.//$_{27}$

याव∀द्भागं धनुस्तावद्भागान्बाणासनान्ततः ।[6] B5b
संख्याय तत्र सूत्रं च स्थापयित्वा मृगास्यकम् ॥ ३.२८ ॥[7]
वृत्तार्धरेखासंलग्नसूत्रस्थाने निवेशयेत् ।
ततः प्राचीनपाश्चात्यरेखोपरि गुणं न्यसेत् ॥ ३.२९ ॥[8]
यस्मिन्भागे मृगास्यं तल्लग्नं तदवधि क्रमात् ।[9]
रेखान्ताद्गणयेदेवं यावत्संख्याः स्युरंशकाः ॥ ३.३० ॥
जायते धनुषस्तस्य तावदंशमितः शरः ।[10]

As many degrees are the measure of a [given] arc, after having counted so many degrees from the end of the arc and placed the string there, the cursor (*mṛgāsyaka*)//$_{28}$ should be moved to the point of the string where it touches the line of the semicircle. From that point, one should [rotate and] put the string on the east-west line.//$_{29}$ Whichever degree the cursor (*mṛgāsya*) has touched, up to that point successively

[1] V: पूर्वसूत्रं

[2] B: समाश्लिखेत्, i.e. *kha* for *ṣa*.

[3] B: कौटेर्या॰

[4] Em: दक्षिणोत्तरे This emendation, however, does not resolve the mismatch of gender.

[5] In other words, one should fix the cursor on the string.

[6] V: ॰सनांततः B: ॰सनात्ततः

[7] B: संख्याय ते तत्र सूत्रं स्थापयित्वा

[8] B: प्राचीनपाचीनपश्चात्य॰

[9] B: यस्मिद्भागे

[10] V: जायंते

from the end of the [east-west] line, one should count the degrees. Thus as many degrees there are,$//_{30}$ the arrow (*śara*) of that arc will have a measure of so many degrees.

अथ प्राचीप्रतीचीनरेखायां योजयेद्गुणम् ॥ ३.३१॥
यावद्भागः शरस्तावद्भागावधि विचक्षणः ।
संख्याय रेखायाः प्रान्तान्मृगास्यं तत्र योजयेत् ॥ ३.३२॥[1]
अथ वृत्तार्धरेखायां यथा तिष्ठेन्मृगास्यकम् ।
तथा शरासने सूत्रं संस्थाप्य धनुषोऽन्ततः ॥ ३.३३॥
∀सूत्राधिष्ठितभागान्तं गणिते कार्मुकांशकाः ।[2] V5a,
यावन्तस्तावदंशं स्यात्तस्य बाणस्य तद्धनुः ॥ ३.३४॥[3] B6a

Now, one should join the string to the east-west line.$//_{31}$ As many units the arrow (*śara*) has, having counted up to so many degrees from the end of the [east-west] line, the learned should put the cursor (*mṛgāsya*) there.$//_{32}$ Now, one should stretch the string up to the arc in such a way that the cursor rests on the line of the semicircle; from the end of the arc$//_{33}$ up to the point occupied by the string, when counted, as many degrees of the arc there are, the arc for that arrow will have so many degrees.$//_{34}$

यावन्तः कार्मुकस्यांशा भवेयुस्तावतस्त्यजेत् ।
नवत्यामवशिष्टं तत्संपूर्णं कार्मुकं भवेत् ॥ ३.३५॥

As many degrees an arc may have, so many should be subtracted from ninety. The remaining is [the measure of] its complementary arc (*saṃpūrṇaṃ kārmukaṃ*, lit. complete arc, implying 'with this the arc of ninety degrees on the sine quadrant will be complete').$//_{35}$

इति तुर्ये तृतीयोऽध्यायः॥[4]

Thus the third chapter in the *Turya[-yantraprakāśa]*.

II.4 Chapter Four: Meridian altitude of the sun

मध्याह्नसमयसन्निधिमवगम्य करेण यन्त्रमादाय ।
बहुशो रविप्रभाया वेधं विदधीत वेधविधिबोद्धा ॥ ४.१॥[5]

[1] B: संख्यायायाष्प्रातान्मृ°

[2] B: °धिष्टित

[3] V: °वदंशस्य तस्य B: °वदंप्रास्य तस्य

[4] V om. इति B: तुर्यः

[5] B: प्रभायाः B: विधिवौद्धा

यस्माद्भागादधिको भागो भागो चरो न भवेत् ।[1]
सवितुः स एव मध्योन्नतभागो बुधवरैर्बोध्यः ॥ ४.२ ॥[2]

Having understood that it is close to the time of midday, one, who has understood the method of astronomical observation (*vedha-vidhi-boddhā*), should hold the instrument in his hand and make the observation of the sun's light many times.//$_1$ If no degree higher than a certain degree is [observed], and if [that] degree does not vary, then that same [degree] should be understood by the excellent scholars as the degree of the sun's meridian altitude (*madhyonnatabhāga*).//$_2$

पूर्वाभिमुखं यातुश्छाया चेद्वामतः पतति ।[3]
भानुर्दक्षिणदिक्स्थस्तदावगम्योऽन्यथोत्तराशास्थः ॥ ४.३ ॥
∀अथ चेच्छाया न स्यात्तदा शिरःस्थोऽवगन्तव्यः ।[4] B6b
उत्तरतो लङ्कया वेददृगक्षांशकोनदेशेषु ॥ ४.४ ॥
वेददृगक्षांशमिते देशे भास्वान्न वामतो भ्रमति ।[5]
तदधिकदेशेषु पुनर्न वामतो भ्रमति न च मूर्ध्नि ॥ ४.५ ॥[6]

If, for one who proceeds facing the east, the sun's shadow falls to the left, then it should be understood that the sun is in the southern direction, otherwise in the northern direction.//$_3$ If there is no shadow, then it should be understood that [the sun] is at the zenith (*śiraḥstha*) in the localities situated at latitudes lower than 'Vedas-eyes' (24) degrees to the north of equator (*laṅkā*).//$_4$ In the locality situated at the latitude of 'Vedas-eyes' (24) degrees, the sun does not move towards the left; in the localities with higher [latitudes] the sun neither moves towards the left nor is at the zenith.//$_5$

अक्षांशा यावन्तो यत्र स्युस्तावतो नवतिमध्ये ।[7]
त्यक्त्वावशिष्ट∀भागा लम्बांशपदाभिधेयतां प्राप्ताः ॥ ४.६ ॥ V5b
यावन्तः क्रान्त्यंशास्तावद्भिर्योजिताः कार्याः ।
यदि नवतिन्यूनाः स्युस्तदा त एवांशकास्तत्र ॥ ४.७ ॥[8]

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[1]B: °धिको भागो चरो

[2]B: सवितुस B: वुधरैर्वोध्यिः

[3]B: भास्करमभिप्रयातुःछायाचैद्वामतः

[4]VB: शिरस्थो

[5]B: वर्णदृ°

[6]B: तदाधिक

[7]V: यत्र स्युस्तावंतो B: यत्रः स्युस्तावतो

[8]B: न्यूना

मध्योन्नतभागाः स्युर्दक्षिणतश्चापि भास्करो ज्ञेयः ।[1]
यदि नवतिसंमिताः स्युर्लम्बांशाः क्रान्तिभागयुताः ॥ ४.८ ॥[2]
ज्ञेयास्त एव मध्योन्नतभागाः शिरसि ∀भास्करो ज्ञेयः ।[3] B7a
यदि नवतेरधिकाः स्युस्तदाभ्रवसुवसुमतीषु संत्यक्ताः ॥ ४.९ ॥[4]
मध्योन्नतांशकाः स्युस्तत्र च तीव्रांशुरुत्तराशास्थः ।[5]
उत्तरगोलविहर्तरि दिनभर्तरि विधिरयं बोध्यः ॥ ४.१० ॥

As many are the degrees of latitude at a certain place, so many are subtracted from ninety; the remaining degrees shall be designated by the term 'degrees of co-latitude' (*lambāṃśa*, lit. 'degrees of perpendicular').//$_6$ The [degrees of co-latitude] should be increased by so many as many are the degrees of declination [of the sun]. If [the sum is] less than ninety, then these degrees at that place//$_7$ are the degrees of the meridian altitude [of the sun]; it should also be known that the sun is in the south. If the degrees of co-latitude increased by the degrees of declination are measured by ninety,//$_8$ it should be known that the same [sum] is the degrees of the meridian altitude and that the sun is at the zenith. If these are more than ninety, then these are subtracted from 'sky-Vasu-earth' (180);//$_9$ [the remainder] will be the degrees of the meridian altitude and the sun will be in the northern direction. This must be understood as the [proper] procedure when the sun is moving in the northern [hemi-]sphere.//$_{10}$

अथ दक्षिणगोलस्थे सवितरि लङ्कापसव्यदेशेषु ।[6]
पूर्वाभिहितं सर्वं विज्ञेयं वैपरीत्येन ॥ ४.११ ॥[7]

If the sun is in the southern [hemi-]sphere, all that has been said previously should be known [here also but with the sun's position] reversed, in localities which are to the right (i.e., south) of the equator.//$_{11}$

अपि च क्रान्तेर्भागा निपातिता लम्बभागेषु ।[8]
मध्योन्नतांशकाः स्युः पूर्ववदन्यो विधिः सर्वः ॥ ४.१२ ॥

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[1] V: भागः B: भागा

[2] B: संमिता

[3] B: ज्ञेया

[4] B: तदाभ्रवसुमतीषु B: संत्यक्ता

[5] B: ◦शका B: ◦शुरुक्तरा◦

[6] B: दाक्षिण

[7] B: ◦भिहतं

[8] B: निपतिता

Also, [in two other cases], the degrees of declination are subtracted from the degrees of co-latitude; there will be the degrees of the meridian altitude. All the rest of the procedure is as before.$//_{12}$

लङ्कायामथ गोलद्वितये क्रान्त्यूननवतिपरिशेषम् ।
मध्योन्नतभागाः स्युर्लंबांशाः क्रान्त्यभावे तु ॥ ४.१३ ॥[1]

Now, on the equator, in both [hemi]-spheres, the remainder after subtracting the declination from ninety will be the degrees of the meridian altitude; if there is no declination, the degrees of co-latitude [themselves are the meridian altitude].$//_{13}$

इति चतुर्थः ॥[2]

Thus the fourth [chapter].

II.5 Chapter Five: Declination

त्रिंशत्त्रिंशद्भागैः शरासनादेः क्रमेण प∀रिकल्प्यम् ।[3] B7b
मेषादि∀त्रयमेवं कर्कादित्रितयमुत्क्रमशः ॥ ५.१ ॥ V6a
अथ पूर्ववत्तुलादित्रितयं परिकल्प्य तदनु मकराद्यम् ।
व्युत्क्रमशः परिकल्प्यं क्रान्त्यानयनोपयोगवशात् ॥ ५.२ ॥

From the beginning of the arc (*śarāsana*), at distances of thirty degrees each in regular order, the three [signs] starting from Aries should be arranged; likewise [at thirty degree intervals] the three [signs] starting from Cancer in the reverse order.$//_{1}$
Now, the three [signs] beginning with Libra should be arranged [in regular order] as before; thereafter the three [signs] beginning with Capricorn should be arranged in the reverse order, for the sake of [their] use in determining the declination (*krānty-ānayana*).$//_{2}$

उत्तरदक्षिणरेखान्यस्तगुणक्रान्तिवृत्तसंयोगे ।[4]
संस्थापयेन्मृगास्यं ततोऽर्कसंपर्कभाजि राश्यंशे ॥ ५.३ ॥
आरोपिते गुणे यां क्रमजीवामुल्लिखेन्मृगास्यं तत् ।
सा यत्र धनुषि लग्ना तद्भागावधि शरासनस्यादेः ॥ ५.४ ॥
गणयेद्यावद्भागांस्तावद्भागा भवेत्क्रान्तिः ।

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[1] B: भागा

[2] B: इति तुर्यचतुर्थो ध्य

[3] B: °कल्प्य

[4] B: संयोग

The cursor (*mṛgāsya*) should be placed at the intersection of the string that is stretched along the north-south line and the declination circle; next, place the string on the degree of the zodiacal sign which is occupied by the sun;$//_3$ the cursor scratches (i.e., intersects) whichever horizontal line (*krama-jīvā*); where that [horizontal line] touches the arc [at a point], up to that degree from the beginning of the arc,$//_4$ the degrees are to be counted; as many degrees there are, of so many degrees will be the declination.

