Galen's Computation of Medical Weeks:
Textual Emendations, Interpretation History,
Rhetorical and Mathematical Examinations

Stephan Heilen

Universität Osnabrück

Viro doctissimo Wolfgang Hübner octogenario d.d.d.

Abstract

The seven steps of Galen's unit-fractional computation in *On Critical Days*, pp. 932.5–933.12 Kühn, which presents corruptions in the manuscript tradition, are here for the first time ever presented in a satisfactorily emended Greek version with English translation. Moreover, this paper provides a survey of the history of scholarly engagement with Galen's computation from antiquity through Syriac, Arabic, Hebrew, and Latin sources to the 17th century. This survey covers, but is not limited to, the medical school of Alexandria, Sergius of Rēšʿaynā, Ḥunayn ibn Isḥāq, Qusṭā ibn Lūqā, Ibn Sīnā, Ibn al-Nafīs, al-Nasawī, Gerard of Cremona, Pietro d'Abano, Niccolò da Reggio, Pierre d'Ailly, Giovanni Pico della Mirandola, Luca Gaurico, Jean Lalamant, Jean Guijon, Giovanni Manardi, Girolamo Cardano, and Antonio Magini. Two further sections are devoted to Galen's Platonizing rhetoric of mathematical accuracy and ten questions that arise from his computation. Thus, Galen's complex motives, his astronomico-mathematical education, his astrological development of a new chronometric concept, the single steps of his computation, and the merits (and limits) of his interpreters are comprehensively elucidated within a perspective including Egyptian, Greek, and Roman mathematics. The results are summarized in the final section.

I. Introduction

While many works of the vast Galenic corpus have, since the late nineteenth century, been made available in splendid critical editions,[1] the situation is still bleak in the case of one of Galen's most influential works, *On Critical Days* (Περὶ κρισίμων ἡμερῶν).[2] This is particularly true with regard to the unique computation of so-called 'medical weeks' in chapter 3.9.[3] By 'unique' I mean that the entire Galenic corpus seems to lack a parallel (that is, any similarly complex sum of unit fractions) whose treatment could elucidate the present case. In the most recent and supposedly authoritative edition by Kühn (1825, henceforth 'K.'), Galen's computation (pp. 932.5–933.12) is unintelligible, in both the Greek text and the

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[1] Mostly in the *Corpus Medicorum Graecorum* (CMG), section V.
[2] For a summary of its contents, cf. Cooper 2011a, 11–12.
[3] Information on its purpose will be given below.

Latin translation, due to corruption of several numerical values. On closer inspection, how-
ever, one finds that neither the Greek text nor the Latin translation are Kühn's work: In-
stead both are verbatim reproductions of Chartier's Paris edition of 1679.[4] But even Chart-
ier drew on earlier Renaissance editions. It turns out that the numerical corruptions in the
Greek text of our passage go back, with minor variants, through the edition of Basel (1538)
to the Aldine *editio princeps* (Venice 1525), and beyond that to MS Venice, Marc. gr. app.
V 8.[5] Occasionally, Kühn's text is even inferior to the Aldine.[6] Moreover, the Latin transla-
tion printed by Chartier and Kühn, which in various details seriously fails to match their
Greek text,[7] turns out to be a slightly modified version of the Latin translation made by a
German student, Johannes Winter of Andernach, first published in the Paris edition of
1529.[8] In other words: There is no modern critical edition of Galen's *On critical days*, and
the early modern editors and translators did not understand Galen's mathematical reason-
ing.[9]

Two new editions of the complete Greek text of *On Critical Days* have been an-
nounced, one by Bengt Alexanderson[10] and another, more recently, by Glen M. Cooper in
his meritorious edition of the Arabic translation made by Ḥunayn ibn Isḥāq in the 9th cen-
tury.[11] While it is to be hoped that one of these editions will appear soon, there is reason to
be concerned that the computation in question (pp. 932.5–933.12 K.) will not be dealt with
adequately: Cooper has made it clear through two anticipated statements that he plans to
follow earlier editors in adopting a manuscript reading which will here be shown to be an
inauthentic and mathematically impossible intrusion into the text.[12] Surprisingly, Cooper

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[4] On Chartier's and Kühn's editions, see pp. 235–236 below. For an up-to-date yet incomplete survey of Greek
and Latin editions of *On Critical Days* see Fichtner 2017, 54–55, no. 68 (with several hyperlinks to digitized
copies). The *Catalogus Translationum et Commentariorum* (CTC) does not include an article on Galen (see
http://catalogustranslationum.org/index.php), but that matters little in view of the meritorious work of Durling
1961 and Fortuna's database for the Latin translations at http://www.galenolatino.com/index.php?id=2&clean=1.

[5] See below, n. 34. For a concise account of how all three later editions of Galen's Greek *opera omnia* depend
on the Aldine edition, "il vero e unico lavoro originale," see Perilli 2005, 428.

[6] Cf. the example adduced by Cooper 2012, 633–634, regarding p. 806.6–7 K. οὐ γὰρ αὕτη βάσανος ὀρθὴ τῶν
κριτικῶν εἶναι πιστὰς, where ἡμερῶν τὸ καὶ χωρὶς τῶν ἄλλων σημείων τῶν κριτικῶν (transmitted by MS Ven.
Marc. gr. app. V 8 and translated by Ḥunayn in Cooper 2011a, 163.13–14) is missing after τῶν κριτικῶν (cf.
Cooper 2011a, 510).

[7] Cf., for example, p. 504C Ch. = p. 932.7 K. εἴκοσιν ἓξ ἡμερῶν καὶ ἡμισείας / *dies vigesimussextus et dimidius*
and p. 504E Ch. = p. 933.6–7 K. ἑξήκοντά τε καὶ ἑκατοστὰ εἰκοστὰ καὶ σμ′ / *sexagesima parte, centesima vi-
gesima et ducentesima quadragesima*.

[8] See Durling 1961, 255 (no. 1529.3), Fortuna 2012b, 321, and the edition of Winter's translation below, p. 226
(**t3**). See also n. 290 below.

[9] This is equally true of the only existing modern full translation of Galen's *On Critical Days* which faithfully
reproduces in Spanish the wording and the mathematical mistakes of Kühn's Greek edition (García Sola 2010,
174–175), adding one more mistake by omitting p. 932.10 K. ἡμίσεος. There is not discussion of Galen's math
in the notes to that translation. I thank R. Wittwer (BBAW Berlin) for sending me scanned pages of this very
rare book.

[10] Cf. Garofalo 2003, 52, n. 36.

[11] Cooper 2011a, 81. Cf. Cooper's personal site at https://independentscholar.academia.edu/GlenCooper where
he announces two more volumes to follow: "Galen's De diebus decretoriis: Greek Edition, Translation, Histor-
ical and Philological Commentary, Graeco-Arabic Lexicon" (vol. 2), and "Galen's De Crisibus: Translation,
Commentary, and Graeco-Arabic Study" (vol. 3).

[12] Cf. p. 213 with n. 95 below.

did so despite his awareness of an Italian contribution by Garofalo (2003) in which the flawed manuscript reading in question is athetized.[13]

Garofalo's contribution is of mixed value: He begins with important observations regarding Galen's solid astronomical education, his clearly documented belief in soli-lunar and, to a lesser extent, planetary influences, and the problem that Galen tries to solve, namely the discord within the so-called 'rationalist' sect regarding the critical days, with himself and the Hippocratic tradition on one side, for whom the 20th and 27th days of an illness are critical, and, on the other, the strongest rationalist medical school of his time, the pneumatics (headed by Archigenes), for whom the 21st and 28th days are critical.[14] Garofalo's explanation of Galen's strategy for solving this problem by inventing an artificial, astronomically-based medical week, leads to a synopsis[15] that consists of four columns, the first two containing an emended version of the Greek text (pp. 932.5–933.12 K.) with Italian translation, the third an Italian translation of the corresponding passage of Ḥunayn's Arabic translation,[16] on which Garofalo's Greek emendations are based,[17] and the fourth the Latin translation of the Venetian edition of 1490 (= **t1** in my list of sigla below). Regrettably, each of these four columns contains indisputable mistakes which look, for the most part, philological rather than typographical and (regardless of this distinction) impair the usefulness of his synopsis.[18] Moreover, Garofalo neglects to explain why he selected just these two sources, Ḥunayn and **t1**, for comparison (that is, omitting Greek

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[13] Cf. Cooper 2011a, 569 (bibliography). See also Cooper 2012, 634 with n. 91, where he explicitly adopts a mathematical correction of Kühn's Greek text from Garofalo's article (p. 54) but soon after (Cooper 2012, 640–641) repeats his own erroneous view on the inauthentic manuscript reading mentioned above from Cooper 2011a, 372, this time without reference to Garofalo's correction on the previous page (Garofalo 2003, 53).

[14] Among the three major medical sects in antiquity, that of the rationalists (or dogmatists) was less clearly defined and less coherent than those of the empiricists and the methodists. The rationalists were united in their belief that it is not sufficient to base medical activity on empirical knowledge but that one must find rational explanations of the antecedent causes of illnesses. These rational explanations tended to be more or less speculative, as in Galen's case here. Cf. Nutton 1997, Nutton 1999, 1111, and Siraisi 1990, 3–5, especially p. 5: "Galen took a moderate position. He had no use for the methodists but tried to harmonize the views of the rationalists and the empiricists. Thus, while his works are mainly rationalist in their approach, they also contain empiricist elements."

[15] Garofalo 2003, 52–54.

[16] Since Cooper's edition was not published at that time, Garofalo's translation is based on MS Madrid, Escorial arab. 797 (dated 1217 CE by Cooper 2016, 39).

[17] Cf. Garofalo 2003, 52: "testo greco ... emendato sulla base della traduzione araba."

[18] Col. 1: three times τρίτον instead of τρίτου; ἑξακοστὸν; διακοστὸν; τεσσερακοστόν (according to the TLG, this morphologically incorrect form is attested only in a few works by late antique Christian authors of the 3rd to 7th centuries CE, while the Galenic corpus contains 89 attestations of τεσσαρακοστ-); ξ v (read ξ′); col. 2: "mancante di ¼" (lapsus for ⅙ = ἕκτῳ in the Greek); col. 3: "ventisei giorni e un terzo" (the first word is a lapsus for 'ventisette', that is, twenty-seven, the mathematically correct reading of the Arabic manuscript used by Garofalo [Madrid, Escorial arab. 797]; I thank Petra Schmidl for verifying this based on the *apparatus criticus* of Cooper 2011a, 372) and "cento venti" (read 'una parte di cento venti'), "traveremo" (typo, read 'troveremo'); col. 4: *et fortis* (read <*et*> *fortis*); <*non*> *fit* (wrong assumption of a polar error in **t1** due to Garofalo's loss of two full lines of the text of **t1**, caused by his *saut du même au même* from lines 32/33 *utras/que* to lines 34/35 *utros/que* in **t1**); *mediata* (read *medietas*), *.xlviii. hoc* (read *.xlviii. et hoc*); *.cc. et* [*pars*] *xl.* (read *.cc. et .xl.*, there is no *pars* in **t1** that could be athetized, nor does the sense allow its conjectural insertion); footnotes to the synopsis: n. 45: διακοστὸν, τεσσερακοστόν, σ′ (read ο′) and διακοσιά; n. 51: ξ′ (read ρκ′); n. 54 "emendando in ἑξεκοστὸν καὶ ἑβδομηκοστόν" (the last word is mathematically impossible and contradicts Garofalo's emendation in the main text); n. 55 "$(6 + \frac{35}{48}) \times 3 = 20 + \frac{1}{6}$" is wrong (omission of "$+ \frac{1}{48}$"). Moreover, Garofalo is inconsistent in partly following the Aldine's reading ἡμίσεως, partly following Kühn's change to ἡμίσεος (ibid. notes 42, 43, 47).

manuscripts and other Latin translations), and what their respective characteristics and heuristic values are. Garofalo seems to be unaware that **t1** was translated from Ḥunayn's Arabic version, which is equally included in the synopsis, thus making the inclusion of **t1** questionable, if not useless. These methodological shortcomings and the aforementioned philological and typographical mistakes justify the assertion that the Greek text of Galen's computation has, to the present, never been published in a satisfactorily emended version.

One of the main goals of the present article is to address this philological *desideratum.* However, it would be a disproportionate effort to identify, procure, collate, and stemmatically analyze all (at least sixteen)[19] extant Greek manuscripts of the short passage in question, especially in view of the critical editions announced by Alexanderson and Cooper. Hence I shall not provide a full-fledged critical edition of Galen's computation but rather a provisionally emended text meant to improve the mathematical and philological deficiencies of the hitherto available editions. These corrections will benefit from three manuscripts that seem to have escaped editorial attention to the present. One is Vat. Barb. gr. 221 (15th/16th century), ff. 40v–41r (henceforth: **B**) which occasionally gives a better text than the printed editions.[20] The other two are what seem to be the only extant manuscripts of the Latin translation made by Niccolò da Reggio in the early 14th century.[21] This translation, which has not been taken into account by any of the scholars who wrote about Galen's computation, is especially important because Niccolò translated *verbatim* from a now lost Greek manuscript of high quality (and age, too: it was older than most, if not all, extant Greek manuscripts). It is to be hoped that the emendations proposed here will be useful for future editions of *On Critical Days*.

The second goal is to provide a survey of the history of scholarly engagement with Galen's computation from antiquity to the 17th century, when interest in medical astrology began to fade. This survey will elucidate the historical importance of *On Critical Days* in general and of our computation in particular. Since Galen's works were known mainly through Latin translations, these will play an important role in the survey. Moreover, it will pay due credit to individual insights of earlier scholars while simultaneously showing the limits of their investigations (and perhaps of their interest): In fact, some questions that pertain to our mathematical examination were never asked by any of them, nor by later scholars to the present day.[22]

Finally, I wish to offer an analysis of Galen's Platonizing rhetoric of mathematical accuracy and a thorough mathematical examination of ten leading questions in ten subsections, incorporating current research in the history of astronomy and mathematics.
This holistic approach, based on interconnected sections, will enable us to address fully the complexity of Galen's passage. All results will be summarized in the final section ('Conclusions').

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[19] Diels 1905, 90–91, lists fifteen manuscripts that are relevant to our passage, to which at least one more (**B**, see below) must be added.

[20] Online: https://digi.vatlib.it/view/MSS_Barb.gr.221. This manuscript was unknown to Kühn (vol. I, Leipzig 1821, p. CVII). Its readings are partly better than Kühn's text, partly wrong (for the latter, cf., for example, n. 64 below).

[21] See below, p. 222.

[22] See the list of questions below, p. 240.

II. Galen's Motivation and Computational Approach

The third book of *On Critical Days* is devoted to astrological medicine.[23] Astrology and medicine were, in Galen's age, closely related sciences that shared the same empirical method and were in many cases practiced by one and the same individual.[24] Galen agrees with the traditional medical view[25] that diseases have decisive turning points for better (recovery) or worse (death), the most important of these so-called 'crises' (κρίσεις, literally 'judgments') occurring after one, two, and three weeks from the time the patient took to his bed (the so-called 'decumbiture'). Galen's problem is that the simple scheme of subdividing a month into four weeks by subdividing 28 days into four times seven days does not exactly match the astronomical reality (the sidereal lunar month comprises 27 ⅓ days, the synodic month 29 ½ days) nor, more importantly, the medical reality because physicians had long noted that, apart from the crises on the 7th and 14th days, on which there was medical consensus, it was more often the 20th than the 21st day that was critical. This had led to a disagreement, not only among ancient physicians generally but also among members of the 'rational' school (which included Galen), as to which of the debated days was actually critical, the 20th or the 21st, or whether both were critical. This problem called for a solution because the correct definition of critical days was of paramount importance for medical diagnosis, therapy, and prescription. This explains the leading question of chapter 3.9 of our work: "Why are both days critical, the 20th and the 21st, but much more [*as already Hippocrates thought*] the 20th?"[26]

Galen's solution is not to reject the traditional medical theory of critical days but to modify it by giving it a new, astrological foundation:[27] the 'critical days,' he argues, originate in the course of the moon (the astrological symbol of biological growth and bodily functions), more precisely: from a combination of two distinct lunar effects, one based on its motion through the zodiac, that is, on the sidereal month (27 ⅓ days), the other on specific geometrical configurations (square aspects and oppositions) of the moon in relation to the sun, that is, on the synodic month (29 ½ days). With regard to the latter, Galen advances the original idea that the synodic period must be shortened by three full days because this is about the time span the moon is invisible at the time of its conjunction with the sun (new moon) and thus not producing any luminosity-based astrological effects. The arithmetic mean of the sidereal month (27 ⅓ days) and the curtailed synodic month (26 ½ days) is the new artificial 'medical month' of 26 $^{11}/_{12}$ days. Since each of these months can be divided into four medical 'hebdomads' (that is, weeks, literally 'periods of seven'), these will end at times that are again different from the respective sidereal or synodic hebdomads.