पूर्वानीतक्रान्तेर्भागानादाय कार्मुकस्यादेः ॥ ५.५ ॥
चिह्नं च तत्र कृत्वा सूत्रं संयोजयेद्गणकः ।[1]
∀अथ पूर्वापररेखास्थपञ्चपञ्चाश ५५ दंशजा रेखा ॥ ५.६ ॥[2] B8a
सूत्रे यत्र विलग्ना तत्संस्पर्शिन्यनुक्रमज्या स्यात् ।[3]
यत्रैव धनुषि लग्ना तदवधि चापादितो गणितैः ॥ ५.७ ॥
अंशैर्मिता खलु भवेदसंशयं क्रान्तिरानीता ।[4]
सर्वत्र पञ्चपञ्ञाशदंशजरेखा बुधैर्ग्राह्या ॥ ५.८ ॥[5]

Having measured out the degrees of declination, which have been obtained previously, from the beginning of the arc$//_5$ and having made a mark there, the astronomer should join the string [to that mark]. Then, the [vertical] line that arises at fifty-five, i.e., 55, degrees on the east-west line$//_6$ touches the string at a point; there will be a horizontal line (*anukrama-jyā*) that touches (i.e., passes through) that point; that [line] touches the arc at a point; up to that [point] from the beginning of the arc,$//_7$ the degrees counted will no doubt measure the declination, which is [thus newly] obtained. The wise should always take the [vertical] line arising from fifty-five degrees [of the east-west line].$//_8$

पूर्वैः परमक्रान्तिर्वेददृगंशात्मिका लब्धा ।[6]
∀सार्द्रीन्दुपल १७ त्रिंशद्वल्य ३० धिकत्र्यक्षि २३ संमिताधुनिकैः॥ ५.९ ॥[7] V6b

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[1]B: सृत्रं B: संयोद्ग°

[2]B: °स्थपंचाशदंशजा

[3]B: तत्स्पर्शिन्युत्क्रमज्या

[4]V: क्रांतिसानीतु In V, in the left margin is written, almost in the same hand, *=yavanamatakrāntiḥ 2*, with the same mark '=' above *krāṃ* and *ti*. This remark ('the declination according to the Muslims') presumably refers to the value given in the next verse (5.9).

[5]B: °शरेखा B: ग्राह्य

[6]B: °दृगंशत्मिका सर्वत्र काल्ध्या (*sarvatra* crossed out)

[7]V: त्रिंशद्वटिव्य ३० B: त्रिंशत् घघ B: °क्षिरडसंमिताधुविकैः

The maximum declination that consists of 'Vedas-eyes' (24) degrees was obtained by the ancients (*pūrva*). [The same] by the moderns (*ādhunika*) is tantamount to 'three-eyes' (23) [degrees] increased by thirty minutes (*ghaṭī*) and 'mountains-moon' (17) seconds (*palas*).//$_9$

इति क्रान्तिविचारः पञ्चमः ।[1]

Thus the fifth [chapter], the deliberation on the declination.

II.6 Chapter Six: Solar longitude

मध्योन्नतभागेभ्यः क्रान्तिं विज्ञाय दैवज्ञः ।
धनुरादेरथ परमक्रान्तिं परिगृह्य विन्यसेत्सूत्रम् ॥ ६.१॥
क्रममौर्वी धनुरादेर्गृहीततात्कालि∀कक्रान्तेः ।[2] B8b
यत्राश्लिष्यति सूत्रं तत्र मृगास्यं नयेद्गणकः ॥ ६.२॥
उत्थाप्य दक्षिणोत्तररेखामधिरोहिते सूत्रे ।
स्पृशति मृगास्याधिष्ठितदेशोत्था यत्र कार्मुकं रेखा ॥ ६.३॥[3]
तदवधि शरासनादेर्गणयेद्दिनभर्तुरंशाः स्युः ।[4]
धनुरन्ततस्तु गणयेत्क्रान्तौ संक्षीयमाणायाम् ॥ ६.४॥[5]

The astronomer (*daivajña*), after knowing the declination from the degrees of [the sun's] meridian altitude and having measured the maximum declination from the beginning of the arc, should place the string [at that point].//$_1$ Where the horizontal line (*kramamaurvī*) of the declination of that moment counted from the beginning of the arc touches the string, up to that point the astronomer (*gaṇaka*) should move the cursor (*mṛgāsya*).//$_2$ When the string is lifted up and placed on the south-north line, the [horizontal] line that arises from that point occupied by the cursor touches the arc at a point,//$_3$ up to which point from the beginning of the arc he should count [the degrees of the arc]; there will be the degrees of the sun's [longitude]. When the declination is decreasing, he should count from the end of the arc.//$_4$

अथ कार्मुकादिभागाद्गृहीततात्कालिकक्रान्तेः ।
क्रमजीवा यत्रैव क्रान्तेर्वृत्तं परामृशति ॥ ६.५॥
तत्र न्यस्तं सूत्रं शरासनं यत्र संस्पृशति ।
तदवधि शरासनादिमकोटेरात्ता रवेरंशाः ॥ ६.६॥[6]

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[1] B: इति तु क्रांतिविचाराध्याय

[2] B: ॰मोर्वी

[3] B: म्पृशति B: ॰धिष्टित V: om. यत्र

[4] B: ॰रंशा

[5] B: ॰मानायां

[6] B: ॰कोटेरंतेरंशाः

Again, the horizontal line (*kramajīvā*) of the declination at that time counted from the first degree of the arc touches the declination circle at a point;$//_5$ the string placed there touches the arc at a point; up to that point from the first extremity of the arc, are obtained the degrees of [the longitude of] the sun.$//_6$

क्रान्त्युत्थक्रमजीवा क्रान्तेर्वृत्तं न चेत्स्पृशति ।[1]
तस्यां विधाय चिह्नं तदा तु सूत्रं ∀नयेद्गणकः ॥ ६.७ ॥ B9a
दक्षिणरेखां च चतुर्विंशांशोपरि मृगा∀स्यमादध्यात् ।[2] V7a
संचालितेऽथ सूत्रे स्पृशेत्क्रमज्यां यदैव हरिणास्यम् ॥ ६.८ ॥
संस्थापयेत्तदैव क्वचिदपि धनुरंशके सूत्रम् ।[3]
सूत्रावधि धनुरादेर्भवेयुरंशाः सहस्रांशोः ॥ ६.९ ॥

If the horizontal line (*kramajīvā*) arising from the declination does not touch the declination circle [for some reason], the astronomer (*gaṇaka*) should put a mark on it (the horizontal line) and take the string$//_7$ to the south[-north] line. He should place the cursor (*mṛgāsya*) at twenty-four degrees. Now, when the string is moved [to and fro], whenever the cursor (*hariṇāsya*) touches the horizontal line [marked before],$//_8$ then he should place the string at some place in the degrees of arc. From the beginning of the arc up to the string will be the degrees of [the longitude of] the sun.$//_9$

इति षष्ठोऽध्यायः ॥[4]

Thus the sixth chapter.

II.7 Chapter Seven: Terrestrial latitude

उत्तरगोलं गतवति भास्वति मध्योन्नतांशेषु ।[5]
क्रान्तौ निपातितायां शेषांशा लम्बभागतां प्राप्ताः ॥ ७.१ ॥
अन्तर्नवतिनिपातितपरिशिष्टास्ते स्युरक्षांशाः ।

When the sun is in the northern hemisphere, when the declination is subtracted from the degrees of the sun's meridian altitude, the remaining degrees become the degrees of co-latitude.$//_1$ When these are subtracted from ninety, those remaining are the degrees of the terrestrial latitude (*akṣāṃśa*).

[1] B: क्रांत्युथ॰

[2] B: मृगप्स्य॰

[3] B: सस्थाप॰

[4] B: इति तुर्यः षष्टोध्याय

[5] B: मध्योल्नतां॰

धक्षिणगोलस्थेऽर्के मध्योन्नतभागयोजितक्रान्तेः ॥ ७.२ ॥[1]
अंशास्त एव लम्बांशका नवत्यां निपातिताः कार्याः ।[2]
अवशिष्टानथ भागानवगच्छेदक्षभागाख्यान् ॥ ७.३ ॥[3]

When the sun is in the southern hemisphere, the degrees of declination increased by the degrees of meridian altitude [of the sun]//$_2$ are the degrees of co-latitude. These should be subtracted from ninety. The remaining degrees should be understood as those called the degrees of latitude.//$_3$

यदि मध्योन्नतभागा मूर्धन्या$^\forall$दुत्तरत्र रविरत्र ।[4] B9b
क्रान्तेस्तदा तु मध्योन्नतभागैर्योजितेषु भागेषु ॥ ७.४ ॥[5]
नूनं निपातिताया नवतेः परिशेषमक्षांशाः ।[6]

When the sun having the degrees of meridian altitude (i.e., when the sun is on the meridian) is to the north of the zenith (*mūrdhanya*), then from the degrees of the declination increased by the degrees of the meridian altitude//$_4$ ninety are subtracted; the remainder are the degrees of latitude.

मध्योन्नतभागाः स्युर्लम्बांशाः क्रान्त्यभावे तु ॥ ७.५ ॥[7]

In the absence of the declination, the degrees of the meridian altitude [of the sun] will be the degrees of co-latitude.//$_5$

यस्मिन्काले भानुर्यन्नगरशिरःपरिभ्रमं भजति ।
तस्मिन्नगरेऽक्षांशा ज्ञेया तात्कालिकी क्रान्तिः ॥ ७.६ ॥

When the sun is performing the revolution at the zenith of a certain town at a certain time, the degrees of latitude of that town should be known as the declination at that time.//$_6$

न्यूनाः परमक्रान्तेरक्षांशा यत्र तत्र तु च्छाया ।
उत्तरतो दक्षिणतो भवति कदा$^\forall$चिच्च तदभावः ॥ ७.७ ॥ V7b

[1] V: मध्योन्नतः B: योजिताः

[2] B: निपाताः

[3] B: भागानवनवग॰

[4] Em: ॰भागो B: सूर्धन्या॰ B: रविवृत्तान्

[5] B: भागषु

[6] B: नूनां B: ॰शेषतोक्षांशाः

[7] B: ॰भागास्युर्लंवाशाः

If the degrees of latitude are less than the maximum declination at any place, there the shadow will be to the north or to the south; sometimes it is absent.//$_{7}$

अधिकाक्षभागभागिनि देशे छायोत्तरत एव ।
दक्षिणतो लंकायाः सर्वमिदं वैपरीत्येन ॥ ७.८॥

In a locality having the degrees of latitude higher [than the maximum declination] the shadow will only be in the north. All this will be the reverse in the south of the equator.//$_{8}$

ऋक्षाणि यानि शश्वद्ध्रुवपार्श्वपरिभ्रमणभाञ्जि ।[1]
बोध्यास्तेषां मध्योन्नतभागा दीर्घलघवश्च ॥ ७.९॥[2]
तद्वययोगस्यार्द्धं तद्देशीया$^{\forall}$क्षभागाः स्युः ।[3] B10a
रविमध्योन्नतभागक्रान्त्यज्ञाने प्रकारोऽयम् ॥ ७.१०॥[4]

Of the stars that always revolve around the vicinity of the Pole, degrees of meridian altitudes, both larger and smaller, should be known [by observation].//$_{9}$ Half the sum of these two will be the degrees of the latitude of those territories. This is the procedure when the degrees of the meridian altitude and the declination of the sun are not known.//$_{10}$

इत्यक्षांशविचारः सप्तमः ॥[5]

Thus the seventh [chapter], the deliberation on the terrestrial latitude in degrees.