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[23] For its history, see the excellent survey by Greenbaum 2015. On Galen's attitude towards astrology and astrologers, see Cooper 2011b, 125–127, who gives a useful survey of recent scholarship.

[24] Cf. Ptol. *tetr.* 1.3.18–19 (Robbins 1940, 30–33), and Tester 1987, 60–64.

[25] One finds it already in the Hippocratic corpus.

[26] Διὰ τί δ' ἄμφω κρίνουσιν, ἥ τε εἰκοστὴ καὶ εἰκοστὴ πρώτη, πολὺ δὲ μᾶλλον ἡ εἰκοστή; (p. 928.11–12 K.).

[27] He emphasizes the novelty of his approach thus (Gal. *in Hipp. progn. comm.* 3 vol. XVIIIb p. 241.1–3 K.): τίς δέ ἐστιν αἰτία τούτων, πεπειράμεθα ἡμεῖς εἰπεῖν ἐν τῷ τρίτῳ τῶν Περὶ κρισίμων ἡμερῶν οὐδενὸς τῶν ἔμπροσθεν οὐδ' ἐπιχειρήσαντος εἰπεῖν. I agree with Garofalo 2003, 51, who sees no reason to disbelieve Galen's claim.

Galen expresses these various values as sums of unit fractions of 24-hour-days.[28] In the following table, they are supplemented in parentheses with their more reader-friendly equivalents in days and hours:[29]

Definition	End points of the hebdomads (H1–H4) from the time of the decumbiture			
	H1	H2	H3	H4
a: curtailed synodic month	– (6d 15h)	– (13d 6h)	– (19d 21h)	$26\,\frac{1}{2}$ (26d 12h)
b: medical month	$6\,\frac{1}{2}\,\frac{1}{5}\,\frac{1}{60}\,\frac{1}{120}\,\frac{1}{240}$ (6d 17 $\frac{1}{2}$ h)	– (13d 11h)	$20\,\frac{1}{6}\,<\frac{1}{48}>$[30] (= 20d 4<$\frac{1}{2}$>h)	$26\,\frac{1}{2}\,\frac{1}{3}\,\frac{1}{12}$ (26d 22h)
c: sidereal month	– (6d 20h)	– (13d 16h)	– (20d 12h)	$27\,\frac{1}{3}$ (27d 8h)

Table 1: End points of the hebdomads
based on the different definitions used by Galen

Since the moment H3b exceeds twenty full days (that is, 20×24 hours from the time of the decumbiture) by no more than a few hours, the last day of the third medical week coincides almost entirely with the twentieth day.[31] Therefore Galen thinks he has satisfactorily explained why the 20th day is, in compliance with Hippocrates' teaching, far more often than the 21st day a critical day.[32]

III. Emended Text and Translation

As emphasized above (Section I), the following text is *not* a new critical edition including systematic *recensio* and *examinatio* of all extant manuscripts but a provisionally emended text. The corruptions in the hitherto available Greek editions originated mostly from misinterpretations of Greek number-letters in the course of textual transmission.[33] These misinterpretations concern the grammatical case of certain number-letters and, more detrimen-

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[28] The reason for this will be discussed below (p. 247, Question 4). I follow the usual mathematical notation of ancient, especially Egyptian sums of unit fractions; cf. Gillings 1972, 104: "no plus sign is included in the unit fractions, mere juxtaposition implying addition."

[29] Cf. *On Critical Days*, pp. 929–933 K. Galen omits seven values, which are here indicated with en-dashes and supplied among the equivalents in days and hours, based on his clear mathematical rationale.

[30] On this addendum, see n. 69 below and p. 257, Question 6.

[31] More on this below, p. 265, Question 10.

[32] He emphasizes this in a concluding statement (ch. 3.12, p. 937.10–13 K.): ἐπέδειξα δὲ καὶ τῶν παρεμπιπτουσῶν ἡμερῶν τὰς αἰτίας καὶ διὰ τί τὰς τρεῖς ἑβδομάδας οὐκ εἰς τὴν κα΄, ἀλλ' εἰς τὴν κ΄ μᾶλλον ὁ Ἱπποκράτης ἀξιοῖ περιάγεσθαι.

[33] It is not known whether these number-letters go back to Galen himself or to later copyists.

tally, confusions between cardinal and ordinal numbers. Moreover, one arithmetic value in our passage turns out to be an intrusion that must be deleted (see n. 67), while another value, which is missing in all Greek witnesses, must be supplied in order to restore the original text (see n. 69).

The respective page and line numbers of Kühn's text (pp. 932.5–933.12) will be indicated in parentheses (for example, '932.5'); discrepancies between my text and Kühn's edition will be underlined. Since Galen's computational procedure consists of seven steps, these will be marked as **S1** to **S7**. For the sake of clarity, I shall (like the scribe of **B**) write out all Greek numbers which are, instead, without a discernable rationale partly written out and partly expressed with number-letters in the manuscripts **PV** (see below) and in the four hitherto available editions.

The footnotes will record variant readings of the four editions of the Greek text in full, including variants between number-letters and prose number-words. Moreover, the notes will record noteworthy variants of three select Greek manuscripts I have examined: (1) the important Barberini manuscript (**B**), not mentioned in any publication on Galen's *On Critical Days* to the present, (2) MS Paris, BnF gr. 2272 (**P**), closely related to **B** and mentioned by Cooper as the main representative of one of his four manuscript families, and (3) MS Venice, Marc. gr. app. V 8 (**V**), which represents another of Cooper's manuscript families and, according to his research, the ancestor of all four printed editions.[34] My own very limited collation (that is, limited to three manuscripts and about one page of text) tends to confirm Cooper's assertion, but I found two disagreements between **V** and the Aldine edition (**v**) where **V** is right and **v** (as well as the three later Greek editions) wrong.[35] While the first case may be nothing more than a lapsus in **v**, the second is stemmatically significant because **v** adds words to the text of **V** that are both mathematically wrong and matched by **B** (and possibly other manuscripts as well). Hence, future editors may wish to investigate whether the Aldine edition was based on **V** and at least one additional manuscript (presumably one among the other three extant Venetian manuscripts).[36]

The manuscripts **BPV** are marred by a considerable number of obviously incorrect readings and/or mathematically irrelevant *minutiae* which will not be recorded in the notes to the Greek text in order not to overload them.[37] Nonetheless, the value of **BPV** in our

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[34] Cf. Cooper 2016, 20, n. 72: "I am in the process of editing the Greek text of the *Critical Days*, so here is presented the *tentative* results of the collation. As near as I can determine, the eleven extant manuscripts belong to two major families, with four significant branches, which are represented by manuscripts B, H, M, and P." These sigla denote, according to Cooper's appendix (ibid., 38), MS London BL Harley 6305, ff. 83r–148v, MS Oxford, Bodleian, Laud. gr. 58, ff. 319r–364v, MS Paris, BnF gr. 2272, ff. 101v–148v, and MS Venice, Marc. gr. app. V 8 (coll. 1334), ff. 62r–102r. Cooper continues (ibid.): "It is also noteworthy that the Venetian M is virtually identical to Kühn's Greek edition (1825), which strongly suggests that it was the exemplar for the original Aldine edition (1525), which was produced in the Venetian Aldine publishing house, of which Kühn's edition is a direct descendant." The last words call for correction to "an indirect descendant," that is, via the editions of Basel 1538 and Paris 1679.

[35] Cf. notes 61 and 67 below.

[36] These are Marc. gr. 282 (saec. XV), Marc. gr. app. V 4 (saec. XV), and Marc. gr. app. XI 5 (saec. XIV); cf. Diels 1905, 91.

[37] The obviously wrong readings are mostly (but not limited to) these: p. 932.8 K. μέσος] μέσον **V**; p. 932.11 K. οὗτος] αὐτὸς **B**; p. 932.12 K. δωδεκάτῳ] δωδεκάτου **P**; p. 932.13 K. μέρει om. **P**; ibid. αὐτὸν] αὐτῶν **P**; p. 932.14 K. ἑβδομάδος] εὐδομάδος **B**; ibid. μόνον] μόνῳ **B**; p. 933.5 K. τετάρτῳ] δ^ου/ (that is τετάρτου) **P**; p.

context is indisputable: Among all manuscripts and editions that are here taken into account, **P** alone provides the correct Greek reading in three cases,[38] **B** alone in one particularly important case,[39] and both of them together in at least four cases.[40]

Number-letters employed by the manuscripts and editions will be recorded in the notes whenever there is the slightest uncertainty regarding their meaning or when these data shed light on stemmatic filiations. However, since number-letters in our manuscripts and editions are written in three different ways, namely with an upper right vertical stroke or with a horizontal superstroke or without any graphic distinction from normal (that is, non-numerical) letters, and since none of our witnesses does so with a discernable rationale (we often find two or, as in the case of **V**, all three options used randomly by the same witness), all number-letters will, in the interest of conciseness, meaningful groupings, and typographical convenience, be quoted with a vertical stroke.

Moreover, the notes, whenever pertinent, will record the corroborating evidence from the 9th century Arabic translation of Ḥunayn ibn Isḥāq[41] and from the 14th century Latin translation of Niccolò da Reggio.[42] These sigla will be used:

Greek manuscripts
B = Vatican (BAV), Barb. gr. 221 (saec. XV/XVI), ff. 40v–41r[43]
P = Paris, BnF gr. 2272 (saec. XV), ff. 145v–146r
V = Venice, Marc. gr. app. V 8 (coll. 1334, saec. XIV), f. 100r–v

Greek editions
v = ed. Veneta (Aldina) 1525 (ed. G.B. Opizzoni et al.),[44] f. 134r
b = ed. Basileensis 1538 (ed. Hieronymus Gemusaeus),[45] p. (*sic*) 449 ("459," typo)
p = ed. Parisina 1679 (ed. René Chartier), p. 504
l = ed. Lipsiana 1825 (ed. Karl Gottlob Kühn), pp. 932–933[46]

933.5–6 K. ἔτι πρὸς] πρὸς ἔτι πρὸς **B**; p. 933.9–10 K. ἀκριβολογουμένοις ὀλίγον om. **B** (*saut du même au même*: λόγον ... ὀλίγον; see n. 71); p. 933.11 K. ταύτης] ταύτη **P**. The fact that **V** is mentioned here only once does not mean that this manuscript contains fewer mistakes than **B** and **P**; on the contrary: It contains more mistakes, but almost all of them concern words that require notes to the Greek text and will accordingly be recorded there. As to *minutiae*, I shall neither record omissions of καὶ or *iota subscriptum* or *ny ephelkystikon* nor insignificant transpositions of words such as p. 932.7 K. where ἡμερῶν precedes the numeral in **BP** and is corrupted to ἡμῶν in **B**.

[38] See notes 49, 52, and 59 below.

[39] See note 62 below.

[40] See notes 51, 56, 60, 63, and maybe also n. 71 below.

[41] For details on this translation, see below, pp. 213–215. Kühn was aware of it (vol. I, 1821, p. CVII) yet apparently unable to make use of it.

[42] For details, see below, pp. 221–223.

[43] Cf. p. 204 above. I take the paleographical date of this miscellaneous MS from Mogenet 1989, 65.

[44] On this edition see below, pp. 224–225. All number letters (whether cardinal or ordinal) are marked with a vertical stroke (′) in **v** but with a horizontal stroke (‾) in **b**. In the following notes, the vertical stroke will be used for both **v** and **b**.

[45] On this edition (VD 16 G 121), which was published by Andreas Cratander and Johann Bebel, see Gundert 2006, especially 83–84.

[46] On Chartier's and Kühn's editions, see pp. 235–236 below. While **p** always prints the number letter stigma as ϛ′, **l** always has στ′. In the following notes, both writings will be quoted as ϛ′. Moreover, **p** has a dot after each number-letter (while **l** never has one), which will here be omitted.

Arabic translation

h = Ḥunayn ibn Isḥāq (9th century CE), ed. and trans. Cooper 2011a, 372/374

Latin translations

t1 = Gerard of Cremona (12th century), from the Arabic (**h**), quoted below, pp. 217–218

t2 = Niccolò da Reggio (13th century), from the Greek, edited below, pp. 222–223

t3 = Johannes Winter of Andernach (1529), from the Greek, quoted below, pp. 226–227

t4 = Jean Lalamant (1559), from the Greek, edited below, pp. 227–228.

(932.5) ἐπεὶ τοίνυν ὁ μὲν τῆς φάσεως ἐστι τῆς ἐναργοῦς καὶ δραστηρίου χρόνος εἴκοσιν ἓξ [47] ἡμερῶν καὶ ἡμισείας, [48] ὁ δὲ τῆς ζῳδιακῆς περιόδου <u>εἴκοσι καὶ ἑπτὰ καὶ τρίτου</u>, [49] δῆλον ὡς ὁ μέσος αὐτῶν εἴκοσι καὶ ἓξ[50] ἡμερῶν ἔσται καὶ προσέτι <u>μορίου μάλιστα</u>[51] μιᾶς ἡμέρας (932.10) ἡμίσεος <u>τρίτου</u> καὶ <u>δωδεκάτου</u>. [52] **S1** συνθεὶς γὰρ ἀμφοτέρους τοὺς χρόνους, **S2** εἰ τὸ ἥμισυ λάβοις, [53] εὑρήσεις τὸ μέσον. **S3** ἀλλ᾽ οὗτος ὁ χρόνος ὁ μέσος ἀπολείπεται τῶν ἑπτὰ καὶ εἴκοσιν[54] ἡμερῶν δωδεκάτῳ μάλιστα[55] μέρει μιᾶς ἡμέρας. **S4** εἰ <u>τοίνυν</u>[56] τέμοις αὐτὸν εἰς τέτταρα, τὸν ἀκριβῆ χρόνον εὑρήσεις τῆς ἑβδομάδος οὐχ ἕκτῳ μόνον (932.15) μιᾶς ἡμέρας ἀπολειπόμενον, [57] ἀλλ᾽ ἔτι[58] πλέον. λαμβανέσθω γὰρ τῶν

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[47] Orthographical variants: εἴκοσιν (-σι **B**) ἓξ **Bpl** : κϛ´ **PVvb** (misinterpreted as *vigesimussextus* in **t3pl**).

[48] ἡμισείας **Bvbpl** : ἡμίσεος **PV**.

[49] εἴκοσι καὶ ἑπτὰ καὶ τρίτου scripsi, cf. κζ´ καὶ γ^ου/ **P** (cf. **t1** .*xxvii. dies et tertia* [scil. *pars*], **t2** *27 et tercie*, **t3** *vigintiseptem dies cum tertia unius parte*, **t4** *dierum vigintiseptem et tertiae praeterea partis diei*) : εἴκοσι καὶ ἑπτὰ καὶ τρίτον **B** : κ´ καὶ ζ´ καὶ τρίτον **Vvb** : εἰκοστὸν καὶ ἕβδομον καὶ τρίτον **pl**.

[50] εἴκοσι καὶ ἓξ scripsi (cf. **t1** .*xxvi. dies*, **t2** *26 dierum*, **t4** *dierum vigintisex*) : εἴκοσι ἓξ **B** : κϛ´ **P** : κ´ καὶ ϛ´ **Vvbpl** (misinterpreted as *vigesimumsextum* in **t3pl**).

[51] μορίου μάλιστα **BP** (cf. **t2** *porcionis maxime*) : μάλιστα μορίου **Vvbpl**. I follow **BP** because of the usual post-position of μάλιστα with numbers, cf., for example, ibid. p. 931.11 K. ἡμίσει μάλιστα μιᾶς ἡμέρας and p. 932.12–13 K. δωδεκάτῳ μάλιστα μέρει. On Galen's reason for modifying the numerical value with μάλιστα, although the sum of 26½ and 27⅓ days divided by 2 equals *exactly* 26½ ⅓ 1/12 (= 26 11/12) days, see p. 241 below (Question 1).

[52] μιᾶς ἡμέρας ἡμίσεος τρίτου καὶ δωδεκάτου (γ^ου/ καὶ ιβ^ου solvi) **P** (cf. **t1** *medietas diei et tertia et pars ex .xii. partibus diei*, **t2** *porcionis ... dimidie et tercie et duodecime unius diei*, **t4** *partis unius diei dimidiae, item tertiae et duodecimae*) : μιᾶς ἡμέρας ἡμίσεος τρίτον καὶ (καὶ om. **V**) ιβ´ (δωδέκατον **B**) **BVvbpl** : *tertiam maxime et duodecimam unius diei dimidii partem* **t3** (corrected to *dimidium et tertiam maxime et* [*et* om. **l**] *duodecimam unius diei partem* in **pl**).

[53] λάβοις **Ppl** : λάβης **B** : λαβὼν **Vvb**. For the syntax, cf., for example, Gal. *de dign. puls.* vol. VIII p. 901.1–2 K. εἰ δ᾽ ἄμφω λάβοις, οὐ χαλεπῶς συλλογιῇ περὶ τῆς διαστάσεως.

[54] Only orthographical variants: ἑπτὰ καὶ εἴκοσιν (-σι **B**) **Bpl** : ζ´ καὶ κ´ **P** : ἑπτὰ καὶ κ´ **Vvb**.

[55] On this second use of μάλιστα, cf. again p. 241 below (Question 1).

[56] τοίνυν **BP** : μέντοι **Vvbpl**. For τοίνυν, cf. **h** "then" (Cooper 2011a, 372) as well as **t2** *igitur*. A similar construction occurs in the same work p. 897.13–15 K. εἰ τοίνυν τις ἀρξάμενος φρενιτίζειν, ἤτοι πεμπταῖος, ἢ ἑβδομαῖος, ἢ ἑπτακαιδεκαταῖος, ἢ εἰκοσταῖος κριθείη, πρόδηλον ὡς κτλ.

[57] These words refer back to p. 932.1–2 K. (just a few lines before the beginning of our passage) where Galen had given a first rough estimate of the ʿmixed (= medical) hebdomad,ʾ saying that it shortens the seven-day-span by more than a sixth of a day: ὁ μέσος ἀμφοῖν ὁ οἷον μικτὸς ἐξ αὐτῶν ἐπὶ πλέον ἀφαιρήσει τῆς ἑβδομάδος ἕκτου μέρους. That passage, in its turn, refers back to p. 929.14–15 K. where Galen had correctly defined one hebdomad of the sidereal (!) month as seven days minus a sixth (ἑκάστης ἑβδομάδος ἑπτὰ γινομένης ἡμερῶν ἀπολιπουσῶν ἕκτον, because 27⅓ ÷ 4 = 6⅚). Unaware of the latter reference to p. 929.14–15 K., Lalamant (**t4**, p. 504) wrongly conjectures τετάρτου for the transmitted ἕκτου at p. 932.2 K. and τετάρτῳ for the transmitted ἕκτῳ here at p. 932.14 K.

[58] ἀλλ᾽ ἔτι **Vvbpl** (cf. **t2** *sed etiam plus*) : ἀλλὰ **BP**.

εἴκοσιν ἓξ καὶ ἡμίσεος <u>τρίτου δωδεκάτου</u> τὸ τέταρτον μέρος·[59] ἔσται <u>δὴ</u>[60] χρόνος ἡμερῶν ἓξ ἡμίσεος <u>πέμπτου</u>[61] καὶ προσέτι μορίων ἄλλων ἐπικειμένων μικροτέρων, ἅπερ ἐστὶν <u>ἑξηκοστόν τε καὶ ἑκα</u>(933.1)<u>τοστὸν εἰκοστὸν καὶ διακοσιοστὸν τεσσαρακοστόν.</u>[62] **S5** ὁ δ' αὐτὸς χρόνος καὶ <u>οὕτως</u>[63] ἂν λέγοιτο, ἡμερῶν ἓξ καὶ ἡμίσεος καὶ ἕκτου[64] καὶ εἰκοστοῦ τετάρτου καὶ τεσσαρακοστοῦ ὀγδόου.[65] **S6** ὁ δὲ τηλικοῦτος χρόνος ἀπολείπεται τοῦ[66] (933.5) τῶν ἑπτὰ ἡμερῶν τετάρτῳ μέρει μιᾶς ἡμέρας καὶ ἔτι πρὸς τὸν ἀκριβῆ λόγον <u>ἑξηκοστῷ</u> τε [<u>καὶ ἑκατοστῷ εἰκοστῷ</u>] <u>καὶ διακοσιοστῷ τεσσαρακοστῷ,</u>[67] **S7** καὶ τούτων οὕτως ἐχόντων αἱ τρεῖς ἑβδομάδες ἡμερῶν εἴκοσιν ἔσονται καὶ προσέτι μορίου τινὸς ἡμέρας μιᾶς[68] ἕκτου <<u>καὶ τεσσαρακοστοῦ ὀγδόου</u>>,[69] ὥστε [<u>καὶ</u>][70] κατὰ τοῦτον τὸν

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[59] τῶν – μέρος scripsi, cf. τῶν κϛ´ ἥμίσεος γ^ου/ ιβ^ου. τὸ δτ^// μέρος **P** (cf. **t1** *quartam* [scil. *partem*] .*xxvi. dierum et medietatis et tertie diei et partis diei ex .xii.*, **t4** *ex vigintisex diebus, diei item dimidio, ex tertia itidem et duodecima diei partibus pars quarta*) : τῶν εἴκοσι ἓξ ἥμισυ τρίτου. δωδεκάτου τὸ τέταρτον μέρος **B** : τῶν κϛ´ καὶ ἡμίσεος (-ως **v**) τρίτου. ιβ´ (τρίτον .ιβ´. **v**; τρίτον, ιβ´ **pl**) τὸ τέταρτον (-υ **l**, typo) μέρος **Vvpl** (*ex vigintisex diebus et dimidio tertia et ex duodecim quarta pars* **pl**) : τῶν κϛ´ καὶ ἡμίσεος τρίτου ιβ´. τὸ τέταρτον μέρος **b** : *de 26ᵃ* [sic] *et media et tercia et xiiᵃ quarta pars* **t2** : *ex vigintisex diebus et dimidio pars quarta, et unius diei tertia et duodecima* **t3**.

[60] δὴ **BP** : δὲ **Vvbpl**. For δὴ, cf. **h** "then" (Cooper 2011a, 372, = **t1** *ergo*) and LSJ s.v. δή IV.2 "in summing up numbers, γίγνονται δὴ οὗτοι χίλιοι, these *then* amount to 1,000" (Xen. *Cyr.* 1.5.5). Even if Galen's computation is not an addition but a division, this entry of LSJ seems pertinent because in both cases δὴ introduces the result of a mathematical operation. See also Denniston 1954, 239, who remarks that in Hippocrates, whose works Galen had studied intensely, progressive δή is common. I changed Kühn's comma after μέρος to a semicolon.

[61] ϛ´ ἡμίσεος πέμπτου **PV** (cf. **t1** *sex dies et medietas diei et .v.* [= *quinta*] *diei*, **t2** *dierum sex et dimidii et quinte*, **t4** *sex et dimidij, quintae item partis*) : ἓξ ἥμισυ πέμπτου **B** : ϛ´ ἡμίσεος (-ως **v**) πέμπτου **vbpl** : *sex dimidii quinta pars* **t3pl**. Garofalo 2003, 53, n. 42, remarks that the form ἡμίσεως is well attested in Galen. Moreover, he emends πέμπτον to πέμπτου (ibid. n. 44) independently, not knowing the correct reading of **PV**.

[62] *Sic* (with ἐξεικ- pro ἐξηκ-) **B** (cf. **t1** *et pars ex .lx. et pars ex .cxx. et pars ex .cc. et .xl.*, **t2** *sexagesima, centesima uicesima, et ducentesima quadragesima*, and the 16th-century emendation by Guijon, n. 247 below) : ἑξηκοστὸν πρῶτον καὶ ἑκατοστὸν εἰκοστὸν δεύτερον καὶ διακοσιοστὸν τεσσαρακοστὸν **P** : ξ´ πρῶτα καὶ ρκ´ τὰ (αἱ **Vv**) δεύτερα σ´ (ο´ **v**) δὴ καὶ μ´ **Vvb** (cf. **t4** *sexaginta prima, centum viginti secunda, ducentesima quadraginta*) : ἑξήκοντα πρῶτα καὶ ἑκατοστὰ εἰκοστά· τὰ δεύτερα διακόσια δὴ καὶ τεσσαράκοντα **pl** : *sexagesima prima dein centesima vigesima, ducentesima quadragesima* **t3pl**.

[63] οὕτως **BP** and (independently by way of conjecture) Garofalo 2003, 54, n. 46 (cf. **t2** *ita* and **h** "in other words," Cooper 2011a, 372; hence **t1** *sermone alio*) : οὗτος (and no punctuation after λέγοιτο) **Vvbpl** : *idem tempus et hoc* **t3pl** : *idem hoc ipsum tempus* **t4**.

[64] ἓξ (ϛ´ **PVvb**) καὶ ἡμίσεος (-ως **v**) καὶ ἕκτου **PVvbpl** (cf. **t1** *vi dies et medietas diei et sexta diei*, **t2** *sex et dimidie et sexte*, **t3pl** *dierum sex et dimidii et sextae partis diei*, **t4** *dierum sex et dimidii dici queat, sextae partis diei*) : ἓξ ἥμισυν καὶ πέμπτου **B**.

[65] Only orthographical variants: καὶ εἰκοστοῦ τετάρτου καὶ τεσσαρακοστοῦ ὀγδόου **BPpl** : καὶ κδ´ καὶ μη´ **Vvb** (cf. **t1** *et pars ex .xxiiii. et pars ex .xlviii.*, **t2** *et xxiiiiᵗᵉ et 48ᵉ* [cf. n. 191], **t3t4pl** *et* [*et* om. **l**] *vigesimaequartae* [*vice-* **t4**] *et quadragesimaeoctavae*).

[66] τοῦ **PVvbpl** : om. **B**.

[67] ἑξηκοστῷ (ξ´ **V**) τε καὶ διακοσιοστῷ τεσσαρακοστῷ (σμ´ **V**) **PV** and (by way of conjecture) the humanists Guijon (ξ´ τε καὶ σμ´, cf. n. 251) and Manardi p. 412.35 (cf. p. 232 below) as well as Garofalo 2003, 54 (ξ´ τε καὶ σμ´), all independently of each other (cf. **t1** *pars ex .lx. et pars ex .cc. et .xl.*, **t2** *in sexagesima et ducentesima quadragesima parte*) : ἑξηκοστῷ τε καὶ ἑκατοστῷ εἰκοστῷ καὶ διακοσιοστῷ τεσσαρακοστῷ **B** (cf. **t3pl** *sexagesima parte, centesima vigesima, et ducentesima quadragesima*) : ξ´ τε καὶ ρκ´ καὶ σμ´ **vb** : ἑξήκοντά τε καὶ ἑκατοστὰ εἰκοστὰ καὶ σμ´ **pl**. Presumably the inauthentic fraction ¹⁄₁₂₀ originated as an 'error of perseveration' (Delz 1997, 60) in step **S4**, p. 933.1 K. Lalamant (**t4**) translates *quadragesimaoctava* (scil. *parte abest*), which implies that he added up the two authentic fractions ($\frac{1}{60} + \frac{1}{240} = \frac{1}{48}$).

[68] Only orthographical variants: μιᾶς **BP** : α´ **Vvbpl**.

[69] I supplied καὶ – ὀγδόου following Garofalo 2003, 52 <καὶ μη´>. Cf. **h** "then three weeks amount to twenty days, one sixth, and one forty-eighth of a day" (Cooper 2011a, 372–374). **t1** is here (for the first time) wrong in the printed version of 1490: *Cum ergo res ita sit, tres septimane sunt .xx. et .lx. diei et pars ex .xlviii.* (Garofalo 2003, 54, n. 56, emends *.lx.* to <*pars ex*> .*vi.*), but correct in MS London, Wellcome 285 (saec. XIVⁱⁿ, cf. n. 116 below): *Cum ergo res ita sit, tres septimane sunt .xx. dies et sexta diei et pars ex 48.* The two reprints of **t1**

τρόπον [71] ἀκριβολο(933.10)γουμένοις ὀλίγον ὑπερβάλλει τῆς εἰκοστῆς [72] ἡμέρας ὁ τῶν τριῶν ἐβδομάδων ἀριθμός, καὶ ταύτης ἔσται πολὺ μᾶλλον ἢ [73] τῆς εἰκοστῆς πρώτης [74] οἰκεῖος.

Translation:[75]

(932.5) So, since the time the Moon is visible and (thus astrologically) efficacious is twenty-six days and a half, and the time of its sidereal period is <u>twenty-seven</u> days and <u>a third</u>, it is clear that their average will be twenty-six days and in addition about a half and (932.10) <u>a third</u> and <u>a twelfth</u> part of one day.[76] **S1** For if you add both times together **S2** and take half, you will find the middle (that is, the average). **S3** But this average time is less than twenty-seven days by about the twelfth part of one day. **S4** If you <u>then</u> divide this (average time) by four, you will find that the exact time of the (medical) week is less (than seven days) not only by a sixth (932.15) of one day[77] but even more. For, take the fourth part of those twenty-six and a half and <u>a third</u> and <u>a twelfth</u> days; the (resulting) time will <u>then</u> be six and a half and <u>a fifth</u> days and in addition other smaller parts on top (of that), namely <u>a sixtieth</u> and (933.1) <u>a hundred-twentieth and a two-hundred-fortieth</u> (of a day). **S5** The same time could also be expressed <u>thus</u>: six days and a half and a sixth and a twenty-fourth and a forty-eighth. **S6** This time is less than (933.5) seven days by a fourth part of one day plus—to be accurate—<u>a sixtieth</u> and <u>a two-hundred-fortieth</u>. **S7** And since these (data) are such, the three (medical) weeks will comprise twenty days plus about a sixth <u>and a forty-eighth</u> part of one day. Hence, if we calculate accurately in this manner, the number of three (medical) weeks exceeds the twentieth day (only) by a little and will belong much more to this (day) than to the twenty-first.

(1515 and 1528) have the same passage in partly corrected wording: *tres septimane sunt .xx. et sexta diei et pars .lxvii.* (sic, read: *.lxviii.*; I owe the reading of the edition of 1528 to Pennuto 2008, 92, n. 108).

[70] καὶ (**BPVvbpl**, cf. **t2** *et*) seclusi; vide infra p. 258.

[71] τρόπον **Vvbpl** : λόγον **BP** (maybe preferable; cf. **t2** *rationem*; **t2** had used the same word before to translate p. 933.6 K. λόγον).

[72] εἰκοστῆς **Bpl** : κ′ **Pv** : η′ **V** (cf. the correct paleographical explanation in n. 235 below) : κ′ (et in marg. var. l. η′) **b**.