II.8 Chapter Eight: Shadow of the gnomon

ऋज्वी विपरीता च छाया द्विविधा बुधैरुक्ता ।
सूत्रं शङ्कोः शिरसश्छायाग्रावधि भवेत्कर्णः ॥ ८.१॥[6]

The wise say that the shadow is of two types: straight (*ṛju*) and reverse (*viparīta*). The line from the top of the gnomon up to the tip of the shadow is the hypotenuse (*karṇa*).//$_{1}$

[1] B: अक्षाणि

[2] B: वौध्या◦ B: टीर्घ◦

[3] B: तद्वय◦ B: भागा

[4] B: ज्ञान

[5] B: ◦विचारो नाम सप्तमो ध्यायः

[6] V: शिरसःश्छाया◦ B: शिरसस्छाया◦

उक्तस्त्रिविधः सप्ताङ्गुलस्तथा द्वादशाङ्गुलः शङ्कुः ।[1]
षष्ट्यङ्गुलश्च तस्य छाया घटिकावबोधाय ॥ ८.२॥

The gnomon is said to be of three types: that having [a length of] seven *aṅgulas*, that having [a length of] twelve *aṅgulas* and that having [a length of] sixty *aṅgulas*. Its shadow is [employed] for knowing the *ghaṭikā* (i.e., for measuring time in *ghaṭikās*).//$_2$

सप्ताङ्गुलं तु शङ्कुं सार्धषडङ्गुलमितं कुर्यात् ।[2]
अथवा साभ्रपयोधिस्वषष्टिभागैः षडङ्गुलिभिः ॥ ८.३॥[3]

The gnomon of 'seven *aṅgulas*' should however be made with a measure of six and half *aṅgulas* or, otherwise, six *aṅgulas* and forty of its own sixty divisions.//$_3$

गणितेषु कार्मुकादेर्भानोर्विद्धोन्नतांशेषु ।[4]
दध्यात्सूत्रमथोदग्दक्षिणरेखागृहीतेभ्यः ॥ ८.४॥
शङ्क्वङ्गुलिसंख्येभ्यो भागेभ्यः प्रोत्थिता क्रमज्या तु ।[5]
∀यत्राश्लिष्यति सूत्रं तद्देशादुत्थितोत्क्रमज्याख्या ॥ ८.५॥[6] B10b
प्राचीपश्चिमरेखां संगच्छेद्यत्र तत्र खलु केन्द्रात् ।
यावन्तो भागाः ∀स्युश्छाया तावद्भिरङ्गुलिभिः ॥ ८.६॥[7] V8a

At the degrees of the sun's observed (*viddha*) altitude, which are counted from the beginning of the arc, the string should be placed. Then, where the horizontal line arising from the units counted on the north-south line,//$_4$ whose number is the same as the *aṅgulas* of the gnomon, touches the string, from that point a vertical line arises//$_5$ and joins the east-west line at a point; up to that point from the center as many units there are, by so many *aṅgulas* the shadow is [measured].//$_6$

धनुरादेर्विद्धोन्नतभागानादाय योजयेत्सूत्रम् ।[8]
शङ्क्वङ्गुलिसंख्येभ्यो दक्षिणरेखादितो गृहीतेभ्यः ॥ ८.७॥[9]

---

[1] B: सप्तागुलसूथा

[2] B: सप्तागुलं तु शंकु

[3] B: ॰भागेः B: ॰लिभि

[4] B: गणितेषु कादेएर्भाताविद्धो॰

[5] B: भागेभ्यो

[6] V: ॰त्थिताक्रम॰

[7] B: यावतो भागा स्युच्छाया B: ॰लिभि

[8] B: ॰देविद्धो॰ B: योजयेशूत्रं

[9] B: शंक्कगुलि॰

भागेभ्यः क्रमजीवा गुणं स्पृशेद्यत्र तत्र च मृगास्यम् ।
संस्थाप्य दक्षिणोत्तररेखायां योजयेत्सूत्रम् ॥ ८.८॥[1]
यत्र मृगास्यं तिष्ठेत्तद्देशावधि च केन्द्रतो गणिताः ।
यावन्तो भागाः स्युः कर्णस्तावद्भिरङ्गुलिभिः ॥ ८.९॥[2]

Having counted the degrees of the observed altitude [of the sun] from the beginning of the arc, one should put the string there. From [the point of] the degrees equal to the *aṅgulas* of the gnomon counted from the beginning of the south-north line//$_7$ a horizontal line [arises and] touches the string at a point. Having placed the cursor at that point, one should join the string to the south-north line.//$_8$ Up to the point where the cursor rests, as many units are counted from the center, by so many *aṅgulas* the hypotenuse is [measured].//$_9$

विपरीतच्छायामपि जानीयादेवमेव सुधीः ।[3]
ग्राह्याः शरासनान्तादुन्नतभागा इति विशेषः ॥ ८.१०॥

The intelligent person should understand the reverse shadow also in the same manner. [But] the difference is that the altitude degrees [of the sun] should be counted from the end of the arc [of the quadrant].//$_{10}$

∀स्वल्पेषून्नतभागेषु कार्मुकादेर्गृहीतेषु ।[4] B11a
सूत्रं विनिहितमाद्यां क्रमजीवां यत्र संस्पृशेच्च ततः ॥ ८.११॥[5]
उदितोत्क्रमजीवा प्राक्पश्चिमरेखां समुल्लिखेद्यत्र ।[6]
तद्देशावधि केन्द्राद्गणयेद्भागान्क्रमेण गणितज्ञः ॥ ८.१२॥
ते शङ्क्वङ्गुलिसंख्यानिहताश्च भवन्ति यावन्तः ।
तत्कालीना छाया ज्ञेया तावद्भिरङ्गुलिभिः ॥ ८.१३॥[7]
सूत्रं यदा द्वितीयक्रमजीवां संस्पृशेत्तदा भागान् ।
अङ्गुलिसंख्यार्धहतान्कुर्यात्परतोऽपि कल्पयेदेवम् ॥ ८.१४॥

The string, placed [on the arc] at the small number of altitude degrees counted from the beginning of the arc, touches the first horizontal line at a point, from which point//$_{11}$ a vertical line arises and intersects the east-west line at a point, up

[1] B: संस्थाय्प

[2] B: भागा B: वद्भिःरंगुलि॰

[3] VB: ॰तच्छाया॰ B: ॰देवमैव

[4] B: ॰कादेगृही॰

[5] V: ॰हितमाद्या B: ॰हितामाद्यां B: ॰जीवा

[6] B: ॰जीव

[7] B: ॰लिभि

to which point, from the center, the mathematician should count the units one by one.//$_{12}$ As many as these [units will be when] multiplied by the number of *aṅgulas* in the gnomon, by so many *aṅgulas*, it should be known, the shadow at that time is [measured].//$_{13}$ When the string touches the second horizontal line, then one should multiply the units by half of the number of *aṅgulas* [in the gnomon]. After that also, one should proceed in the same way.//$_{14}$

विद्धोन्नतभागान्धनुरादे∀रादाय तन्तुमाधाय । V8b
तानेव धनुःप्रान्ताद्गणयेत्तन्निर्गतोत्क्रमज्या तु ॥ ८.१५ ॥
यत्रालिङ्गति सूत्रं तत्र मृगा∀स्यं निधातव्यम् ।[1] B11b
सूत्रेऽथ दक्षिणोत्तररेखामधिरोहिते मृगास्यं तत् ॥ ८.१६ ॥
यत्रैव संस्थितं तद्देशावधि केन्द्रतो गणिताः ।[2]
यावद्भागास्तावच्छाया षष्ट्यङ्गुलस्य भवेत् ॥ ८.१७ ॥[3]

Having taken (counted) the number of the observed altitude degrees [of the sun] from the beginning of the arc, and having placed the string [at that point], one should count the same [number] from the end of the arc. The vertical line that arises from there//$_{15}$ embraces (touches) the string at a point, where the cursor (*mṛgāsya*) should be placed. Then, when the string is [rotated and] mounted on the south-north line, that cursor//$_{16}$ is situated at a point; up to that point, from the center, as many units are counted, so many [*aṅgulas*] will be [the length of] the [reverse] shadow of the gnomon of sixty *aṅgulas*.//$_{17}$

इति छायाविचारोऽष्टमोऽध्यायः ॥[4]

Thus the eighth chapter, the deliberation on the shadow [of the gnomon].

II.9 Chapter Nine: Altitude of the sun

शङ्क्वङ्गुलिसंख्येभ्यो दक्षिणरेखादितो गृहीतेभ्यः ।[5]
भागेभ्यो निर्याता क्रमजीवासंज्ञिता रेखा ॥ ९.१ ॥
तच्छङ्कुच्छायाङ्गुलिसंख्यांशेभ्यश्च पूर्वरेखादेः ।[6]
गणितेभ्यो निर्यातामुत्क्रमजीवां समुल्लिखेद्यत्र ॥ ९.२ ॥
सूत्रं तत्र विनिहितं यत्र स्थितिमेति कोदण्डे ।

[1] B: निध्यातव्यं

[2] B: गणितः

[3] B: °द्भागस्तावछाया

[4] B: इति तुर्ययंत्रे छायाविवारो नामाष्टमो ध्यायः

[5] V: °रेखागृहीतेभ्यः

[6] B: तछंकु°

तदवधि शरासनादेर्गणिता भागाः समुन्नतांशाः स्युः ॥ ९.३॥[1]
विपरीतच्छायायाः सू∀र्योन्नतभागजिज्ञासुः । B12a
तानेव कार्मुकान्ताद्गणयेद्भागानिति विशेषः ॥ ९.४॥

The horizontal line called *kramajīvā*, which arises from the degrees equal in number to the *aṅgulas* on the gnomon and counted on the south[-north] line from its beginning,$//_1$ intersects the vertical line, which arises from the point of the units equal in number to the *aṅgulas* of the shadow of that gnomon and counted on the east[-west] line from its beginning, at a point;$//_2$ the string placed over that point rests on the arc; up to that point [indicated by the string], from the beginning of the arc, the degrees are counted; they will be the altitude degrees [of the sun].$//_3$ One who wishes to know the degrees of the sun's altitude from the reverse shadow of the gnomon should count the same degrees [of altitude as mentioned above] from the end of the arc; this is the difference.$//_4$

सप्ताङ्गुलस्य शङ्कोरयं विधिर्द्वादशाङ्गुलस्यापि ।[2]
न तु षष्ट्यङ्गुलशङ्कोर्विज्ञेयो गणिततत्त्वज्ञैः ॥ ९.५॥[3]

This is the procedure for the gnomon of seven digits (*aṅgulas*) and also for the gnomon of twelve digits (*aṅgulas*), but not for the gnomon of sixty digits (*aṅgulas*); this should be known to those who are experts in mathematics.$//_5$

इति सूर्योन्नतांशविचारो नाम नवमोऽध्यायः ।[4]

Thus the ninth chapter entitled the deliberation on the the sun's altitude in degrees.