[73] ἢ **BVvbpl** : om. **P**.

[74] εἰκοστῆς πρώτης **Bpl** : κα′ **Vvb** : κ′ ἡμέρας **P**.

[75] My translation includes some expressions borrowed from Cooper's translation (2011a, 372–374) of the Arabic text.

[76] On Galen's reason for modifying the result here and also in **S3** with μάλιστα ('about'), see p. 241 below (Question 1).

[77] These words are explained in n. 57 above.

IV. Historical Survey

The following survey focuses on material relevant to the recovery of Galen's original text and its mathematical elucidation, not to its medical, astrological, philosophical, or other appreciation, because that would go beyond the scope of the present study. Also, this survey does not pretend to be complete, which would obviously be impossible.

By about 500 CE, the medical school of Alexandria had a firmly established Galenic curriculum reported (by Arabic sources) to have comprised sixteen treatises, one of them being *On Critical Days*.[78] The Alexandrian origin of this collection is certain thanks to an extant *Summary of the Sixteen* (scil. *Books*) *of Galen* by Yaḥyā al-Naḥwī, that is 'John the Grammarian'.[79] The Greek original of this work is lost; its Arabic translation is extant only in one manuscript.[80] It is the work of a highly competent scholar who gives a coherent account of Galen's teaching with numerous cross-references between the various works and occasional anatomic drawings.[81] Since only John's prologue is available in a modern edition with translation,[82] I cannot tell if this short summary contains noteworthy remarks on Galen's computation of the medical week.

The Arabic world also received a group of significantly longer, anonymous summaries of the sixteen Galenic works in question.[83] These summaries are considered by experts to be of Greek, especially Alexandrian origin, even if no Greek originals are extant and no Greek or Christian name is attached to them.[84] In the case of *On Critical Days*, there must actually have been "at least two quite different versions" of this longer, anonymous summary circulating in Arabic. One of these is extant, the other must have existed because a Hebrew translation was made from it in 1322 by Shimshon ben Shlomo.[85] Regrettably, both versions skip the passage here under scrutiny.[86]

That the canon of sixteen Galenic works existed by 500 CE is clear also because Sergius of Rēšʿaynā (who died in 536 CE) came into contact with it during his medical studies in Alexandria around 500 CE and translated all its works with the exception of *De sectis*,

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[78] Overwien 2017, 2. Cf. Mattern 2013, 283: "Modern scholars debate whether the list may be properly called a canon and whether any such formal institution as 'the medical school of Alexandria' [...] existed, but most do not dispute that these titles were favored and widely read at Alexandria."

[79] Garofalo 2000, 137–138, and Overwien 2017, 2, both list the names of all sixteen works (some works actually comprise a bundle of works).

[80] British Library, Arund. or. 17, ff. 2v–139r; the summary of *On Critical Days* is on ff. 111v–115r. Cf. Garofalo 2000, 135 and 138, who presumes (ibid., 143) that John's summary of *On Critical Days* is not based on the Galenic work itself but rather on a commentary to that work written by the same John.

[81] Garofalo 2000, 139–140.

[82] Ibid., 144–146 (Italian translation) and 150–151 (Arabic original).

[83] Langermann 2008, 102.

[84] Cf., for example, Garofalo 2000, 143, n. 28.

[85] Langermann 2008, 103–104 (cf. ibid., 111). These two versions were edited by Bos and Langermann 2015.

[86] In the Arabic version, it would fall between § 75 and § 76; cf. the respective notes 154 and 155 of Bos and Langermann 2015, 86. For the Hebrew version, cf. ibid., 117, § 26. Langermann 2008, 113, remarks that these two versions take a far more favorable stance towards Pythagorean arithmology than Galen himself does in the third book of *On Critical Days*. With regard to all three summaries (that is, including that of John the Grammarian), he remarks (ibid., 100): "Their authors all seem to be more firmly convinced of the astrological explanation than Galen was; this suggests to us that, in the centuries after the demise of its founder, Galenism became more and more receptive to astrology."

De pulsibus ad tirones und *De sanitate tuenda* into Syriac.[87] The importance this curriculum came to assume in the Arabic world is well captured in the words of Mattern: "At Baghdad, intellectual center of the Muslim world and home of the 'House of Wisdom,' which perhaps imitated Alexandria's famous Museum, medical education was modeled on the Alexandrian curriculum and included works of Hippocrates and the 'sixteen books' of Galen. This remained the foundational canon of Greco-Arabic medicine for a thousand years and profoundly influenced the later development of medieval Islamic medicine."[88]

One of the most important scholars in this process of cultural transfer from the Greek to the Arabic world was the Nestorian Ḥunayn ibn Isḥāq (808–873 CE).[89] His mother tongue was Syriac; he learned Greek in Alexandria and Arabic in Basra. As Ḥunayn reports in his important *Risāla* ('Epistle'),[90] he first corrected Sergius of Rēšʿaynā's Syriac translation of *On Critical Days*, and later translated 'it'—either the Syriac translation or the Greek original—into Arabic.[91] In any case, it seems safe to infer that Ḥunayn had access to at least one Greek manuscript because otherwise he would have been unable to correct the Syriac translation.

Based on a comparison of Ḥunayn's Greco-Arabic (or Greco-Syriac-Arabic?)[92] translation and the Greek manuscript tradition, Cooper makes two claims that are important for our leading question, namely that the fraction $\frac{1}{120}$ in the Greek text of **S6** (p. 933.6 K.) must be supplied in the Arabic translation, where it is missing,[93] and that the fraction $\frac{1}{48}$ in the Arabic text of **S7** must be restored in the Greek original (p. 933.9 K.), from whose extant manuscripts it is missing.[94] The first claim can here, despite Cooper's emphasis on its importance, be dismissed because it originates in a mathematical lapsus made by this otherwise praiseworthy editor.[95] The second claim, however, is valid, as our mathematical

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[87] Overwien 2017, 3. A very late antique Greek appreciation of Galen's astrological explanation of the critical days is found in the Commentary on Hippocrates' aphorisms 4.34 by Stephanus of Athens, who was active at Alexandria in the 6th/7th century CE (ed. Westerink 1992, 302.4–9): τὴν δὲ αἰτίαν, δι' ἣν καὶ ποίῳ λόγῳ καὶ διὰ τί ὑπὸ τὴν σεληνιακὴν κίνησιν καὶ τὴν ἔλλαμψιν τῶν οὐρανίων σωμάτων κρίνονται τὰ πάθη, εἴτε κατὰ περίοδον εἴτε ἐν ταῖς παρεμπιπτούσαις ἡμέραις, οὐκ ἔστιν ἡμῖν καιρὸς νῦν ἐπεξιέναι καὶ ζητεῖν ταῦτα διά τε τὸ μῆκος τῶν λεγομένων, ἀλλὰ δεόντως καὶ τελείως εἴρηται τῷ Γαληνῷ, ὡς θεοῦ [δε]διδοῦντος καὶ βοηθοῦντος ἐν τῇ Περὶ κρισίμων ἡμερῶν μαθησόμεθα. (trans. ibid., 303: "As regards the cause and the ratio and the reason why diseases are brought to a crisis by the motion of the moon and the influence of the celestial bodies, whether periodically or on intercident days, we have no time now to discuss and examine this question, because there is too much to be said, but it has been explained duly and adequately by Galen, as God permitting and helping we shall see in Critical Days.") In his Galen courses, Stephanus followed the typical Alexandrian curriculum mentioned above (ibid., 12).

[88] Mattern 2013, 283.

[89] Cf. Strohmaier 1971.

[90] Cf. Cooper 2011a, 14: "In it, Ḥunayn described 129 of the works of Galen, and the history of their translation into Syriac and Arabic; including the Critical Days, by whom, for whom, and often under what circumstances."

[91] Cf. the English translation of the decisive passage of the *Risāla* by Cooper 2011a, 15, as well as Cooper 2016, 5: "It is unclear from this passage whether Ḥunayn translated it [*that is, Galen's work on critical days*] from Syriac into Arabic, or directly from Greek."

[92] In the following, I shall omit references to this uncertainty.

[93] Cooper 2011a, 372; repeated by Cooper 2012, 640–641.

[94] Cooper 2011a, 374 and 527; repeated by Cooper 2012, 634.

[95] He translates thus (Cooper 2011a, 372): "This time is less than seven days by a fourth of a day and is something that is, according to correct reckoning, one sixtieth, <one hundred-twentieth>, and one two-hundred-fortieth of a day"; with this footnote (no. 1132): "< > : added from Greek. This addition is absolutely essential for understanding the passage." Cf. the same view, expressed in almost identical words, in Cooper 2012, 640–

examination will show.[96] It remains to be asked, however, whether Cooper is right to assume that the fraction $\frac{1}{48}$ was expressed in Ḥunayn's (now lost) Greek manuscript (siglum Ψ in Cooper's edition).[97] If one compares Cooper's edition (2011a) with two of his later articles (Cooper 2012 and 2016), it becomes clear that he underwent a profound change of mind (which he does not, however, make explicit) from the unquestioned assumption (2011a) that whenever Ḥunayn's translation seems superior to the Greek manuscript tradition we are allowed to restore the reading (or at least the sense) of Ψ, to a much more cautious assessment: Cooper 2016 now adopts Brock's useful distinction between text-oriented and reader-oriented translations[98] and shows that Ḥunayn's method "was 'reader-oriented,' meaning he added whatever he thought necessary to help his readers understand the text and its complex subject matter, rather than 'text-oriented,' which adhered closely to the original."[99] Using several examples classified in a working typology, he shows "how caution must be used when deriving Greek textual variants from Arabic."[100] Ḥunayn's interventions are of various kinds: Not only did he correct his source texts when he saw fit;[101] he also made sometimes long additions or expanded the original while translating with the intention of conveying "as much meaning and context as he feels is needed."[102] His analysis of numerous examples leads Cooper to the following conclusion:

> Ḥunayn's purpose was to produce texts that were as useful as possible to his associates and his patrons, and this meant expanding or contracting them, as needed. [...] For the purposes of reconstructing a Greek source, where there are differing readings between Greek and Arabic, we must in many instances remain uncertain whether the differences represent true variant readings, or are the product of the translator's interventions, or arising from differences between the languages. One of the only cases where we can confidently apply the usual principles of textual criticism is when the Arabic text corroborates a variant reading in the Greek tradition, in which case, the common reading must have been present in the archetype from which both that Greek manuscript and the non-extant Greek manuscript on which the Arabic translation was made, derived. In

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641 (where "Add after" is to be deleted). Cooper touches upon or summarizes our Galenic computation several more times in other articles, but without adding anything that changes his views expressed in the book (2011a, 64–65, 68, 76, and 372–374), and without addressing the question whether the fraction $\frac{1}{120}$ (p. 933.6 K.) is authentic: see Cooper 2004, 54–55; Cooper 2011b, 129; Cooper 2013, 540; Cooper 2014, 88 (with a lapsus: $20\frac{9}{48}$ days are not "20d 4h 31m," but 20d 4h 30m); Cooper 2018, 47–49 (with two lapsus: Galen's medical week is not "$26\frac{11}{16}$ days," but $26\frac{11}{12}$ days, and on p. 49, l. 24, correct "synodic month" to 'sidereal month').
[96] See p. 256 below, Question 5.
[97] He explicitly does so in his Graeco-Arabic apparatus (2011a, 527). Note that Ḥunayn may have used more than a single Greek manuscript. We know that he used to procure as many manuscripts as possible and establish the Greek text philologically before starting any translation (Strohmaier 1993, 158). See Ḥunayn's comment on Galen's *On Sects* (Lamoreaux 2016, 10–11).
[98] Brock 1983, 4–5.
[99] Cooper 2016, 1 (abstract).
[100] Ibid., 1.
[101] Ibid., 6.
[102] Ibid., 6–7. Cf. ibid., 10: "Ḥunayn provided explanations or amplifications that seem to be absent in the original" (with examples ibid., 10–12); then (ibid., 13–19) he provides two examples where Ḥunayn's translation is significantly longer than the Greek version of the same passage.

most other cases, as we have seen, the difference could have been the result of Ḥunayn's expansions or judicious alterations as he prepared a reader-oriented text.[103]

Applied to p. 933.9 K. (**S7**), where the Greek manuscripts unanimously omit the fraction $\frac{1}{48}$, Cooper's conclusion implies that it is no longer legitimate to infer, as he had done earlier, from the presence of the fraction $\frac{1}{48}$ in Ḥunayn's translation that it had also been present in Ψ, and was lost in the later transmission of the text. It seems equally possible that Ψ did not have the fraction $\frac{1}{48}$, that Ḥunayn checked Galen's math, noticed the error, and added the missing fraction. With regard to the other fraction in question ($\frac{1}{120}$ in **S6**, p. 933.6 K.), the situation is similarly ambiguous: The absence of this fraction from Ḥunayn's translation may mean that it was absent from Ψ, but it is at least equally possible in view of the strong presence of this fraction in the Greek manuscripts that Ḥunayn found it in Ψ and athetized it.[104] For reasons that will be explained in Section VI below, I am convinced that Galen wrote the second fraction ($\frac{1}{48}$) but not the first ($\frac{1}{120}$). The fact that Ḥunayn did the same is a token of the overall quality of his translation, but we have no reliable means to ascertain whether this quality, in the present two cases, is due to the quality of his Greek manuscript(s) or to his own competence as a mathematician and textual critic. As we will see, the heuristic value for the constitution of the Greek text, too quickly given by Cooper (2011a) to Ḥunayn's Arabic translation, is to be found instead in a Latin translation unconsidered in the scholarship to date.[105]

Not long after Ḥunayn ibn Isḥāq, the Baghdad physician Qusṭā ibn Lūqā (he died about 912/3 CE) discussed, in his *Questions on the Critical Days in Acute Illnesses*, the question (no. 15) "For what reason does the third week end with twenty days and not twenty-one?"[106] In this context, he explains what spurred Galen to investigate the whole phenomenon of the critical days. However, "the crisis doctrine seems to have remained fairly stable, in the basic form that Ḥunayn's translation cast it, until the *Canon* of Ibn Sīnā (d. 1037 CE),"[107] who "simplified Galen's theory by ignoring the sidereal month completely, and derived the half week parameter directly from the 26 and a half day (effective) synodic month, by taking the eighth part of it, which is $3\frac{5}{16}$ or 3.3125 days, which is equivalent to $3\frac{30}{96}$ days for the medical half week ($\frac{5}{96}$ day less than Galen's figure). He then derived all

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[103] Ibid., 37. In his previous article (Cooper 2012), the author had presented numerous other passages that contain expansions (617–618) or "textual additions that are likely variant readings" (621–633) or "textual additions that restore a lacuna in Greek" (633–634), but that article had a different focus and requires more caution on the reader's part because it still lacks the sharp conceptual and methodological foundation of the later one (2016). Basically the same insights as Cooper's were reached independently by Overwien 2012, 169: "Although they [*that is, Ḥunayn's group*] endeavoured to collect multiple manuscripts of a single text in order to constitute a good working text and carefully considered the quality of their source text, they did not intend to preserve verbatim the 'archetype' which they reconstructed in this way. Rather, they aimed at expanding their base text for the purpose of usefulness, instruction, and perhaps also reading pleasure [...]." Cf. ibid., 155: "The text of the Greek original was not sacrosanct to Ḥunayn."

[104] Cooper's belief in this fraction's importance (cf. n. 95) may have contributed to his not envisaging the possibility that Ḥunayn examined Galen's math critically and emended the Greek text accordingly. Cooper does not return to either problem (that of $\frac{1}{48}$ and that of $\frac{1}{120}$) in his 2016 article.

[105] See below, p. 221, on Niccolò da Reggio.

[106] Cooper 2014, 87–89. Cf. the almost identical title of Galen's chapter 3.9, quoted above (p. 205 with n. 26).

[107] Cooper 2018, 28.

critical days from this figure, with no stated theoretical explanation nor justification, nor acknowledgement why he had departed from the ancient scheme."[108] The most famous commentator on Ibn Sīnā's *Canon*, Ibn al-Nafīs (he died in 1288 CE), "justifies Ibn Sīnā's use of the synodic month alone as the basis for his medical week, in contrast to Galen [...]. All of the medically significant lunar periods are derived by successively halving this reduced month of 26.5 days: 13 ¼ days, 6 ⅝ days, etc."[109] Note that this approach leads to a consequence not mentioned by Cooper, namely that three weeks will span 19 ⅞ (= 19.875) days, that is, less than twenty full days.[110] Whether this tradition had any followers in the West remains to be investigated.

Cooper points out an interesting *desideratum*, that is, to examine a short work on the critical days by al-Nasawī (ca. 1010–1075 CE), a mathematician who wrote a summary of Euclid's *Elements* and an influential treatise on arithmetic.[111] Since Cooper suspects that al-Nasawī's work on the critical days is arithmological in nature, it may contain a discussion of Galen's computation of medical weeks.

Among European scholars, knowledge of Galenic medicine had been very limited until a vast body of translations, largely made in Toledo by Gerard of Cremona (he died in 1187 CE) and his school, became available in the 12th century.[112] This material has fittingly been called "the new Galen" by García-Ballester (1982). Jean de St. Amand, in his *Revocativum memoriae* (soon after 1285), provides students of medicine with summaries of nine of the most important works in the 'new Galen,' one of them being *De creticis diebus*.[113]

The first extant full translation (**t1**) is from the pen of Gerard of Cremona and based on the Arabic translation of Ḥunayn ibn Isḥāq (possibly with sporadic use of a Greek manuscript, too).[114] **t1** will here be quoted (with three editorial corrections)[115] according to its *editio princeps* of 1490,[116] with all variants of the reprints of 1502[117] and 1515[118] in the

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[108] Ibid., 48 (with a typo: "from this figures").

[109] Ibid., 36.

[110] Since Ibn Sīnā did not explain his departure from Galen, one may wonder if one reason for him was the same question raised by Cooper 2011a, 496, which will be discussed here on pp. 263–264 (n. 426).

[111] Cooper 2011a, 49–50.

[112] On the previous situation and the assimilation of "the new Galen" cf. McVaugh 1990, 62–69.

[113] McVaugh 1990, 64. The orthography *cretic-* is widespread in Latin texts. It is based on an erroneous etymology from *cernere*. Cf. Isid. *orig.* 4.9.13: *Creticos dies medici uocant, quibus, credo, ex iudicio infirmitatis hoc nomen inpositum est, quod quasi iudicent hominem, et sententia sua aut puniant aut liberent* and Barney 2005, 114 ad loc., as well as Ricci 1952, Weiss 1953, and especially MLW II (1999), col. 2027 s.v. *criticus* with reference to Godescalcus Saxo (saec. IX) gramm. 2 p. 484.6 *non critici sed cretici a cernendo* and so forth. Nothing on ʿcreticusʾ in Maltby 1991 and Marangoni 2007.

[114] More on this below, n. 125.

[115] Cf. notes 124, 126, and 131 below. See also one dubious case in n. 123.

[116] *Galeni opera*, vol. I, Venice: Philippus Pincius, 27 Aug. 1490, ff. pp<i>r–qq<v>v, here quoted: f. qq<v>r, lines <30>–<52>. This incunabulum is GW no. 10481 = ISTC no. ig00037000. The editor, Diomedes Bonardus (a physician from Brixen), informs the reader in the explicit of each Galenic treatise of the respective translator's identity, if he knows. In this case, he does not, yet the identity of the translator is fairly certain because some of the numerous manuscripts of this translation are attributed to Gerard and a translation of *De diebus decretoriis* features in a list of Gerard's works that was compiled by his students after his death. I owe these arguments to Stefania Fortuna (http://www.galenolatino.com/index.php?id=11&L=&uid=12; cf. Fortuna 2005, 487, where she identifies the present translation without doubt as Gerard's). That this translation was made from Ḥunayn's Arabic translation is certain because they agree in significant modifications, shortenings, and

notes.[119] Moreover, the notes will give a few select variants of two (out of more than fifty extant) manuscripts, namely Vat. Pal. lat. 1092, f. 143va (here: **G**), and Vat. lat. 2375, f. 160va (here: **C**), both written around 1300.[120] The edition of 1490 is the first of twenty-two editions of Galen's *opera omnia* in Latin that were published from the invention of printing to the last such enterprise in 1625.[121] The same long period saw only two editions of the original Greek text of Galen's *opera omnia* (Venice 1525 and Basel 1538). This huge disproportion shows that Galen's works were read and known in early modern Europe almost exclusively in Latin.[122] Gerard's translation reads thus:

Cum ergo sit tempus visionis lune manifeste fortis[123] .xxvi. dies et medius et tempus revolutionis eius in orbe signorum .xxvii. dies et tertia, fit tempus medium inter utraque[124] quasi[125] .xxvi. dies et medietas diei et tertia et pars ex .xii. partibus diei. Et non scitur tempus

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expansions of the original Greek text, for example, in adding, after p. 932.15 K. ἀπολειπόμενον, the figure of seven days that is only implicit in the original (*minuitur ex xxvii diebus*, cf. **h**: "is less than seven days," Cooper 2011a, 372), in omitting p. 932.17–18 K. προσέτι μορίων ἄλλων ἐπικειμένων μικροτέρων, ἅπερ ἐστὶν (Cooper ibid.), in reading, instead of οὕτως or οὗτος (p. 933.2 K., cf. n. 63 above), *sermone alio* (cf. **h**: "in other words," Cooper ibid.), and especially in adding ¹⁄₄₈ day to the sum of three weeks (cf. n. 69 above). I have also collated one manuscript of this translation, London, Wellcome 285 (early 14th century), ff. 98v–99r, which features some minor variants that have no bearing on the mathematical reasoning. This witness has been quoted above (n. 69), on the occasion of a textual corruption in the 1490 edition, where the Wellcome manuscript is correct.

[117] *Secunda impressio Galieni* [...], Venice: Bernardinus Benalius Bergomensis, ed. Hieronymns Surianus, vol. 1, 1502, f. ddv *verso* (= p. 542 of the digitized copy of the SUB Göttingen at http://resolver.sub.uni-goettingen.de/purl?PPN548120684). This edition is missing in Durling 1961, 284, no. 39. I thank S. Fortuna (personal communication) for directing my attention to this edition and its digitized copy.

[118] Galenus. *Quarta impressio ornatissima continens omnes Galeni libros*, Pavia: Rusticus Placentinus, vol. 2, 1515, f. LXXXIIIIr–v. Most of the variants of this edition are mistakes, but note the correction in n. 137 as well as the intentional, mathematically correct substitution of ¹⁄₆₀ + ¹⁄₂₄₀ with ¹⁄₄₈ in notes 130 and 135 below.

[119] I did not see the last reprint (Venice 1528).

[120] I chose the sigla **G** and **C** with reference to Gerard of Cremona (sigla **V** and **P** were no longer available, cf. p. 208 above). Systematic information on all variants of **G** and **C** is not desirable because they both contain numerous trivial errors.

[121] Fortuna 2012a, 391 and 394, as well as Fortuna 2012b, 304.

[122] Fortuna 2012a, 391–392 and 394.

[123] Sic edd. 1490, 1502, 1515 as well as **G** and **C**. One would expect *manifeste et fortis.*

[124] *utraque* scripsi (scil. *tempora*): *utrasque* edd. 1490, 1502, 1515 as well as **G** and **C**.

[125] The presence of *quasi* here is surprising. It is present in all witnesses that I saw (edd. 1490, 1502, 1515, as well as **G** and **C**) and equals μάλιστα ('about') in the Greek original, where a second μάλιστα with the same meaning follows in **S3**, which has no equivalent here in **t1**. Ḥunayn's Arabic translation omits both those instances of μάλιστα (Cooper 2011a, 372–373; they are absent from Cooper's Arabic text and from its *apparatus criticus*, as P. Schmidl kindly informed me). Hence, the surprising point is that **t1** contains one word that has no equivalent in the translator's Arabic model and that the context does not suggest at all, because Galen's math is *exactly* correct (Galen's reason for modifying the two numerical values with μάλιστα will be discussed below, p. 241, Question 1). It is interesting to note that **t1** had on an earlier occasion in the same third book, and with reference to the same astronomical figure (that is, the length of the sidereal month), rendered the Arabic equivalent of μάλιστα, *naḥwa* ("approximately," Cooper 2011a, 338), with *quasi* (cf. ch. 3.5 p. 911.7–8 K.: [...] ἑπτὰ καὶ εἴκοσιν ἡμέραις [...] μετά τινος ἐπικειμένου μορίου, τρίτου μάλιστα μιᾶς ἡμέρας and **t1**, ed. 1490, f. qqiiirb: *in .xxvii. diebus et quasi tertia diei*; I disagree with Cooper's identification [2011a, 524] of *naḥwa* with μετά τινος ἐπικειμένου μορίου instead of μάλιστα). This shows that *quasi* here in **S1**–**S2** is not an obscure intrusion into the Latin text but the lexical equivalent of μάλιστα. This makes me wonder if Gerard of Cremona, while doubtlessly translating from Arabic, had access to a Greek manuscript, too, and made at least occasional use of it. (Or did any hitherto unknown early reviser of Gerard's translation check it against a Greek manuscript? Or did Gerard translate from an Arabic manuscript which contained equivalents of μάλιστα in **S1**–**S3** that happen to be missing from the Arabic manuscripts used by Cooper?) Already Cooper 2012, 613, had noticed that **t1** transliterates a significant word in the first book *On Critical Days*, σοφίσματα (ch. 1.7 p. 804.5 K.),

medium inter utraque,[126] nisi **S1** *quando aggregantur duo tempora* **S2** *et accipitur medietas,* **S3** *et hoc tempus medium est minus .xxvii. diebus per unam partem ex .xii. partibus diei.* **S4** *Nam si diviseris hoc in quartas, invenies, quod quantitas septimane certe minuitur ex septem diebus plus .vi.[127] diei. Accipe ergo quartam .xxvi. dierum et medietatis et tertie diei et partis diei ex .xii.,[128] ergo erit hoc tempus sex dies et medietas diei et .v.[129] diei et pars ex .lx. et pars ex .cxx. et pars ex .cc. et .xl.[130]* **S5** *Et tibi quidem possibile est, ut narres hoc tempus sermone alio, et est, ut dicas .vi. dies et medietas[131] diei et sexta[132] diei et pars ex .xxiiii.[133] et pars ex[134] .xlviii.,* **S6** *et hoc tempus minuitur ex .vii. diebus in quarta et re, que est secundum computationem certam pars ex .lx. et pars ex .cc. et .xl.[135]* **S7** *Cum ergo res ita sit, tres[136] septimane sunt .xx. et .lx.[137] diei et pars ex .xlviii.[138] Et cum nos computaverimus computationem certam secundum hunc modum, inveniemus superfluitatem trium septimanarum super .xx. dies superfluitatem parvam, et fit crisis dignior die .xx. valde quam in[139] die .xxi.*

The first scholar who (to my knowledge) engaged with the details of our Galenic passage is the author of the anonymous treatise *Aggregationes de crisi et creticis diebus* ('Collection on crisis and critical days') in the latter part of the thirteenth century.[140] This date implies that the author is drawing on the Arabo-Latin translation **t1** (see above), whose existence by that time is certain thanks to two 13th-century manuscripts.[141] According to its

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as *sophysmata*, while the Arabic translation *hādhihi 'l-aghālīṭ* means 'these captious questions.' Cooper rightly wondered how the Latin translator knew the correct word, but he did not envisage the explanation proposed here (cf., however, ibid., 619, on the Latin rendering of Arabic *mumawwihin* ["impure"] as *sophystica*), partly, one may assume, because he was unaware of the similarly surprising translation *quasi* for μάλιστα in **S1**–**S2** (he does not register Ḥunayn's double omission of μάλιστα in his Graeco-Arabic apparatus, 2011a, 527). It is a *desideratum* for future research systematically to check whether there are any further instances of agreement between **t1** and the Greek original against the Arabic translation and thus reach a reliable assessment of the source(s) of **t1**.

[126] *utraque* **C** : *utrasque* **G** : *utrosque* edd. 1490, 1502, 1515.

[127] *.vi.* edd. 1490, 1502 : *quarta* ed. 1515.

[128] *ex .xii.* edd. 1490, 1502 : *duodecime* ed. 1515.

[129] *.v.* edd. 1490, 1502 : *octava* ed. 1515.

[130] *pars ex .lx. et pars ex .cxx. et pars ex .cc.* (*.c.* ed. 1502) *et .xl.* edd. 1490, 1502 : *pars duodecima et pars quadragesimaoctava* ed. 1515.

[131] *medietas* **GC** : *medietatem* edd. 1490, 1502, 1515.

[132] *sexta* edd. 1490, 1502 : *octava* ed. 1515.

[133] *ex .xxiiii.* edd. 1490, 1502 : *duodecima* ed. 1515.

[134] *ex* edd. 1490, 1502 (item **GC**) : om. ed. 1515.

[135] *pars ex .lx. et pars ex .cc. et .xl.* edd. 1490, 1502 : *pars quadragesimaoctava* ed. 1515.

[136] *tres* edd. 1490, 1515 : *res* ed. 1502.

[137] *et .lx.* edd. 1490, 1502 : *et sexta* corr. ed. 1515 (item **GC**) : *et <pars ex> .vi.* coni. Garofalo 2003, 54.

[138] *.xlviii.* ed. 1490 (item **GC**) : *.xlvii.* edd. 1502, 1515.

[139] *in* edd. 1490, 1502 : om. ed. 1515 (item **GC**).

[140] The date of composition (not much before the date of the earliest manuscript, paleographically dated to about 1280 CE) and the identity of the author (the manuscripts provide contradictory information) are discussed by O'Boyle 1991, 16–20.

[141] Cf. O'Boyle 1991, 19, who does not mention the Arabic provenance and states that "there is only one translation of the Latin text" (probably a lapsus for 'by that time there was only one Latin translation of the text;' note that he wrongly dates that translation to not much before 1260 CE). O'Boyle's words about the Latin translation are criticized by Cooper 2011a, 55, in similarly obscure terms: "Although O'Boyle seems to imply that the *Aggregationes* quotes from the Greek-derived Latin version [*which one? they are all later, and neither O'Boyle nor Cooper knows the first Greco-Latin translation by Niccolò da Reggio which postdates the* Aggre-

editor, this work "is of special importance, first, because (if the number of surviving manuscripts is anything to go by) it appears to have been one of the most widely read works on medical astrology, and, second, because it marks the introduction into Western medical prognosis of a highly sophisticated and mathematically very technical medical astrology that was derived from Galenic sources new to the thirteenth-century Latin West."[142] In his adaptation of our passage, which is part of the second chapter, the unknown author flawlessly converts all fractions of days, without mentioning them, into full and half hours. He demonstrates intellectual independence by occasionally departing from Galen's computational method.[143] His text reads thus:[144]

Est etiam mensis communis impressionis sive manifeste visionis. Et completur in viginti sex diebus et duodecim horis. Et iste mensis durat a principio, quo incipit luna nobis apparere illuminata a sole, usque ad tempus, in quo nobis amplius non apparet. Et secundum istum mensem est luna principium impressionis communis, que fit per naturam luminis et splendoris: ex luce splendor, et ex splendore calor, et calor est principium vite. Et quia, ut probatum est supra, ratione proprie influentie et communis luna facit ad crisim,[145] ideo medicus componit unum mensem ex istis duobus mensibus, cuius compositionis causam solus deus sublimis et gloriosus novit, ut dicit Galienus. Iste autem 'mensis medicinalis' nuncupatur. Iste autem ex predictis duobus sic componitur: Mensis proprie impressionis habet viginti septem dies et octo horas. Mensis manifeste visionis habet viginti sex dies et duodecim horas. Et sic mensis proprie impressionis superat mensem manifeste visionis in viginti horis. Divide ergo in decem et decem et da mensi manifeste visionis decem.[146] Et cum habeat viginti sex dies et duodecim horas, erunt viginti septem dies duabus horis minus, cum dies naturalis habeat viginti quatuor horas. Et iste mensis est medicinalis, qui superat mensem manifeste visionis in decem horis et a mense peragrationis superatur in decem. Et sic est medius ex utrisque compositus. Et cum iste mensis sit imperfectus,[147] similiter septimane, ex quibus integratur, sunt imperfecte, quas proportionaliter partiamur. Cum in hoc mense medicinali sint viginti septem dies duabus horis minus, ratione viginti quatuor dierum debemus cuilibet septimane sex dies. Et sic remanent tres dies duabus ho-

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gationes *by several decades*], I found the opposite [*actually not the opposite but dependence on the Arabic translation*] to be the case."