II.10 Chapter Ten: Length of the day and the night

आरोपयेत्समुन्नतभागत्वं तावदक्षभागेषु ।
अथ षष्ट्यङ्गुलशङ्कुच्छायापरिमाणमवगच्छेत् ॥ १०.१॥[5]
∀छायांशानपराशाप्राचीरेखादितो गणयेत् ।[6] V9a
तज्जातोत्क्रमजीवा धनुरादेः परिगृहीतेषु ॥ १०.२॥[7]
क्रान्त्यांशेषु विनिहितं सूत्रं यत्रैव संस्पृशेत्तत्र ।

[1]B: शराशना॰ B: ॰तांशा

[2]B: ॰लस्य वाशंकोः

[3]B: ॰शंकेर्वि॰

[4]V: इति सूर्योन्नतांशविचारो नवमः । B: विवारो

[5]B: ॰माणमाणमव॰

[6]B: रेखादिभागातो

[7]B: तज्जातेत्क्रम॰

चिह्नं कुर्वीत ततः क्रमजीवा यत्र कार्मुके तिष्ठेत् ॥ १०.३ ॥[1]
∀तद्देशावधि गणिता धनुरादेरंशकाश्चरार्धं स्यात् । B12b
उत्तरगोलस्थेऽर्के तन्नवत्या योजितं द्विगुणितं च ॥ १०.४ ।[2]
वासरशरासनं स्यात्षष्ट्यधिकशतत्रितयमध्ये ।[3]
तत्पातितावशिष्टं रात्रिधनुर्भवति नियमेन ॥ १०.५ ॥

One should first apply the state of being altitude degrees to the [co-]latitude degrees (i.e., treat the colatitude as the altitude) and know the length of the shadow of the sixty-*aṅgula* gnomon.$//_1$ One should count the units in the shadow from the beginning of the west-east line. The vertical line (*utkramajīvā*) arising from these$//_2$ touches the string, placed at the degrees of declination counted from the beginning of the arc, at a point, where one should put a mark (*cihna*); the horizontal line (*kramajīvā*) [arising] from there rests on the arc at a point;$//_3$ the degrees counted up to that point from the beginning of the arc will be half of the ascensional difference (*carārdha*). When the sun is in the northern hemisphere, that (*carārdha*), increased by ninety and doubled,$//_4$ will be the arc of the day. When this is subtracted from 360, the remainder will be the arc of the night inevitably.$//_5$

दक्षिणगोलस्थेऽर्के त्यजेन्नवत्यामिति विशेषः ।
लङ्कादक्षिणभागे विपरीतं धनमृणं कुर्यात् ॥ १०.६ ॥

When the sun is in the southern hemisphere, [the *carārdha*] should be subtracted from ninety: this is the difference. In the south of the equator (laṅkā), the positive and negative signs should be reversed [in the procedure taught above].$//_6$

इति दिवसरात्रिविचारो दशमः ।[4]

Thus the tenth [chapter], the deliberation on [the length of] the day and night.

III Commentary

III.1 Chapter One: Construction of the instrument

Verses 1.7-14ab: Construction of the surface of the quarter circle

[1] B: विह्नं

[2] B: सनवत्या, which is metrically better but which would cause disagreement of gender. V: originally योजिते द्विगुणितश्च, which has been corrected to योजितं द्विगुणितं च. B: योजितो द्विगुणितश्च.

[3] B: वासरं शरासरं शरासनं V: ॰घिकशतत्रयमध्ये

[4] B: इति वटार्द्धविवारो नाम दशमो ध्यायः ।

Bhūdhara's instructions for constructing the sine quadrant can be understood from the reconstruction below.

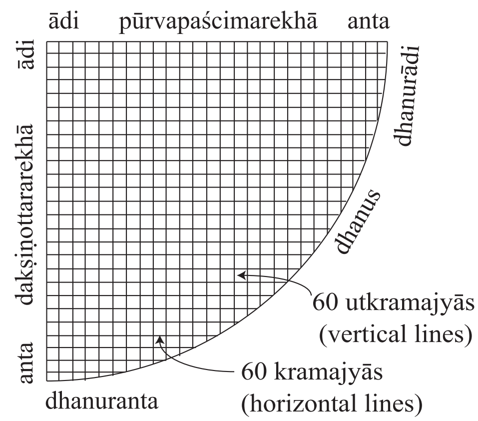

Fig. 1

The arc of the quarter circle is graduated into 90 degrees and each group of 5 or 6 degrees are marked by distinct lines. The following is a tentative reconstruction.

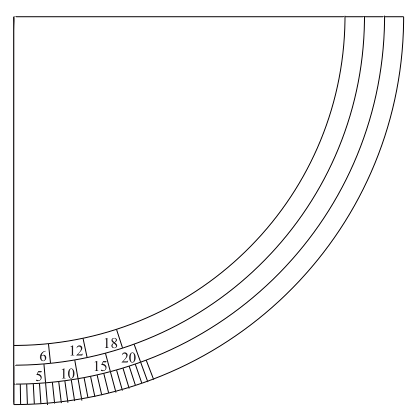

Fig. 2

Verses 1.14cd-18ab: Construction of the attachements

Here Bhūdhara instructs that, after marking the quarter circle with a series of lines parallel to the two radii and a graduated scale on the arc (see verses 1.7-14ab above), a plumb line with a weight should be suspended from the center of the quarter circle and two sighting vanes (*kūṭa*) be attached to one of the radian sides. About the sighting vanes, he cites three different practices: some people equip both the sighting vanes with holes, some others attach a sighting tube (*nalikā*), and yet some others bore a hole only in the sighting vane close to the center and observe the stars through that single hole. The third practice is clearly wrong and the text is corrupt here.

The text is also incomplete, because it does not mention the declination circle (*krānti-vṛtta*), the half circle (*vṛttārdha-rekhā*) drawn with the south-north line as the diameter, and the bead or cursor (*mṛga-āsya*) that slides along the string of the plumb-line. But these elements are essential for the various types of procedures described from the third chapter onwards. These are shown in the following figure.

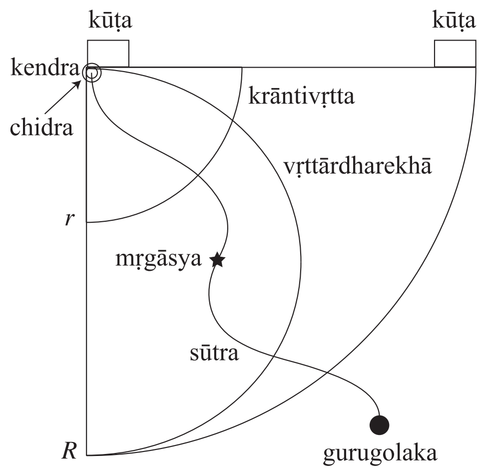

Fig. 3

Technical terms

anta: The end.
ādi: The beginning.
utkramajyā: A vertical line. See *N.B.* below.
kārmuka: The arc of the quadrant. See *dhanus* below.
kūṭa: A projection having a hole or a groove or a tube for sighting.
kendra: The center.
kramajyā: A horizontal line. See *N.B.* below.
krāntivṛtta: The declination circle whose radius (r) is $R \sin \varepsilon = 60 \sin 23;30,17 \approx 24$. See verses 5.5cd-8 and 5.9.
gurugolaka: A heavy round weight or plumb.
chidra: The hole for tying the string.
dakṣiṇottararekhā: The south-north line graduated into R ($= 60$) units.
dhanuranta: The end of the arc.
dhanurādi: The beginning of the arc.
dhanus: The arc of the quadrant graduated into 90 degrees. *ādimadhanus* (the first or original arc) for differentiation.
pūrvapaścimarekhā: The east-west line graduated into R ($= 60$) units.
mṛgāsya/hariṇāsya: A cursor that slides on the string (lit. 'the face, i.e., the beginning, of Capricorn'), a term used originally for the pointed projection at the first point of Capricorn on the rete of the astrolabe.
vṛttārdharekhā: The line of the semicircle whose diameter lies on the south-north line. See verse 3.22.

vyāsa: Usually means the diameter of a circle but here used for the two orthogonal radii, namely, the east-west and the south-north lines of the quadrant.
sūtra: A string with the weight and the cursor.

N.B. The terms *kramajyā* and *utkramajyā* usually mean the 'sine' and the 'versed sine' respectively, but in this work they denote respectively 'horizontal' and 'vertical' lines, which are orthogonal to each other just like the 'sine' and the 'versed sine'. Note that the author uses simply *jyā* or *jīvā* or *maurvī* ('chord') and *śara* or *bāṇa* ('arrow') respectively for 'sine' (see 3.2-7,24, etc.) and 'versed sine' (see 3.11, 12, 14, 16, etc.).

III.2 Chapter Two: Measuring the altitude

Verses 2.1-3: Measuring the sun's altitude

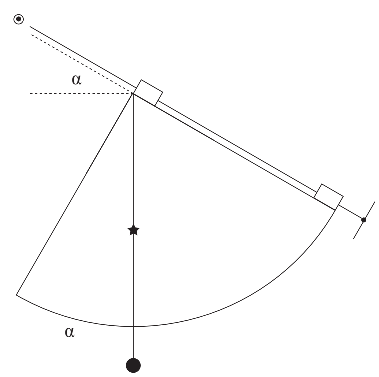

Fig. 4

III.3 Chapter Three: Arc, chord (sine) and arrow (versine)

Verses 3.1-2: arc ($\theta < 90$) $\rightarrow$ chord ($J = R\sin\theta$)

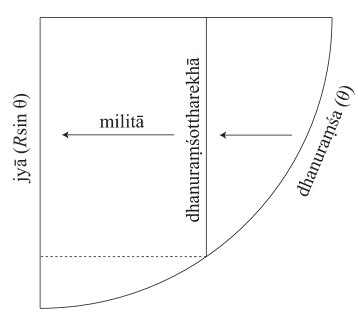

Fig. 5

jyā: Chord, i.e., half chord or sine.
dhanuraṃśa: Degrees of the arc.
dhanuraṃśottharekhā: Line that arises from the degrees of the arc.
militā: Merged with or mapped onto.

Verse 3.3: arc ($90 < \theta < 180$) $\rightarrow$ chord ($J = R\sin\theta$)

If $\alpha + \beta = 180$ ($\alpha < \beta$), then

$$R\sin\beta = J = R\sin\alpha.$$

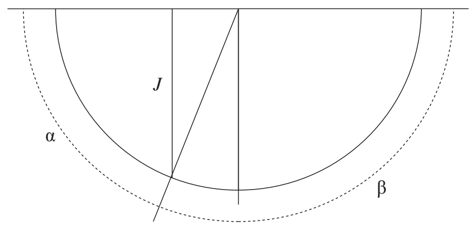

Fig. 6

Verses 3.4-5: chord ($J = R\sin\theta$) $\rightarrow$ arc (θ)

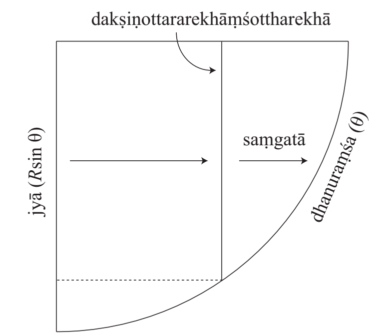

Fig. 7

saṃgatā: associated, united, met.

Verse 3.6: Chord for 90 degrees and interpolation

$R\sin 90 = 60$, where $R = 60$. The rule of three (and hence linear interpolation) should be employed for the interstices between the 60 lines.

Verses 3.7-10ab: Treatment of arcs greater than 90 degrees

These verses prescribe the conversion of arcs greater than 90 degrees to those smaller than 90 degrees:

1. If $90 < \theta < 180 \quad \rightarrow \quad \theta' = 180 - \theta$;
2. If $180 < \theta < 270 \quad \rightarrow \quad \theta' = 270 - \theta$;
3. If $270 < \theta < 360 \quad \rightarrow \quad \theta' = 360 - \theta$.

In these cases, we have the following relations,

1. $R\sin\theta = R\sin\theta'$ (cf. verse 3.3 above)
2. $R\sin\theta = -R\cos\theta'$
3. $R\sin\theta = -R\sin\theta'$

However, the second case actually meant by the author may be:

$$2'.\text{ If } 180 < \theta < 270 \quad \rightarrow \quad \theta' = \theta - 180,$$

in which case we have the relation,

$$2'. \quad R\sin\theta = -R\sin\theta',$$

and one can use the *jyā* (Rsine) with a positive or a negative sign for any value of θ after the conversion, although the two manuscripts used for the present edition do not support this reading. These rules are necessary for the quadrant, which has only the first 90 degrees.

Verses 3.10cd-12ab: arc $(\theta < 90) \rightarrow$ arrow (s)

Procedure. Given an arc (θ) that is less than 90 degrees, count the degrees of the given arc on the arc from its beginning and find the vertical line that arises from the that point. Count the units of the east-west line from its end up to the point where the vertical line touches. Obtained is the arrow or versed sine (s) of the given arc.

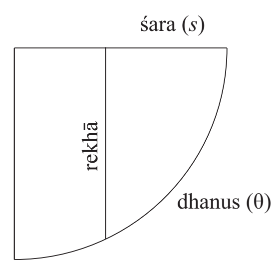

Fig. 8

Verses 3.12cd-14ab: arc $(90 < \theta < 180) \rightarrow$ arrow (s)

Procedure. Given an arc (θ) that is greater than 90 degrees, subtract 90 from it. Obtain the [half-]chord (Rsine) of the remainder $(\theta - 90)$ and add 60. Obtained is the arrow of the given arc.

That is to say,

$$s = R\,\text{vers}\,\theta = R\sin(\theta - 90) + R, \quad \text{where} \quad R = 60.$$

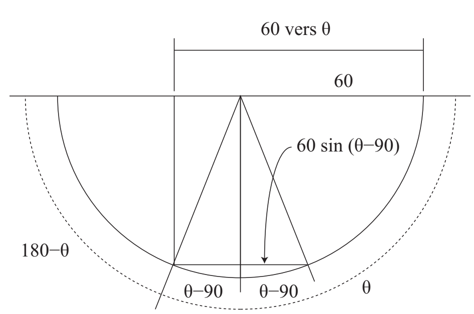

Fig. 9

Verses 3.14cd-16: arrow ($s = R\text{versin}\ \theta < R$) $\rightarrow$ arc (θ)

Procedure. Given an arrow (*śara*) s ($= R\,\text{vers}\ \theta$) that is less than R, count the s on the east-west line of the sine quadrant from its end and find the corresponding vertical line (*utkramajyā*). Count the degrees (θ) from the extremity (*koṭi*) of the arc of the quadrant (*ādima-dhanus*) up to the point indicated by the vertical line. Obtained is the arc (θ) for the given arrow.

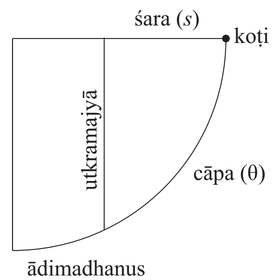

Fig. 10

Verses 3.17-19: arrow ($s = R\text{versin}\ \theta > R$) $\rightarrow$ arc (θ)

Procedure. Given an arrow, s ($= R\,\text{vers}\ \theta$), greater than $R(= 60)$, subtract 60 from it. Find a horizontal line of that length that arises from the south-north line and that touches the arc. Count the degrees of the arc from the the south-north line to the point which the horizontal line touches. Add 90 to the degrees obtained. The sum is the arc (θ) for the arrow.

That is to say,

$$\theta = R\ \arcsin(s - R) + 90, \quad \text{where} \quad R = 60.$$

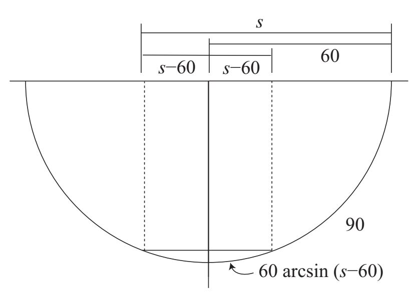

Fig. 11

Verse 3.20: Relationship of arc (*dhanus*) and half chord (*ardha-jyā*)

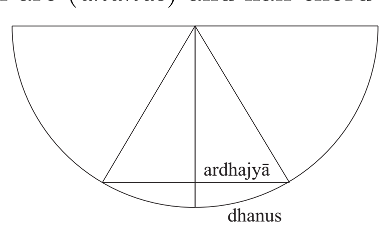

Fig. 12

Verses 3.21-24ab: arc (θ) $\rightarrow$ chord (J)

Procedure. Given an arc (θ), place the string (*sūtra*) at θ on the arc. Slide the cursor called *mṛgāsya* on the string and place it at the intersection of the string and the line of the semicircle (*vṛttārdha-rekhā*). Rotate the string onto the north-south line. Read the point (J) indicated by the cursor. Then, $J = R \sin\theta$.

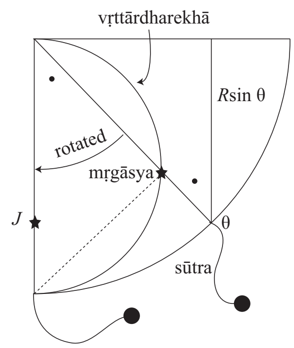

Fig. 13

Verses 3.24cd-27: chord (J) $\rightarrow$ arc (θ)

This is the reverse of the preceding rule (verses 21-24ab).

Procedure. Given a sine J ($= R\sin\theta$), put the string on the south-north line and place the cursor (*mṛgāsya*) of the string at J. Rotate the string so that it may cut the line of the semicircle at the cursor. Read the degrees (θ) on the arc indicated by the string.

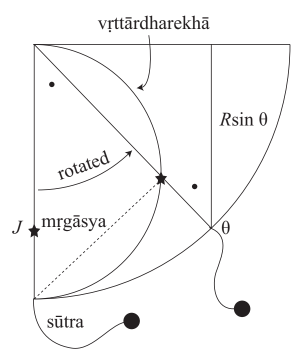

Fig. 14

Verses 3.28-31ab: arc (θ) $\rightarrow$ arrow (s)

Procedure. Given an arc (θ), count the degrees on the arc from its end and fix the point. Stretch the string up to that point and place the cursor at the intersection of the string and the line of the semicircle. Rotate the string up to the east-west line and read the degrees on the line from its end up to the cursor. Obtained is the arrow (*śara*), $s = R \operatorname{vers}\theta$.

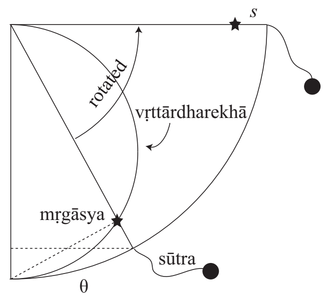

Fig. 15

Verses 3.31cd-34: arrow (s) $\rightarrow$ arc (θ)

This is the reverse of the preceding rule (verses 28-31ab).

Procedure. Given an arrow (*śara*), $s = R \operatorname{vers} \theta$, count the number of the units in s on the east-west line from its end. Stretch the string on the east-west line and place the cursor at that point. Rotate the string so that the string may cut the line of the semicircle at the cursor. Count the degrees on the arc from its end up to the point indicated by the string. Obtained is the arc for the arrow, $\theta = R \operatorname{arcvers} s$.

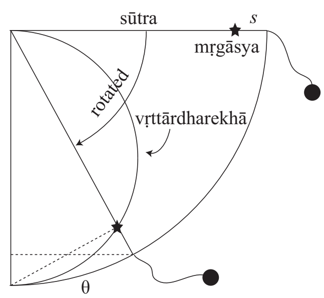

Fig. 16

Verse 3.35: Complementary arc

The relationship of an arc ($\theta < 90$) and 'its complementary arc' ($\bar{\theta}$) is: $\theta + \bar{\theta} = 90$. Its expression in the verse, *tatsaṃpūrṇaṃ kārmukam*, which literally means 'an arc filled with it', is difficult to understand.

III.4 Chapter Four: Meridian altitude of the sun

Verses 4.1-2: Observation of the meridian altitude (α) of the sun

When it is getting closer to the time of midday, the observer measures the sun's altitude many times (see verses 2.1-3 above). Let the values obtained be α_i. Generally, the sequence increases up to a term and decreases after that: $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_{m-1} \leq \alpha_m \geq \alpha_{m+1} \geq \cdots \geq \alpha_n$. Then, the maximum value (α_m) is the sun's meridian altitude (α).

Verses 4.3-5: Possible range of the sun's positions on the meridian

The possible range of the sun's positions on the meridian and the possible directions of the noon shadow depend on the latitude (φ) of the locality. In the following table, ε denotes the maximum declination of the sun, which is regarded as 24 degrees in these verses (cf. verses 5.5cd-8 and 5.9); the p's stand for 'possible'.

	Possible range of the sun's positions on the meridian		
	Left (north)	Zenith	Right (south)
Locality	(shadow to right)	(no shadow)	(shadow to left)
$\varphi < \varepsilon$	p	p	p
$\varphi = \varepsilon$		p	p
$\varphi > \varepsilon$			p

Verses 4.6-10: Latitude (φ) and declination (δ) $\rightarrow$ meridian altitude (α), case 1

The colatitude is defined as: $\bar{\varphi} = 90 - \varphi$.