[142] O'Boyle 1991, 7. Cf. ibid., 4: The medical astrology contained in it is derived almost entirely from Galen's *On Critical Days*. As to the number of manuscripts, the ITER ITALICVM (completed in 1997) does not provide addenda to the list established by O'Boyle (ibid., 15–16).

[143] See below, n. 146.

[144] O'Boyle 1991, 75–77. I have added several commas and changed the editor's punctuation here and there to ease the reading.

[145] My punctuation. The editor misunderstood this passage: "*Et quia, ut probatum est supra ratione, proprie influentie et communis luna facit ad crisim.* [new paragraph] *Ideo medicus*" and so forth (O'Boyle 1991, 75). For *ratione* plus genetive (here: *proprie influentie et communis*) meaning 'with regard to' cf. ibid., 73 *Ratione autem quarte partis supradicte*, ibid. *ratione istius motus*, and here (p. 75) *ratione viginti quatuor dierum*. The textual reference (*supra*) is to p. 66: *duplex est lune influentia. Una est communis* [...]. *Est etiam influentia propria* [...].

[146] That is, the author departs from Galen's mathematical procedure (**S1**–**S2**) replacing $(26\,\tfrac{1}{2} + 27\,\tfrac{1}{3}) \div 2$ with $26\,\tfrac{1}{2} + ((27\,\tfrac{1}{3} - 26\,\tfrac{1}{2}) \div 2)$.

[147] That is, not comprising an integer number of days.

ris minus, qui per quatuor partes divisi quater septendecim horas cum dimidia [p. 76] *constituunt. Et istas septendecim horas cum dimidia <debemus>[148] unicuique septimane, et habebit quelibet septimana dies integros sex et septendecim horas cum dimidia. Et si diviseris septimanam, invenies, quod medietas eius continet tres dies et octo horas et tres quartas hore. Et si ulterius volueris dividere, scies, quot horas habet dies mensis medicinalis et quantum diminuitur a die naturali.[149] Septimana ergo prima habet sex dies et septendecim horas cum dimidia. Et ita ad hoc, quod[150] septem dies compleantur, oportet, ut a sequenti septimana mutuo accipiantur sex hore cum dimidia.[151] [...][152] complentur tres septimane in spatio viginti dierum, et est primus* [p. 77] *peryodus universalis, et remanent tantum quatuor hore cum dimidia.[153] Unde vicesima prima potest dici cretica. Et secundum istam formam debet fieri computatio in secundo peryodo universali et tertio. Et sic apparet secundum,[154] scilicet quare tres septimane medicinales complentur in viginti dierum spatio,[155] et quare non oportet computare secundum integritatem septimanarum et dierum.[156]*

Pietro d'Abano (1250–1316) discusses Galen's medical month towards the end of chapter (*differentia*) 104 of his *Conciliator* (1303).[157] He rejects Galen's computation as 'fanciful' (*secundum phantasiam*).[158] In his mathematically impeccable report of the core data, Pietro tacitly converts all sums of unit fractions into hours. He does not mention or discuss any word of the original Greek text that he translated.[159] Therefore it is unclear if he had access to an uncorrupted manuscript or if his figures are partly based on mathematically derived conjectures or if he had compared the Arabic translation of Ḥunayn ibn Isḥāq. Of the numerical data contained in our passage, he mentions the following: combined length of the sidereal and the curtailed synodic months: *liii dies et viginti hore*; its half: *xxvi dies et vigintidue hore*; one medical week: *ex sex diebus et decem et septem horis cum media*; and three medical weeks consist of *viginti diebus et quatuor horis cum media*.

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[148] My addition, supplying the same word that the Latin author had used two sentences before. One could also supply *damus*.

[149] This would lead to 23$\frac{1}{14}$ hours; cf. n. 434 below.

[150] *ad hoc, quod* means the same as *ut*.

[151] This is Galen's value in **S6** ($\frac{1}{4}$ $\frac{1}{60}$ $\frac{1}{240}$ d).

[152] The author discusses at some length the final days of the first and second medical weeks.

[153] This is Galen's value in **S7** (20 $\frac{1}{6}$ <$\frac{1}{48}$> d).

[154] This refers back to the second point of three that had been announced earlier (O'Boyle 1991, 66: *secundum est quomodo tres septimane in spatio viginti dierum complentur*); this point is actually discussed ibid., 72–77 (editor's summary: ibid., 11–12). All three points belong to the third main question of the treatise's second chapter, namely *de investigatione eorum* [scil. *dierum creticorum*] *via rationis* (ibid., 57).

[155] This is inaccurately phrased. Galen's actual reasoning at this point is elucidated below, Section VI, Question 9 (p. 263).

[156] Cf. Galen's confirmation of Hippocrates' words at p. 933.12–17 K., quoted in n. 391 below.

[157] On Pietro's criticism of Galen's critical days, cf. Pennuto 2008, 80–83, and Cooper 2013, 543–548. Pennuto used the reprint of Venice 1565. The *editio princeps* (Mantua 1472), which I used, lacks foliation.

[158] Pennuto 2008, 81.

[159] On Pietro's translation of Galen's work on critical days from the Greek cf. Thorndike 1942, 650, with n. 8. Pietro had learned Greek during a long stay in Constantinople, from where he brought various Greek medical manuscripts back to Europe. Cf. Touwaide 2017, 46–48.

An undeservedly forgotten milestone in the history of Galen's *On Critical Days* is its translation by Niccolò di Diopreprio da Reggio (ca. 1280 – ca. 1350).[160] This scholar from a southern Italian Greek family[161] was active as a translator from Greek and as a renowned (later even royal) physician[162] at the Neapolitan court of Robert I of Anjou (*1278, king 1309–1343). He translated well over fifty works of Galen,[163] mostly at the direct request of his king, but occasionally also at the request of other individuals, as in the case of *De diebus creticis*, which he translated at the request of Robert's personal physician, Giovanni della Penna (he died in 1349).[164] Niccolò's aim "was definitely not the production of fine literature, but merely to give a painstakingly faithful reproduction of the original Greek, in which every word was substituted by a Latin equivalent."[165] This verbatim translation strategy, which we find confirmed in our passage (pp. 932.5–933.12 K.),[166] allows us to make fairly precise judgments regarding single readings of his Greek manuscript, which must have been at least as old as the three oldest extant Greek manuscripts of our passage recorded by Diels[167] and may be related to (or even the ancestor of?) MSS **B** and **P**.[168] This Greek manuscript almost certainly belonged to the royal library of Robert I.[169] Therefore it probably shared the fate that overtook Robert's library: "The main dispersal of this very valuable collection occurred during the Hungarian invasion of 1347–1348, when its bulk was confiscated by Lewis I of Hungary, who presented it to his court physician, Conversino, the father of the humanist Giovanni Conversini da Ravenna. We are told this by Giovanni himself, from whose autobiography we also learn about the miserable end of part of it by shipwreck in the Adriatic, while on its way to Hungary."[170] It is quite possible that Niccolò's Greek exemplar of *On Critical Days* perished in that shipwreck.

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[160] On this scholar, see Thorndike 1946, Weiss 1950, 216–225, Hankinson 1998, 57–61, and especially Chandelier 2013.

[161] Diopreprio is an Italianized form of the Greek genetive Theoprepus ('worthy of God'). A public notary Nikolaos Theoprepus, probably our translator's grandfather, was active in Reggio di Calabria in 1257 and 1273 (Weiss 1950, 217).

[162] Weiss 1950, 216–218.

[163] Weiss 1950, 221.

[164] Cf. MS London, Wellcome 286, f. 68r: *<E>cce socie Karissime Magister Johannes* [supra lin.: *de penna*] *illustris domini principis tarentini librum Galeni de creticis diebus iuxta requisicionem uestram et instanciam. Ego Nicolaus de regio yd<i>omate greco in latinum trastuli.* The Greek manuscript belonged to Philip, prince of Taranto, the king's brother (Weiss 1950, 218). The present translation is missing from the otherwise meritorious list established by Thorndike 1946.

[165] Weiss 1950, 223 (cf. ibid., 224). Cf. Hankinson 1998, 57–58, who points out that the translators the king employed, in sharp contrast to their humanist successors, did not concern themselves with literary excellence, but aimed rather at literal accuracy, and that Niccolò was the foremost representative of this tradition.

[166] Cf., for example, n. 195 below.

[167] Vat. Pal. gr. 157, Ven. Marc. gr. app. V 8 and XI 5 (all three saec. XIV); cf. Diels 1905, 90–91.

[168] See above, pp. 207–208.

[169] Cf. Weiss 1950, 220: "Similarly, we can safely assume that he [*that is, Niccolò*] must have drawn, at any rate in most cases, the necessary Greek originals from the Royal Library." Note that Hankinson 1998, 58, reports that Robert I "was in contact with the emperors in Constantinople, and sometime between 1332 and 1335 he received a gift of a number of manuscripts from Andronicus III." This assertion could make the reader wonder if Niccolò's Greek manuscript had been part of the Byzantine emperor's donation. Actually, however, Weiss 1950, 208 (Hankinson's declared source), speaks of only one work, namely Galen's *De passionibus uniuscuiusque particule* (ibid., 220 with n. 2), and so does the ultimate source, Niccolò himself, in his description of the emperor's gift and its circumstances (quoted by Delisle 1880, 432–433). Hence, the Greek manuscript of *On Critical Days* was almost certainly not part of that donation.

[170] Weiss 1950, 211–212.

While other translations by Niccolò continued to be appreciated, translated and even printed in the Renaissance,[171] this one seems to have fallen into oblivion. Nevertheless, it would have been more useful to readers than any of those produced later by various humanists, at least regarding Galen's computation of the medical week, because Niccolò's version of this passage is mathematically flawless (except for $<\frac{1}{48}>$ which is missing from **S7** here as it is from all extant Greek MSS). Therefore it deserves to be edited here, based on the only two extant manuscripts, both parchment manuscripts from the first half of the 14th century: London, Wellcome Library MS 286 (**W**),[172] f. 80v, and Madrid, Biblioteca Nacional MS 1978 (**M**),[173] f. 118r. They are both (direct or indirect) copies of Niccolò's lost autograph, as is clear from copying mistakes, more frequent and more serious in **M**. The high material quality of **W**, not only written by professional scribes but also beautifully illuminated (see especially ff. 11r, 84r, 119r, 132r), indicates that this excellently preserved manuscript may have belonged to the Neapolitan royal library.[174] Moreover, **W** (unlike **M**) contains sometimes extensive marginal notes likely written by the same clear gothic hand that copied the main text. Presumably these comments go back to Niccolò, too, who is known to have critically studied the works he translated.[175] These marginal explanations convert the majority of Galen's sums of unit fractions into hours.[176] In the following edition, which is based on **W**, this manuscript's marginal notes as well as variant readings of **M** will be recorded in the notes.

Quoniam igitur apparicionis quidem manifeste et actiue[177] tempus dierum est 26 et dimidie, zodiace[178] uero periodi 27 et tercie, manifestum est, quod medium[179] eorum 26 dierum erit et adhuc porcionis maxime dimidie et tercie[180] et duodecime unius diei.[181] **S1** *Cum enim composueris utraque tempora,* **S2** *si medietatem acceperis,* **S3** *inuenies quod medium, scilicet ipsum tempus medium, deficit a 27 diebus in xiiᵃ maxime parte[182] unius diei.* **S4** *Si igi-*

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[171] Weiss 1950, 223.

[172] Digitized and described at https://wellcomelibrary.org/item/b19294979.

[173] Digitized at http://bdh-rd.bne.es/viewer.vm?id=0000012728&page=1, described at
  http://www.galenolatino.com/index.php?id=12&L=&uid=141.

[174] While I leave this question to codicological experts, the reader may wish to take notice of the fact that this manuscript was purchased in 1934 from the antiquarian Hoepli's sale of the Bibliothèque Joseph Martini (Lucerne). Martini owned various rare manuscripts from the collection of the Neapolitan philosopher and writer Gaspare Selvaggi (1763–1856), which points to a Neapolitan origin. Cf. Fabris 2013, 116–117. An anonymous codicological description in French glued on the inner part of the front cover remarks: "Notre manuscript semble avoir fait partie d'un ouvrage plus étendu, qui dans une époque reculée fut divisé et en partie recopié."

[175] Weiss 1950, 224–225, with examples.

[176] This is true of figures H1a, H1b, H1c, H4a, H4b, and H4c of Table 1, above. See also the marginal table on the previous page of **W** (f. 80r), in which the mathematically correct specification of one medical day as 23 $\frac{1}{14}$ hours deserves attention: *dies medicinalis habet horas 23 et 14ᵃᵐ partem unius hore*. More on this below, n. 434.

[177] *actiue* **W** : *atute* **M**.

[178] Sic **WM** (*zodiace periodi* = ζῳδιακῆς περιόδου; cf. OLD s.v. *zodiacus, -a, -um*).

[179] *medium* ex *medie* **W**.

[180] **W** supra lineam: *idest 8 hore.*

[181] *dimidie – diei* scripsi : *tercie et duodecime unius diei et dimidie* **W** : *dimidie et 3ᵉ et xiiᵉ unius diei et didie* (debuit scribere *dĩdie = dimidie*) **M**. It seems that Niccolò had first placed the word *dimidie* at the end and later (after **W** had been copied?) somehow transposed or repeated it in its proper position after *maxime*.

[182] *in xiiᵃ maxime parte* **W** : *et 12ᵃ parte* **M**.

tur diuiseris[183] id in 4[or], inuenies certitudinale tempus septimane non in sexto solum[184] unius diei deficere, sed etiam plus. Accipiatur enim de 26[a] et media et tercia et xii[a] quarta pars, et erit tempus dierum sex et dimidie et quinte et adhuc aliarum porcionum superpositarum[185] minorum, que sunt sexagesima,[186] centesima uicesima,[187] et ducentesima quadragesima.[188] **S5** *Hoc ipsum tempus[189] et ita[190] dicetur esse dierum sex et dimidie et sexte et xxiiii[te] et quadragesimeoctaue.[191]* **S6** *Id autem tempus deficit[192] in 4[ta] unius diei et adhuc[193] secundum certitudinalem rationem in sexagesima et ducentesima quadragesima[194] parte.* **S7** *Et hijs ita habentibus[195] tres septimane erunt dierum xx et adhuc porcionis cuiusdam scilicet sexte unius diei. Quare et secundum hanc rationem acribologizantibus[196] parum excedit 20um diem trium septimanarum numerus, et huic erit magis proprius quam xxi[e].*

Pierre d'Ailly (1350–1420) explains Galen's medical month in chapter 39 of his *Elucidarium*.[197] He may have followed the *Aggregationes de crisi et creticis diebus* because he departs from Galen's computational steps **S1**–**S2** in the same manner as described above (n. 146). D'Ailly reports the values H1b, H2b, H4a, H4b, H4c of Table 1 above in days and hours without mentioning the original sums of unit fractions. The brief chapter ends thus:

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[183] Comm. in marg. **W**: *Si igitur diuiseris: id scilicet tempus mensis medicinalis, quod est dies 26 et hore 22, per septimanas 4 erunt dies 6 et hore 17 et media per quamlibet septimanam. Et sic deficit istud tempus a vii diebus non in sexto unius diei tantum, quod est hore 4, sicut deficiebat septimana mensis peragracionis, verum etiam deficit in horis vi et media.*

[184] *in sexto solum* **W**: *in 7[o] solummodo* **M**.