If the locality (φ) is in the north and the declination (δ) is in the north:

$\bar{\varphi} + \delta < 90$	$\rightarrow$	$\alpha = \bar{\varphi} + \delta$	sun in the southern hemisphere
$\bar{\varphi} + \delta = 90$	$\rightarrow$	$\alpha = \bar{\varphi} + \delta$	sun at the zenith
$\bar{\varphi} + \delta > 90$	$\rightarrow$	$\alpha = 180 - (\bar{\varphi} + \delta)$	sun in the northern hemisphere

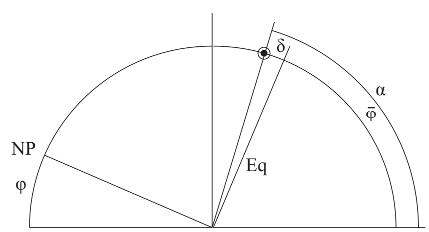

Fig. 17

Verse 4.11: Latitude (φ) and declination (δ) $\rightarrow$ meridian altitude (α), case 2

If the locality (φ) is in the south and the declination (δ) is in the south, the position of the sun is reversed:

$\bar{\varphi} + \delta < 90$	$\rightarrow$	$\alpha = \bar{\varphi} + \delta$	sun in the northern hemisphere
$\bar{\varphi} + \delta = 90$	$\rightarrow$	$\alpha = \bar{\varphi} + \delta$	sun at the zenith
$\bar{\varphi} + \delta > 90$	$\rightarrow$	$\alpha = 180 - (\bar{\varphi} + \delta)$	sun in the southern hemisphere

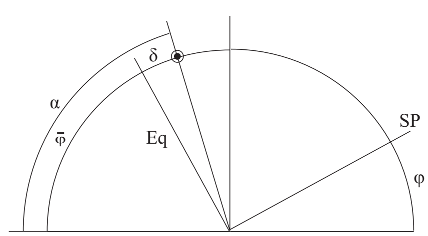

Fig. 18

Verse 4.12: Latitude (φ) and declination (δ) $\rightarrow$ meridian altitude (α), cases 3 and 4

Case 3: If the locality (φ) is in the north and the declination (δ) is in the south, then $\alpha = \bar{\varphi} - \delta$ and the sun is in the southern hemisphere.

Case 4: If the locality (φ) is in the south and the declination (δ) is in the north, then $\alpha = \bar{\varphi} - \delta$ and the sun is in the northern hemisphere.

Verse 4.13: Latitude (φ) and declination (δ) $\rightarrow$ meridian altitude (α), case 5

If the place is on the equator ($\varphi = 0$ and $\bar{\varphi} = 90$), then $\alpha = 90 - \delta$ and the sun is in the northern or southern hemisphere according to whether the declination is in the northern or southern hemisphere; if $\delta = 0$, then $\alpha = 90$.

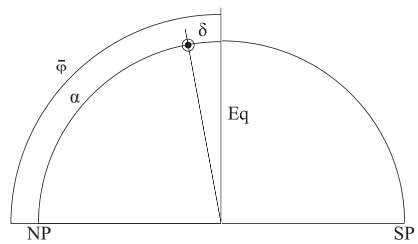

Fig. 19

III.5 Chapter Five: Declination

Verses 5.1-2: Arrangement of the twelve signs on the quadrant

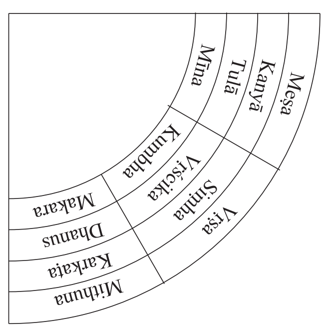

Fig. 20

Verse 5.3-5ab: sun's longitude (λ) $\rightarrow$ declination (δ)

Procedure. Given the longitude (λ) of the sun (*arka*), find the corresponding point on the arc according to the arrangement of the twelve signs described above (verses 5.1-2). Place the cursor (*mṛgāsya*) at the intersection of the string placed on the south-north line and the declination circle (*krānti-vṛtta*). Rotate the string up to the point corresponding to the solar longitude and find the horizontal line (*kramajīvā*) passing through the cursor. Count the number of degrees on the arc from its beginning up to the point indicated by the horizontal line. Obtained is the declination (δ) of the sun.

In this procedure, it is actually not necessary to make use of the cursor as long as the declination circle is drawn on the quadrant, because the horizontal line that indicates the declination to be obtained can be determined, without the cursor, by the intersection of the declination circle itself and the string positioned at the longitudinal point of the arc. Using the cursor, therefore, seems to have been meant for such cases where the declination circle is not entirely available for some reason. Cf. verses 6.7-9 below.

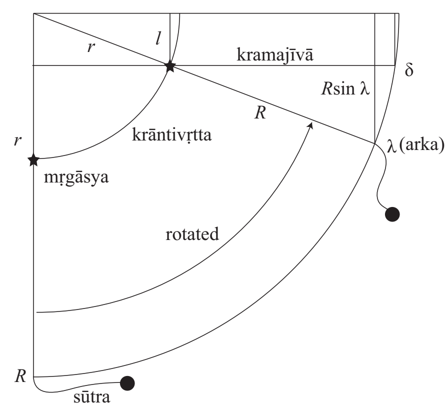

Fig. 21

Rationale. From the similar right trilaterals on the quadrant, we have the relationship, $R : R\sin\lambda = r : \ell$, where $r = R\sin\varepsilon$. On the other hand, from the similar right trilaterals inside the celestial sphere (*gola*), we have the relationship, $R : R\sin\varepsilon = R\sin\lambda : R\sin\delta$, which can be rewritten as $R : R\sin\lambda = R\sin\varepsilon : R\sin\delta$. Hence follows $\ell = R\sin\delta$.

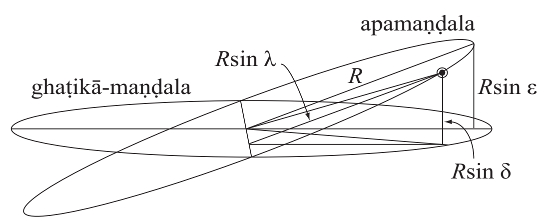

Fig. 22

Verses 5.5cd-8: Conversion of the sun's declination ($\delta \rightarrow \delta'$)

Procedure. Given the declination (δ) of the sun, measure it out on the arc from its beginning and put a mark (*cihna*) there. Stretch the string up to the mark. Find the horizontal line (*anukramajyā*) passing through the intersection of the string and the vertical line at 55 on the east-west line. Count the number of degrees on the arc from its beginning up to the point indicated by the horizontal line. Obtained (δ') is the declination converted to the solstice day.

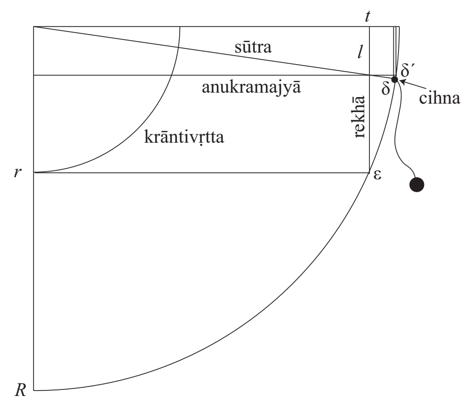

Fig. 23

Rationale. Let $t = R\cos\varepsilon$. Then, $t \approx 55$. From the similar right trilaterals on the quadrant, we have the relationship, $R\cos\delta : R\sin\delta = t : \ell$. Hence follows

$$\ell = R\sin\delta' = R\sin\delta \times \frac{R\cos\varepsilon}{R\cos\delta}.$$

$R\cos\delta$ and $R\cos\varepsilon$ are the radii of the sun's diurnal circles on a given day and on the solstice day respectively. We, however, do not understand the purpose of this 'conversion'.

The last statement of this paragraph (verse 5.8cd: 'The wise should ...') is probably meant for stressing the superiority of 55 as the value of $R\cos\varepsilon$, which seems to have been obtained from the 'modern' value of ε, 23; 30, 17, in comparison with the value obtained from the 'ancient' value, 24. See the next verse.

ε	$R\sin\varepsilon\,(= r)$	$R\cos\varepsilon\,(= t)$
24	$24.4\,(= 24\frac{2}{5})$	$54.8\,(= 54\frac{4}{5})$
23;30,17	$23.9\,(= 23\frac{9}{10})$	$55.0\,(= 55)$

Verse 5.9: Two values of the maximum declination

$$\varepsilon = 24\text{: by the ancients}$$
$$\varepsilon = 23;30,17\text{: by the moderns}$$

Bhūdhara uses the terms, *ghaṭī* and *pala*, respectively for 'minute' and 'second' of arc. This is a most unusual usage which is not attested in any other text. These terms are actually used as sexagesimal time units: 60^2 *palas* = 60 *ghaṭīs* (or *ghaṭikās*) = 1 *dina* (day, i.e., day and night).[1]

This 'modern' (*ādhunika*) value, 23; 30, 17, has been attributed to al-Qūshjī in the *Hayatagrantha*.[2]

[1] See, for example, Bhāskarācārya, *op. cit.*, p. 18 (Grahagaṇitādhyāya, madhyamādhikāra, kālamāna, verses 17d-18a).

[2] *Hayata*, ed. by V. Bhattācārya, Varanasi 1967, p. 24.

अथ नाडीवलयक्रान्तिवृत्तयोः परमान्तरं कर्के मकरे वा मैलकुल्ली संज्ञा परमक्रान्तिः रसदग्रन्थेषु नानाविधा दृष्टास्ति। अल्लाम कौशजी नामा उलूकवेगस्य गुरुपुत्रो वदति अस्मद्रसदग्रन्थेषु परम-क्रान्तिरंशाद्या २३।३०।१७ दृष्टा इति।

Now, the maximum difference between the celestial equator (*nāḍī-valaya*) and the ecliptic (*krānti-vṛtta*) occurs in Cancer (*karka*) and Capricorn (*makara*). The maximum declination called *mailakullī* (*mail kullī*) [in Arabic] has been variously determined in books of observation.[1] The son of the teacher of Ulugh Beg (ulūkavega), by name ᶜAllāmā Qūshjī (*allāmakauśajī*), says: 'The maximum declination [in the units] beginning with degree was determined as 23/30/17 in our books of observation.'

The *Hayatagrantha* is a Sanskrit rendering of al-Qūshjī's *Risālah dar hay'ah.* The above passage has been attributed to an anonymous collaborator of the anonymous translator.[2]

According to Khafri, this value was found in the observations that were undertaken under the auspices of Ulugh Beg at Samarqand.[3]

The maximum declination, '$23;20,17$', that occurs on p. 94 of the *Hayatagrantha* should probably be corrected to $23;30,17$, which value is found in one of the manuscripts (fn. 6). In all other cases, however, the 'ancient' or traditional value, 24, is used in this work also. Moreover, the *Hayatagrantha* uses not *ghaṭī* but the traditional name *kalā* for the sixtieth part of a degree (*aṃśa*). Therefore, the *Hayatagrantha* may not have been Bhūdhara's source.

III.6 Chapter Six: Solar longitude

Verse 6.1-4: sun's meridian altitude (α) $\rightarrow$ longitude (λ)

Procedure. Given the meridian altitude (α), calculate the declination at that moment by

$$\delta = \varphi + \alpha - 90.$$

Count off the obtained degrees of the declination on the arc from its beginning. Stretch the string up to the point of the maximum declination (ε) and place the cursor (*mṛgāsya*) at the intersection of the string and the horizontal line (*kramamaurvī*) that meets the point of the declination (δ). Rotate the string up to the south-north line. Count the degrees of the arc from its beginning up to the point indicated by the horizontal line that arises from the point of the cursor on the

[1] *rasadagrantha* < Ar. *raṣd* + Skt. *grantha*

[2] David Pingree, 'Indian Reception of Muslim Versions of Ptolemaic Astronomy,' *Tradition, Transmission, Transformation: Proceedings of Two Conferences on Premodern Science Held at the University of Oklahoma*, ed. by F. J. Ragep and S. P. Ragep, Leiden 1996, pp. 471-85, esp. 476.

[3] F. J. Rajep (ed.), *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy: al-tadhkira fī ᶜilm al-hay'a*, New York 1993, p. 394.

south-north line. Obtained is the sun's longitude (λ).

When the declination (δ) is decreasing (i.e., $90 < \lambda < 180$ or $270 < \lambda < 360$), count the degrees for λ from the end of the arc (cf. verses 5.1-2).

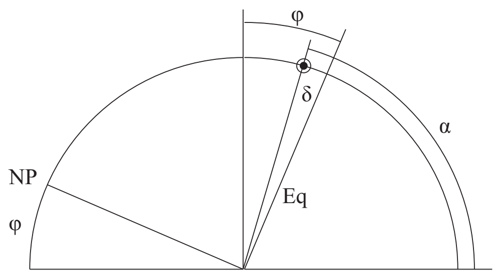

Fig. 24

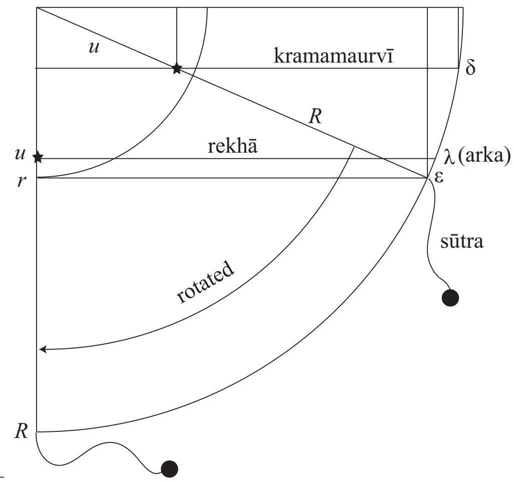

Fig. 25

Rationale. From the similar right trilaterals on the quadrant, we have the relation, $R : R\sin\varepsilon = u : R\sin\delta$. On the other hand, from the similar right trilaterals inside the celestial sphere (cf. Fig. 22), we have the relation, $R : R\sin\varepsilon = R\sin\lambda : R\sin\delta$. Hence follows $u = R\sin\lambda$.

Verse 6.5-6: sun's declination (δ) $\rightarrow$ longitude (λ)

Procedure. Given the declination (δ), measure it out on the arc from its beginning. Stretch the string over the intersection of the declination circle and the horizontal line (*kramajīvā*) that meets the point of the declination (δ). Count the number of degrees on the arc from its beginning up to the point indicated by the string. Obtained is the sun's longitude (λ).

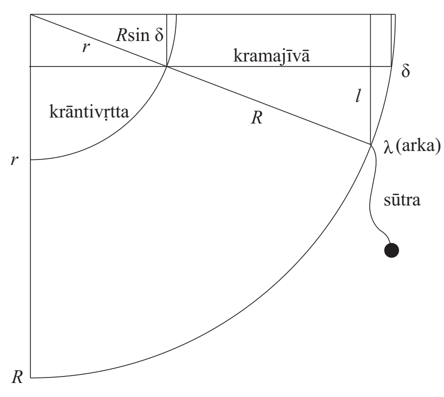

Fig. 26

Rationale. From the similar right trilaterals on the quadrant, we have the relation, $R : \ell = r : R\sin\delta$, or $R : r = \ell : R\sin\delta$, where $r = R\sin\varepsilon$. On the other hand, from the similar right trilaterals inside the celestial sphere (cf. Fig. 22), we have the relation, $R : R\sin\varepsilon = R\sin\lambda : R\sin\delta$. Hence follows $\ell = R\sin\lambda$.

Verses 6.7-9: When the horizontal line that arises from the declination does not touch the declination circle

If the quadrant has the declination circle, it inevitably intersects the horizontal line that arises from the point of declination on the arc. Therefore, the situation intended here must be either (1) the case where the quadrant does not have the declination circle or (2) the case where the declination circle on the quadrant is not entirely available. Cf. verses 5.3-5ab above.

Procedure (cf. Fig. 26). Put a mark for memory on the horizontal line that arises from the point of the given declination. Place the cursor of the string at the point of $\mathrm{Sin}\,\varepsilon$ (= 24 units) on the south-north line. Rotate the string and fix it on a point of the arc so that the cursor may touch the marked horizontal line. The rest of the procedure is the same as above (verses 6.5-6).

III.7 Chapter Seven: Terrestrial latitude

Verses 7.1-2ab: sun's meridian altitude (α) and northern declination (δ) $\rightarrow$ latitude (φ)

Calculation.

$$\varphi = 90 - \bar{\varphi}, \quad \text{where} \quad \bar{\varphi} = \alpha - \delta$$

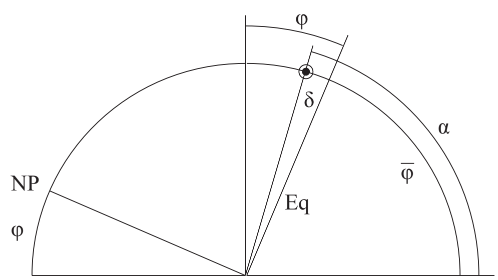

Fig. 27

Verses 7.2cd-3: sun's meridian altitude (α) and southern declination (δ) $\rightarrow$ latitude (φ)

Calculation.

$$\varphi = 90 - \bar{\varphi}, \quad \text{where} \quad \bar{\varphi} = \alpha + \delta$$

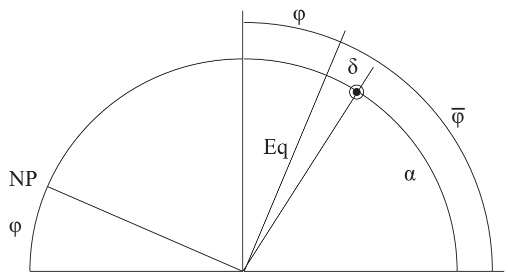

Fig. 28

Verses 7.4-5ab: sun's meridian altitude (α) and northern declination (δ) greater than the latitude $\rightarrow$ latitude (φ)

Calculation.

$$\varphi = \alpha + \delta - 90$$

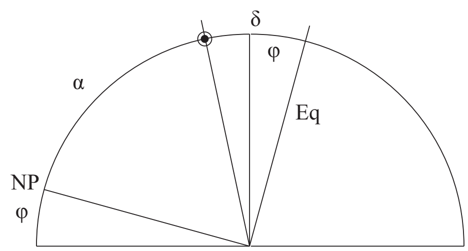

Fig. 29

Verse 7.5cd: When the declination is absent ($\delta = 0$)

When the declination is absent ($\delta = 0$), the sun's meridian altitude is equal to the colatitude of the locality ($\bar{\varphi}$).

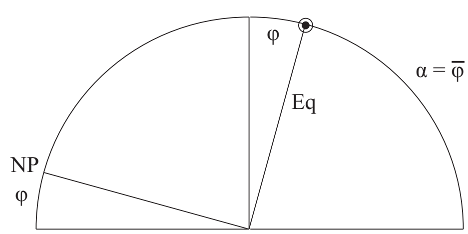

Fig. 30

Verse 7.6: When the sun is at the zenith ($\alpha = 90$)

When the sun is at the zenith ($\alpha = 90$), the declination (δ) of that moment is equal to the latitude of that place.

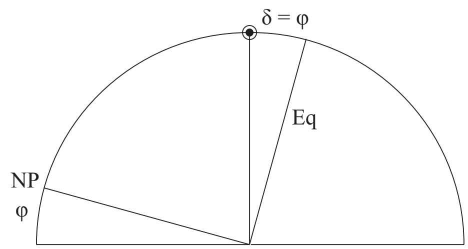

Fig. 31

Verse 7.7: Direction of shadow when the latitude (φ) is smaller than the sun's maximum declination (ε)

The shadow of a gnomon falls sometimes toward north, sometimes toward south, and sometimes disappears.

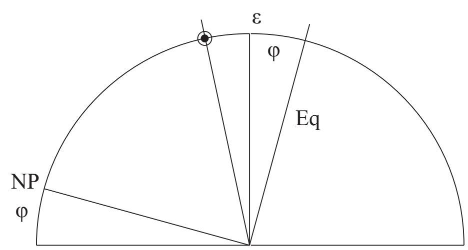

Fig. 32

Verse 7.8: Direction of shadow when the latitude (φ) is greater than the sun's maximum declination (ε)

The shadow of a gnomon always falls to the north.

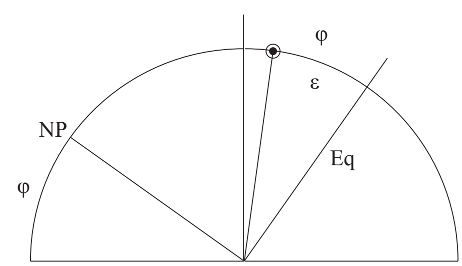

Fig. 33

The latter half of verse 8 says that 'All this will be the reverse in the south of the equator,' but in fact the only differences are the following three cases.

1. For a northern declination, $\varphi = 90 - \bar{\varphi}$, where $\bar{\varphi} = \alpha + \delta$.
2. For a southern declination, $\varphi = 90 - \bar{\varphi}$, where $\bar{\varphi} = \alpha - \delta$.
3. When the latitude (φ) is greater than the sun's maximum declination (ε), the shadow of a gnomon always falls to the south.

Cf. verses 4.11 and 12.

Verses 7.9-10: Determination of the latitude from meridian altitudes of a circumpolar star

Calculation. Let the two meridian altitudes of a circumpolar star be α_1 and α_2. Then,

$$\varphi = \frac{\alpha_1 + \alpha_2}{2}$$

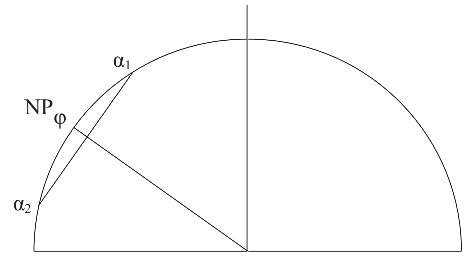

Fig. 34

III.8 Chapter Eight: Shadow of the gnomon

Verse 8.1: Two types of shadow

There are two types of shadow, horizontal and vertical, which Bhūdhara calls 'straight' (*ṛju*) and 'reverse' (*viparīta*), respectively. In both types, the line between the top of the gnomon and the end of the shadow is called hypotenuse (*karṇa*).

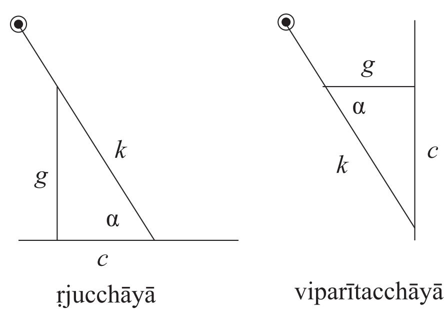

Fig. 35

We have the relationships,

$$c = g\tan(90-\alpha) = \frac{g}{\tan\alpha} \quad \text{and} \quad c = g\tan\alpha,$$

respectively for the horizontal and vertical types, where g is the height of the gnomon, α the sun's altitude, and c the length of the shadow.

Though not mentioned explicitly by Bhūdhara, these relationships are the basis of the rules that follow.