[185] *superpositarum* **W** : *superponitarum* **M**.

[186] Comm. in marg. **W**: *Sexagesima pars unius diei est due quinte unius hore*.

[187] Comm. in marg. **W**: *Centesima <vicesima> est quinta pars unius hore* [correxi; *diei* **W**].

[188] The scribe of **W** wrote *quartagesima* here and twice more in the following lines as well as in the commentary (in marg.): *Ducentesima quadragesima* [*quarta-* **W**] *est media quinta, quibus aggregatis cum sex diebus et media et quinta parte unius diei faciunt in summa dies sex et hore xvii et media, que* [correxi; *mediaque* **W**] *est septimana medicinalis*. In all four cases, the scribe abbreviated the first two syllables as *q[at]a-*. While the adjective *quartagesimus, -a, -um* is not recorded in any dictionary that I know of, a google search retrieves attestions here and there in early modern printed text (both Latin and Italian). In our text, it is probably to be explained as a morphologically (not mathematically) erroneous scribal expansion of Roman numerals or Arabic ciphers in the translator's autograph. That Niccolò had used them (or a mix of them) is indicated by **M**'s corrupt text here (*que sunt 6[a] extensiua 20[a] et cc[a] ɔ[a]*, the final scribble seems to be due to a misreading of the letter *x*) and by **M**'s correct readings below in n. 191 and 194.

[189] Comm. in marg. **W**: *hoc ipsum tempus etc.: idest secundum aliam computacionem similiter fit septimana medicinalis dies 6 et hore 17 et media*.

[190] Niccolò's Greek manuscript obviously read οὕτως.

[191] Correxi : *quartagesimeoctaue* **W** : *48[e]* **M**.

[192] τοῦ τῶν ἑπτὰ ἡμερῶν om. **WM** and apparently the Greek exemplar, too; cf. comm. in marg. **W**: *Id autem tempus deficit scilicet* (!) *a vii diebus in 4[a] unius diei: idest 6 horis et in sexagesima et ducentesima quartagesima parte, que faciunt duas quintas et mediam quintam, idest mediam horam, et sic restat septimanam medicinalem esse dierum sex et horarum 17 et medie secundum ambos modos computacionis*.

[193] Comm. in marg. **W**: *et adhuc etc.: que omnes sunt hore 22 et dies 26*.

[194] Correxi; *quartagesima* **W**. The entire fraction reads *in 60[a] et ccxl[a] parte* in **M**.

[195] *Sic* **WM** (without *se*), a very literal translation of ἐχόντων.

[196] *acribologizantibus* **W** : *acribolozantibus* **M**. ThLL and MLW do not know words beginning with *acribolog-*, but cf. LLN I 119 s.v. *achribologia* (sic). An attestation not mentioned there is Thomas Aquinas, comm. in Arist. metaph. 2.5 no. 336 in Cathala and Spiazzi 1950, 93.1: *'acribologia' idest diligens et certa ratio, sicut est in mathematicis*.

[197] Cf. the *editio princeps*: *Concordantia astronomie cum theologia, Concordantia astronomie cum hystorica narratione, et Elucidarium duorum precedentium domini Petri de Aliaco cardinalis Cameracensis*, Augsburg: Erhard Ratdold, 1490, f. g<7>r–v.

Et [*ad*] *hanc computationem debet considerare* [correxi, *consinuare* ed.] *medicus in iudicando de creticis diebus, unde patet, quod in hoc sicut et in pluribus aliis medicus habere debet astronomice scientie aliquam noticiam sicut et quilibet physicus.*[198]

Giovanni Pico della Mirandola (1463–1494) criticizes Galen's doctrine of the critical days, and especially his invention of an artificial medical week, in chapter 3.16 of his *Disputationes adversus astrologiam divinatricem.* [199] As far as Galen's computation is concerned, Pico converts Galen's fractions correctly into full and half hours, probably following the example of Pietro d'Abano, whose *Conciliator* he quotes in the same chapter.[200] Pico goes beyond both Pietro and Galen himself in specifying all values of Table 1 above except for H1a, H2a und H3a. Incidentally, Pico—who did not own a copy of the Greek text of *On Critical Days*—may well have used manuscript **B**.[201]

Agostino Nifo (c. 1470–1538)[202] explains Galen's computation of the medical month briefly in his *De diebus criticis seu decretoriis aureus liber* 2.13.[203] Like Pietro d'Abano and Pico (he refers to both)[204] before him, he does not speak of fractions of days but of hours. Nifo deems it sufficient to give approximate values for the lengths of the medical week and day (*patet hebdomadam esse ex diebus .vi. horisque circiter .xvii., diem vero ex .xxiii. circiter horis*). He does not tackle any of the philological problems of the Greek text nor does he display any interest in deepening his predecessors' understanding of Galen's computation. He says that the medical month was actually used by physicians of his own time.[205]

The Aldine edition of Galen's *opera omnia* (Venice 1525), that is, of the largest extant corpus by a single ancient author, was a milestone in the history of scholarship. Aldus (born about 1450), who had announced this extremely ambitious project in 1497, died in 1515, a decade before its completion.[206] It eventually appeared in five volumes under the direction of Andrea Torresano (1451–1529, *alias* Andrea Asolano), Aldus' father-in-law and business partner, and under the scholarly supervision of Giovan Battista Opizzoni, a professor of medicine at the university of Pavia, who coordinated a team of young foreign scholars from Oxford and Leipzig.[207] The goal of this project was to recover the original

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[198] That is, whoever investigates natural phenomena. The edition has *phūs*, which could also mean *philosophus*.
[199] The work was published posthumously (ed. princ. Bologna 1496; many reprints). The only modern edition is that of Garin 1946–1952 (ch. 3.16: I 322–348). In view of its shortcomings, a new critical edition of books I to IV is being prepared by Benjamin Topp. For one large textual emendation regarding Pico's criticism of Galen, see Heilen and Topp (forthcoming). On Pico's attack on medical astrology, see also Pennuto 2008, 83–87; Cooper 2013, 550–553; Hirai 2014, 267–269.
[200] Garin 1946–1952, I.324.30.
[201] Cf. pp. 204 and 208 above. The Barberini codex features attention marks on the margins that were typical of Pico but used by some other Renaissance scholars too. I thank Franco Bacchelli (Bologna) for pointing this out to me.
[202] Cf. Palumbo 2013.
[203] Ed. Venice 1519, ff. 7rb–7va, = ed. Strassburg ca. 1530, f. E5r–v.
[204] Ibid. ch. 2.18 (ed. Venice 1519, f. 8vb, = ed. Strassburg ca. 1530, f. F5r).
[205] Ed. Venice 1519, f. 7rb, = ed. Strassburg ca. 1530, f. E5r: *Ultra hosce Galenus adinvenit mensem, in quo medici ipsi in accessionibus ac circuitibus morborum ipsisque pharmacis exhibendis, enumeratione criticorum, predi<c>tionibus ac victibus administrandis uti solent, ob quod medicinalis appellatus est.*
[206] Perilli 2005, 422–423.
[207] Ibid., 424–425. Opizzoni's team comprised John Clement, Edward Wotton, William Rose, Thomas Lupset, and Georg Agricola (Gundert 2006, 81).

Greek text of Galen. Therefore Aldus and his fellow humanists dismissed the (in their view) corrupt Arabic tradition and the medieval Latin translations based thereon. Cooper (2012) shows with manifold examples from *On Critical Days* the loss that this meant to Galenic scholarship. After the publication of the Aldine edition, the Arabo-Latin tradition was almost entirely excluded from subsequent Galenic scholarship.[208] Deplorably, the Aldine edition had serious shortcomings in its use of the Greek manuscript tradition too. Teeming with errors of various kinds and origins, it was scathingly criticized very soon after publication by Erasmus of Rotterdam, and by other humanists in the 16th century.[209] The text of *On Critical Days* was, according to Cooper, based on the Venetian manuscript Marc. gr. app. V 8 (**V**), ff. 62–102.[210]

The need for a Latin translation of the new Aldine edition of *On Critical Days* must, especially among practicing physicians, have been widespread. The first humanist who successfully addressed this need was a young German, Johannes Winter of Andernach (1505–1574).[211] His translation (**t3**) was first published anonymously in Paris in 1529. The following quotation is from this Parisian *editio princeps* (ff. 63v–64r) with all textually relevant variants[212] from the partly revised reprints of 1544 (Venice, pp. 726–727),[213] 1550 (Lyon, pp. 146–148),[214] 1565/1586/1609/1625 (all four Venice, f. 159v),[215] 1679 (Paris, p. 504) and 1825 (Leipzig, pp. 932–933) in the footnotes.[216] Among the various editors of these editions, two deserve special attention: Agostino Ricci (1512–1564) prepared his edition of Galen's *opera omnia* in Latin, and had it printed by Farri's press in Venice in 1541–1545, with rich and accurate annotations based on the collation of Greek manuscripts.[217] Ricci revised the Latin translation of *On Critical Days*, which he explicitly at-

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[208] Cooper 2012, 641.

[209] Perilli 2005, 426–432 (an excellent account), and Fortuna 2012b, 309–310.

[210] Cf. n. 34 above. It is in the final volume: *Galeni librorum pars quinta*, Venetiis: Aldus, 1525, ff. 120v–135r.

[211] *Claudii Galeni Pergameni De diebus decretoriis libri tres*, Parisiis apud Simonem Colinaeum (that is, Simon de Colines), 1529. On Winter, cf. Broemser 1989 and above, p. 202.

[212] That is, orthographical variants will be omitted.

[213] *Galeni operum omnium sectio tertia. Illustriores quam unquam antea prodeunt in lucem omnes hi Galeni libri. Accesserunt enim his ex veterum Graecorum exemplarium collatione adnotationes luculentissimae singulis locis in margine appositae*, Venetiis apud Ioannem Farreum et fratres, 1544, 642–731.

[214] A slightly modified reprint of the Venetian edition of 1544.

[215] These are the fourth, sixth, eighth, and ninth editions published by the Giuntas. The fourth edition (1565) is available online at https://archive.org/details/galeniomniaquaee35gale/page/n6; the ninth (= last) is available at https://hdl.handle.net/2027/ucm.5325107585. In each edition's case *On Critical Days* is in the *quarta classis* of the multi-volume editions. I did not see the first, second, third, fifth, and seventh Venetian editions. The Latin text must have been set again and again for each reprint, as typographical differences in detail show. Curiously, the text of the relevant page (f. 159v) is set almost (yet not entirely) identically in the editions of 1586 and 1625, while the line breaks are different in the intermediate edition of 1609.

[216] This may seem unusual because many of the variants in the footnotes are philologically preferable to Winter's original text, but it is important for the present purpose to consider Winter's translation in its original, imperfect shape because this is what his contemporaries read and what prompted Lalamant (see below, p. 227) to make a new translation from the Greek. I did not see the other reprints of Winter's translation mentioned by Durling 1961, 285.

[217] On the merits of this ambitious editorial project see Fortuna 2012a, 404–405, as well as the proudly phrased title quoted in n. 213 above.

tributes to Johannes Winter of Andernach.[218] The second particularly meritorious editor was Agostino Gadaldini (1515–1575).[219] This physician from Modena edited all four *Giuntines* that appeared in Venice in the years 1541/42, 1550, 1556, and 1565.[220] Gadaldini made use of both Greek editions (Venice 1525 and Basel 1538) and Greek manuscripts. His titanic effort to revise and correct such a vast corpus made him seriously ill: Only after several months of bed rest could he resume his work. His emendations and variant readings from Greek editions and manuscripts were printed in the margins of his *Giuntines*. The last version of these *marginalia* he saw through the press is here recorded in the notes (ed. 1565). His text and *marginalia* were then reprinted without change over the following decades, when Galenic medicine began to decline, until 1625, when the last *Guintine* edition was published. It appears that Gadaldini's notes to our passage are mostly variants which he presents without expressing a critical judgment. The only explicit correction (cf. n. 234 below) concerns the important addition of $\frac{1}{48}$ in **S7**. Regrettably, he did not correct the main text accordingly, and all his *marginalia* to Galen's computation were disregarded by Chartier (1679) who drew on Gadaldini's work. The latter's effort thus remained sterile.

This is Winter's translation (**t3**):

Quum ergo apparitionis manifestae et efficacis tempus sit dies vigesimussextus et dimidius, signiferi[221] vero circuitus expleant vigintiseptem dies cum tertia unius parte, liquet medium ipsorum fore vigesimumsextum diem, ad haec[222] tertiam maxime et duodecimam unius diei dimidii[223] partem. **S1**–**S2** *Siquidem ubi utraque tempora dimidio coniunxeris,[224] medium ipsum invenies.* **S3** *At hoc tempus medium a vigintiseptem diebus duodecima[225] unius diei parte deficit,* **S4** *quia, si in quatuor ipsum dividas, exactum septimanae tempus offendes[226] non sexta[227] modo diei[228] parte, sed etiam maiore indigens.[229] Sumatur[230] enim ex vigintisex*

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[218] Ed. 1544, 642. I did not see the preceding editions of Basel 1533 (Cratander and Bebel), Paris 1534 (Simon de Colines) and Venice 1541–42 (Giunta) mentioned by Durling 1961, 257–258, 280, and 284–285. It is possible that Winter was credited as the translator already in those editions.

[219] I owe the following information on him to Fortuna 2012a, 399–401. See also Garofalo 2004.

[220] This is emphasized by Fortuna 2012a, 411–412, with reference to an explicit statement of the publisher of the Giuntine of 1565, Tommaso Giunta. Gadaldini did not sign any preface in the *Giuntines* of 1556 and 1565, probably because he and his family had been put on trial by the Inquisition; this required caution on the part of the publisher. On Gadaldini's first four *Giuntines* as a group that needs to be distinguished from earlier (1490–1533) and later (1576–1625) editions of Galen's *opera omnia* in Latin cf. Fortuna 2012a, 393.

[221] *signiferi* edd. 1529, 1544, 1550, 1565, 1586, 1609, 1625 : *zodiaci* edd. 1679, 1825.

[222] *haec* ed. 1529 : *haec* (*hǫc* ed. 1609) *dimidium et* edd. 1544, 1550, 1565, 1586, 1609, 1625, 1679, 1825.

[223] *dimidii* edd. 1529, 1544, 1550 : om. edd. 1565, 1586, 1609, 1625, 1679, 1825.

[224] *utraque tempora dimidio coniunxeris* ed. 1529 : *utraque tempora coniunxeris et dimidium acceperis* edd. 1544, 1550 : *utroque tempore coniuncto dimidium acceperis* edd. 1565, 1586, 1609, 1625, 1679, 1825.

[225] *duodecima* ed. 1529 : *duodecima* (-*cim* edd. 1586, 1609, 1625) *maxime* edd. 1544, 1550, 1565, 1586, 1609, 1625, 1679, 1825.

[226] *offendes* edd. 1529, 1544, 1550, 1565, 1609, 1679, 1825 : *offendens* edd. 1586, 1625.

[227] Edd. 1565, 1586, 1609, 1625 in marg.: *Aliter quarta*.

[228] *diei* edd. 1529, 1544, 1550 : *unius diei* edd. 1565, 1586, 1609, 1625, 1679, 1825.

[229] *indigens* edd. 1529, 1544, 1550, 1565, 1586, 1609, 1625 : *mutilum* edd. 1679, 1825.