Verse 8.2: Three types of gnomon

Bhūdhara classifies gnomons according to their lengths: 7 *aṅgulas*, 12 *aṅgulas*, and 60 *aṅgulas*. In India, the length of the gnomon is divided traditionally into 12 *aṅgulas*. The gnomon of 60 *aṅgulas* is just a variant of the former, where each of the 12 *aṅgulas* is further subdivided into 5 units. Gnomons of 7 feet are prevalent in the Islamic world. On the back of the Islamic astrolabes are to be found shadow squares for the gnomon of 7 feet and for that of 12 digits. Following this practice, Sanskrit astrolabes also carry shadow squares for both the types of gnomons.[1] In literature, the gnomon of 7 *aṅgulas* is used for obtaining the shadow to be employed in a formula for the portion of time elapsed before noon or remaining after noon, which is prescribed in an anonymous arithmetical work *Pañcaviṃśatikā* (before AD 1429),[2] and presumably also in the *Gaṇitasārakaumudī* of Ṭhakkura Pherū (ca. 1315).[3]

Verse 8.3: Gnomon of seven *aṅgulas*

The gnomon classified as 7 *aṅgulas* is said to have an actual length of 6;30 or 6;40 *aṅgulas*. It is not clear why it should have a length which is less than 7 *aṅgulas*. One would expect it to be slightly longer than 7 *aṅgulas* so that it could be buried firmly in the ground.

Verses 8.4-6: sun's altitude (α) $\rightarrow$ horizontal shadow (c)

Procedure. Given the altitude (α), measure it out on the arc from its beginning and stretch the string up to that point. Count the number of units in g on the south-north line from its beginning and find the horizontal line (*kramajyā*) at that point. Find the vertical line (*utkramajyā*) passing through the intersection of the string and the horizontal line. Count the number of units on the east-west line from its beginning up to the point from which the vertical line arises. Obtained is the length of the shadow (c).

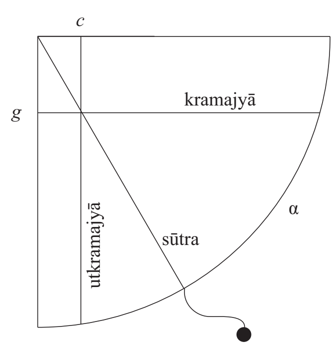

Fig. 36

[1] See, for example, Pingree, *Eastern Astrolabes*, pp. 198-99.

[2] Takao Hayashi (ed. & tr.), 'The *Pañcaviṃśatikā* in its Two Recensions: A Study in the Reformation of a Medieval Sanskrit Mathematical Treatise,' *Indian Journal of History of Science* 26 (4), 1991, 395-448, esp. 441-43.

[3] SaKHYa (ed. & tr.), *Gaṇitasārakaumudī: The Moonlight of the Essence of Mathematics by Ṭhakkura Pherū*, New Delhi: Manohar, 2009, pp. 160-62.

Verses 8.7-9: sun's altitude (α) $\rightarrow$ hypotenuse (k)

Procedure. Given the altitude (α), measure it out on the arc from its beginning and stretch the string up to the point. Count the number of units in g on the south-north line from its beginning and find the horizontal line (*kramajyā*) at that point. Place the cursor (*mṛgāsya*) at the intersection of the string and the horizontal line and rotate the string onto the south-north line. Count the number of units on the south-north line from its beginning up to the point of the cursor. Obtained is the length of the hypotenuse (k).

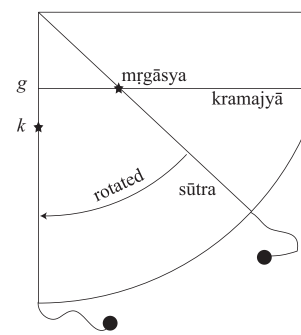

Fig. 37

Verse 8.10: sun's altitude (α) $\rightarrow$ vertical shadow (c)

The procedure is almost the same as in the case of the horizontal shadow (verses 4-6 above), the only difference being that the altitude (α) is counted on the arc from its end.

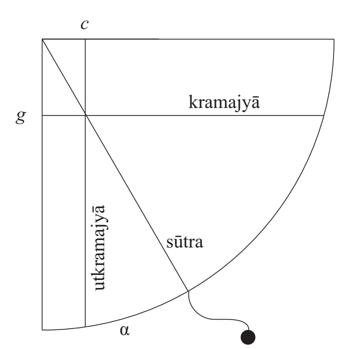

Fig. 38

Verses 8.11-14: sun's altitude (α) $\rightarrow$ horizontal shadow (c) when $R \sin\alpha < g$

If $R \sin\alpha < g$, the string and the horizontal line that arises from g do not intersect. In that case, the n-th horizontal line ($n < g$) is utilized in place of the g-th line.

Procedure. Given the altitude (α), measure it out on the arc from its beginning and stretch the string up to that point. Find the vertical line (*utkramajyā*) passing through the intersection of the string and the n-th horizontal line (*kramajyā*). Count the number of units (c_n) on the east-west line from its beginning up to the point from which the vertical line arises. Then

$$c = c_n \times \frac{g}{n}.$$

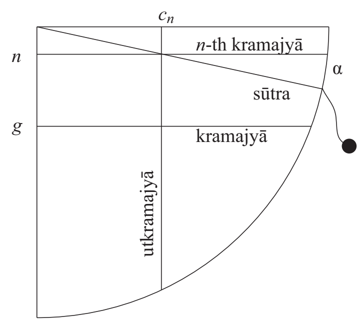

Fig. 39

Rationale. Since $\tan\alpha = n/c_n$ on the quadrant,

$$c = \frac{g}{\tan\alpha} = \frac{g}{n/c_n} = c_n \times \frac{g}{n}.$$

Verses 8.15-17: sun's altitude (α) $\rightarrow$ vertical shadow (c) when $g = R = 60$

Procedure. Given the altitude (α), measure it out on the arc from its end and find the vertical line (*utkramajyā*) that meets that point. Also measure out the same altitude (α) from the beginning of the arc and stretch the string up to that point. Place the cursor at the intersection of the string and the vertical line and rotate the string up to the south-north line. Count the units on the south-north line from its beginning up to the cursor. Obtained (u) is the length of the shadow.

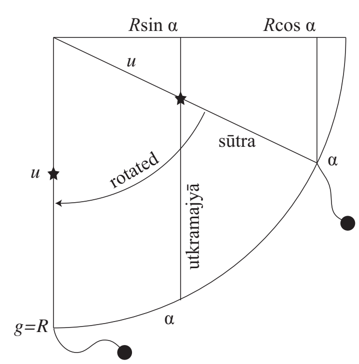

Fig. 40

Rationale. From the similar right trilaterals on the quadrant, we have the relation, $R\cos\alpha : R = R\sin\alpha : u$. Hence follows

$$u = \frac{R \times R\sin\alpha}{R\cos\alpha} = R\tan\alpha = g\tan\alpha = c.$$

III.9 Chapter Nine: Altitude of the sun

Verses 9.1-4: shadow (c) $\rightarrow$ sun's altitude (α)

Procedure. Measure out the units in c on the east-west line from its beginning and find the vertical line (*utkramajīvā*) that arises fron that point. Stretch the string through the intersection of the vertical line and the horizontal line (*kramajyā*) that arises from g on the south-north line. Count the degrees on the arc from its beginning up to the point indicated by the string. Obtained is the sun's altitude (α).

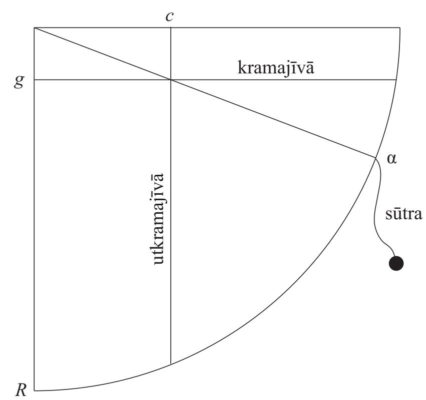

Fig. 41

This is for a 'straight shadow'. For a 'reverse shadow', we have to count the degrees on the arc from its end.

Verse 9.5: Condition

The above rule is for the gnomons of $g = 7$ and 12 *aṅgulas*. When $g = R = 60$ *aṅgulas*, no horizontal line that arises from the point of g exists on the quadrant.

III.10 Chapter Ten: Length of the day and the night

Verses 10.1-5: shadow (c) and declination (δ) $\rightarrow$ half of the ascensional difference (ω) when $\alpha = 90 - \varphi$

Procedure. Measure out the units in c on the east-west line from its beginning and find the vertical line (*utkramajīvā*) that arises fron that point. Measure out the degrees in δ on the arc from its beginning and stretch the string up to that point. Find the horizontal line (*kramajīvā*) passing through the intersection of the string and the vertical line. Count the number of the degrees on the arc from its beginning up to the point indicated by the horizontal line. Obtained (ω) is half of the ascensional difference (*cara*).

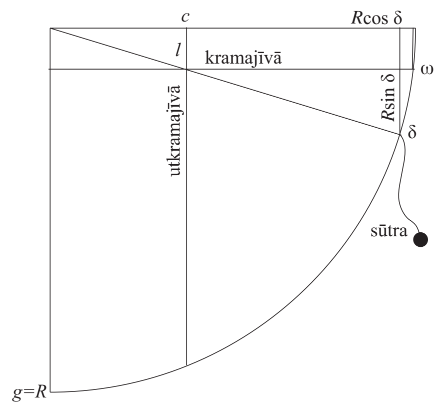

Fig. 42

Rationale. Let $g = R$ and 'apply the state of being altitude degrees to the [co-] latitude degrees', that is to say, $\alpha = 90 - \varphi$. Then, $c = g/\tan\alpha = R\tan\varphi$. From the similar right trilaterals on the quadrant, we have the relation, $R\cos\delta : R\sin\delta = c : \ell$. Hence follows $\ell = c\tan\delta = R\tan\varphi\tan\delta$.

On the other hand, from the next two figures we have the relations, $R\cos\varphi : R\sin\varphi = R\sin\delta : e$ and $R\cos\delta : e = R : E$. Hence follows $E = R\tan\varphi\tan\delta$.

Hence follows $\ell = E = R\sin\omega$, where ω is called *carārdha* ('half of the ascensional difference').

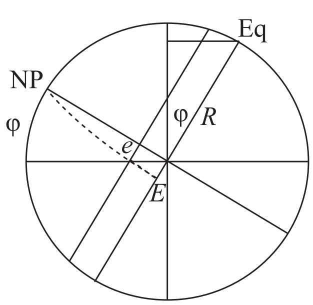

Fig. 43

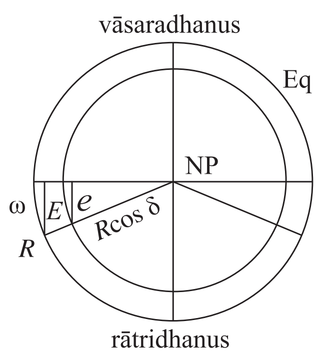

Fig. 44

$$\text{Day-arc } (\textit{vāsaradhanus}) = (90 + \omega) \times 2.$$
$$\text{Night-arc } (\textit{rātridhanus}) = 360 - \text{Day-arc}.$$

Verse 10.6: Other three cases

The sign of ω in the calculation of the day-arc (verses 10.1-5) is plus or minus according to the directions of the declination (δ) of the sun and of the latitude (φ) of the locality.

Day-arc

	δ in north	δ in south
φ in north	$(90+\omega)\times 2$	$(90-\omega)\times 2$
φ in south	$(90-\omega)\times 2$	$(90+\omega)\times 2$

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