[230] Edd. 1565, 1586, 1609, 1625 in marg.: *Aliter: Sumatur enim ex viginti sex diebus et dimidio et ex unius diei tertia et duodecima parte pars quarta; erit sane tempus dierum sex et dimidii et quintae partis et particularum aliarum minorum insuper adiectarum, quae sunt primum sexagesima, dein* (*deinde* edd. 1586, 1609, 1625).

diebus et dimidio pars quarta et unius diei tertia et duodecima;[231] erit autem tempus dierum sex dimidii quinta pars et particulis aliis minoribus insuper adiectis, quae sunt sexagesima prima, dein[232] centesima vigesima, ducentesima quadragesima. **S5** *Idem tempus et hoc dici possit dierum sex et dimidii et sextae partis diei et vigesimaequartae et quadragesimaeoctavae.* **S6** *Abest autem huiusmodi tempus a septem diebus unius diei quadrante, et si exacte computes,[233] sexagesima parte, centesima vigesima, et ducentesima quadragesima.* **S7** *Quae quum ita se habeant, tres septimanae viginti diebus et sexta unius diei parte[234] comprehendentur. Proinde, si hunc in modum exacte rationem ineamus, trium septimanarum numerus paululum[235] viginti dies excedet, atque his multo magis quam viginti et uni erit proprius.*

Luca Gaurico (1475–1558)[236] explains Galen's computation in a summary fashion (that is, without following Galen's text closely) in the first part of his *Super diebus decretoriis (quos etiam criticos vocant) axiomata* (Rome 1546), 14–15.[237] Like Pietro d'Abano, whom he quotes, Pico, and others before him, Gaurico expresses all numerical values in hours and minutes. He actually mentions the values H1b, H2b, H3b, H4a–c of Table 1 above. He does not address problems of textual criticism, nor does he mention Galen's unit fractions.

Our next scholar deserves more attention: Jean Lalamant of Autun,[238] from whose pen we have a Latin translation (**t4**) with an interesting commentary that was first published in 1559 by Guillaume Rouillé (Lyon) and reprinted by the same publisher the following year (I used this reprint).[239] Lalamant's translation reads thus:

Cum ergo apparitionis manifestae et efficacis tempus sit dierum vigintisex et dimidij, tempus vero circuitus zodiaci dierum sit vigintiseptem et tertiae praeterea partis diei, liquet quod medius ipsorum mensis dierum vigintisex erit et insuper partis unius diei dimidiae, item tertiae et duodecimae. **S1** *Siquidem ubi utraque tempora coniunxeris* **S2** *et dimidium acceperis, medium ipsum deprehendes.* **S3** *Sed enim medium hoc tempus a diebus vigintiseptem deficit duodecima potissi*[p. 491]*mum unius diei parte.* **S4** *Et si quidem ipsum in quatuor dividas, iustum et integrum septimanae tempus offendes non sexta modo diei parte,*

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[231] *pars quarta et* (*et ex* edd. 1544, 1550, 1565, 1586, 1609, 1625) *unius diei tertia et duodecima* edd. 1529, 1544, 1550, 1565, 1586, 1609, 1625 : *tertia et ex duodecim quarta pars* edd. 1679, 1825.

[232] Edd. 1529, 1544 in marg.: *In veteri graeco codice habetur ρκβ', idest centesima vigesima secunda, in graeco impresso* [that is, in **v**] *hic locus aliter legitur; alii videant.* The 'old Greek manuscript' was neither **B** nor **P** nor **V**.

[233] Edd. 1565, 1586, 1609, 1625 in marg. (ante *sexagesima*): *etiam quadragesimaoctaua parte.* This is either a misplaced duplicate of the marginal text to **S7** (cf. the next note) or, more likely, a reference to the mathematical variant introduced by Lalamant (cf. n. 67 above and the respective text of **t2** on p. 228 below).

[234] Edd. 1565, 1586, 1609, 1625 in marg.: *Addendum, et quadragesimaoctaua.*

[235] Edd. 1544, 1550 in marg.: *Graecum exemplar* [that is **V**, cf. n. 72 above and Rizzo 1973, 69] *η id est octo habet, sed male, qui quidem error in describendo committi facillime potuit, quod κ litera et nota numeri vigesimi similitudinem quandam habeat ipsius η.*

[236] Cf. Bacchelli 2013.

[237] I did not see the first edition of this work (Rome 1545).

[238] Also Jean Lallemant; for details, cf. Firmin-Didot 1859, 943–944.

[239] *Claudii Galeni Pergameni De diebus decretoriis libri tres recens Latini facti et commentariis inlustrati* [...] *Ioanne Lalamantio apud Heduos* [...] *medico clarissimo auctore*, Lugduni apud Gulielmum Rouillium 1560, 490–491, with commentary ibid., 506–510.

*sed et maiore ad sui integritatem indigere. Sumatur enim ex vigintisex diebus, diei item dimidio, ex tertia itidem et duodecima diei partibus pars quarta; erit sane tempus id dierum sex et dimidij, quintae item partis, et aliarum adhaec [sic] minorum partium adiectarum, quales sunt sexaginta prima, centum viginti secunda, ducentesima quadraginta. **S5** Idem hoc ipsum tempus dierum sex et dimidij dici queat, sextae partis diei et vicesimaequartae et quadragesimaeoctavae. **S6** Huiusmodi autem tempus abest a iusto et integro septem dierum numero unius diei quadrante et exactiori adhuc calculo quadragesimaoctava.[240] **S7** Quae cum ita se habeant, tres septimanae dierum viginti erunt et partis diei aliquotae, ut pote sextae. Proinde si ita exacte rationem inierimus, trium septimanarum numerus paulo viginti dies excedet et huic multo vicinior erit quam vicesimo primo.*

It appears that Lalamant's translation is not thoroughly independent but a modified version of Winter's translation (**t3**): They have significantly more in common than either has with Niccolò da Reggio's (**t2**), who equally worked from the Greek text. Only rarely Lalamant's Latin expressions are remarkably different from Winter's, as is the case with Lalamant's redundant use of *iustum et integrum ... ad sui integritatem* (**S4**) and *iusto et integro* (**S6**) as well as his substitution of Winter's *magis ... proprius* with the less literal *vicinior* (**S7**). Lalamant's independence is more clearly visible at the mathematical level than at the philological one, because he renders the single steps of Galen's computation correctly except for **S4**, where his translation is wrong, and **S6**, where he substitutes (and thus simplifies) Galen's $\frac{1}{60} + \frac{1}{240}$ with $\frac{1}{48}$.

In his four-page-commentary on Galen's computation, Lalamant provides numerous chunks of text from the two Greek editions (**vb**), to which he refers explicitly as *Graecus uterque codex* (p. 506).[241] He also quotes a total of about twelve lines of text from the *vetus interpres* (= **t1**)[242] on pp. 504, 507, and 509, and one line from *Guinterius* (= **t3**) on p. 507. Lalamant also quotes a lot of 'secondary literature,' admittedly not in his commentary on pp. 932.5–933.12 K., but, for instance, on the immediately preceding sentence (p. 932.3–5 K.), where he refers (p. 505) to Pico's *Disputationes* (first published Bologna 1496), ch. 3.16; Giovanfrancesco Pico's *De rerum praenotione* (first published Strasbourg 1506), ch. 5.4; Auger Ferrier's (1513–1588) *Liber de diebus decretoriis secundum Pythagoricam doctrinam et astronomicam obseruationem* (Lyon 1549); Jean François Fernel's (1506[?]–1558) *Monalosphaerium*[243] (Paris 1526), ch. 2.11–12; and Pietro d'Abano's *Conciliator*, ch. 104.

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[240] There follows a raised number *14* which refers to section 14 of the respective commentary where, however, the present mathematically correct fraction $\frac{1}{48}$, which replaces Galen's $\frac{1}{60} + \frac{1}{240}$, is not mentioned.

[241] Cf. ibid., 507 *codices graecos* and p. 508 *Asulanius* (sic, scil. *codex*, = **v**). Before our passage, he spoke explicitly of the *Asulanus codex* as opposed to the *Basiliensis* (p. 501). On the term *codex* denoting a printed book see Rizzo 1973, 7 and 69–75. Lalamant's Greek quotations confirm that he is speaking of **v** and **b**.

[242] He quotes not from the *editio princeps* (1490) but from one of the reprints, probably that of 1515, whose variants recorded in notes 127–134 above he follows in one short and one longer explicit quotations on pp. 507 and 509.

[243] The title is distorted to *monolaspherij.*

While Lalamant has indisputably read extensively, his commentary suffers from two weaknesses, one formal, in that he becomes repetitive and prolix in his effort to be absolutely clear and exhaustive, the other more serious, namely a lack of sound philological judgment, as he tends to propose inept 'emendations' of the printed Greek text.[244] One must admit, however, that he explains correctly the by his time usual conversion of Galen's fractions of days into hours and minutes.[245]

For us, the commentary's importance lies not in Lalamant's own ideas but in his report of a mathematical insight of another scholar, with whom Lalamant was obviously acquainted: Jean Guijon of Saulieu (Bourgogne), a famous physician and orientalist who spent his retirement as a physician in Lalamant's native city Autun.[246] His take on our passage is known only through Lalamant. Guijon emended the printed editions' (**vb**) corrupt Greek text of the second sentence of **S4** thus: λαμβανέσθω γὰρ τῶν κϛ΄ καὶ ἡμίσεως καὶ τρίτου καὶ ιβ΄. τὸ τέταρτον μέρος, ἔσται δὲ χρόνος ἡμερῶν ϛ΄ καὶ ἡμίσεως καὶ πέμπτου καὶ προσέτι μορίων ἄλλων ἐπικειμένων μικροτέρων ἅπερ ἐστὶ μη΄ καὶ ρκ΄.[247] This is mathematically impeccable. My only objection concerns the final words where Guijon should have preserved Galen's concept of adding up $\frac{1}{60}$ + $\frac{1}{120}$ + $\frac{1}{240}$ (writing, for example, ξ΄ καὶ ρκ΄ καὶ σμ΄) instead of replacing that sum of unit fractions with its more concise mathematical equivalent $\frac{1}{48}$ + $\frac{1}{120}$.[248] Curiously enough, Lalamant preferred not to accept Guijon's solution and proposed, instead, at great length one which he claims to have devised before consulting Guijon: It coincides with Guijon's in the first, correct part but takes an implausible turn with ἔσται δέ, at which point Lalamant profoundly changes the transmitted text in order to express the medical week's length (6d 17 $\frac{1}{2}$ h) as 6 $\frac{1}{2}$ $\frac{1}{6}$ days (= 6d 16h) plus ei-

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[244] Instead of φάσεώς ἐστι (p. 932.6 K.), he prefers (saying: *legendum credam*) φάσεώς ὃ ἐστι (p. 506); he resolutely athetizes p. 933.10–11 K. ἡμέρας ὁ τῶν τριῶν ἑβδομάδων ἀριθμός (p. 509), and instead of p. 933.9 K. α΄ ἕκτου he conjectures (saying: *putem scribendum*) ἅπερ ἕκτου (ibid.). His most extensive 'emendations,' which he abandoned thanks to Guijon's timely dissuasion, will be quoted in notes 249 and 254 below.

[245] Cf., for example, p. 506, where he converts Galen's $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{12}$ days of **S2** as the sum of 12 + 8 + 2 hours, or already on p. 502 (= comm. on p. 931.3–6 K., a brief anticipation by Galen of steps **S1**–**S2**), where Lalamant explains Galen's steps **S1**–**S2** correctly, and with specification of the interim result that Galen omits, as 27d 8h + 26d 12h = 53d 20h, which divided by 2 yields 26d 22h.

[246] Cf. Firmin-Didot 1858, 581.

[247] I quote Lalamant's report in full (pp. 507–508): *Caeterum emendandi sunt etiam in his quae sequuntur codices Graeci* [that is, **vb**, especially **b**], *qui sic habent:* λαμβανέσθω γὰρ τῶν κϛ΄ καὶ ἡμίσεως τρίτου ιβ΄. τὸ τέταρτον μέρος, ἔσται δὲ χρόνος ἡμερῶν ϛ΄ ἡμίσεως πέμπτον καὶ προσέτι μορίων ἄλλων ἐπικειμένων μικροτέρων ἅπερ ἐστὶ ξ΄ πρῶτα καὶ ρκ΄ τὰ δεύτερα σ΄ καὶ μ΄. ὁ δ' αὐτὸς χρόνος καὶ οὗτος ἂν λέγοιτο ἡμερῶν ϛ΄ καὶ ἡμίσεως καὶ ἕκτου καὶ κδ΄ καὶ μη΄. ὁ δὲ τηλικοῦτος χρόνος ἀπολείπεται τοῦ τῶν ἑπτὰ ἡμερῶν τετάρτῳ μέρει μιᾶς ἡμέρας καὶ ἔτι πρὸς τὸν ἀκριβῆ λόγον ξ΄ τε καὶ ρκ΄ καὶ σμ΄. *Legendum enim est, vel de Domini Ioannis Guiionij nostratis et medici et mathematici clarissimi consilio, hunc in modum*: λαμβανέσθω γὰρ τῶν κϛ΄ καὶ ἡμίσεως καὶ τρίτου καὶ ιβ΄. τὸ τέταρτον μέρος, ἔσται δὲ χρόνος ἡμερῶν ϛ΄ καὶ ἡμίσεως καὶ πέμπτου καὶ προσέτι μορίων ἄλλων ἐπικειμένων μικροτέρων, *ut sit: 'Sumatur enim ex diebus sex et viginti, ex diei medietate, ex tertia item et duodecima, pars quarta. Erit autem id tempus dierum sex et dimidij, et quintae, partium insuper aliarum minorum adiectarum, quae sunt quadragesimaoctava, et centesima vicesima. At vero idem ipsum enuntiaveris dierum sex et dimidij, et sextae, et vicesimaequartae et quadragesimaeoctavae partium'* etc. *Ceterum ubi in codicibus Graecis legitur* ἅπερ ἐστὶ ξ΄ πρῶτα καὶ ρκ΄ τὰ δεύτερα σ΄ δὴ καὶ μ΄ (= ed. **b**), *putat idem Guiionius collega meus doctissimus legendum* ἅπερ ἐστὶ μη΄ καὶ ρκ΄. *Caetera falso asscripta putat.* The last sentence clearly means that Guijon considered words such as πρῶτα and δεύτερα in the final sum of unit fractions inauthentic. It does not mean that Guijon athetized the following sentence with its alternative sum of unit fractions (**S5**), because that is included in Lalamant's translation of Guijon's emended text (see above).

[248] Note, however, that the resulting value of 6 $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{48}$ $\frac{1}{120}$ days is still not the best possible mathematical solution (cf. n. 374 below).

ther 90 minutes or 5,400 seconds or 60 minutes and 1,800 seconds or 60 minutes and 1<0>8,000 thirds.[249] Far from feeling unsure about his bold changes to the transmitted text, Lalamant takes pride in his 'Herculean' (!) achievement which was needed, he rashly suspects, because the whole complex expression of the medical week's length was not by Galen himself but originated in a learned reader's marginal note later inserted by a copyist into the text only to be horribly corrupted by typesetters.[250] Of higher value for us is the insight that Guijon may have been the first scholar to emend the final words of **S6** by reading καὶ ἔτι πρὸς τὸν ἀκριβῆ λόγον ξʹ τε καὶ σμʹ.[251] I say 'emended' because Lalamant does not mention any manuscript evidence followed by Guijon, who rather appears to have corrected the corrupt text of the printed editions (**vb**) Lalamant had presented to him, based on his own mathematical competence. If so, Guijon unknowingly restored the correct reading of **V**.[252] It is revealing of Lalamant's complacency and lack of judgment that once more[253] he cannot refrain from telling the reader that he had thought, before consulting Guijon, of another solution to the same problem, one that appears thoroughly implausible.[254] In this context one last point deserves attention: Lalamant recognizes correctly that three weeks exceed twenty full days by four and a half hours, but he strangely comments on **S7** as if the 30 minute difference between 4 ½ h and the text's '⅙ day' did not exist: *Nam trium septimanarum numerus (de medio circuitu loquor) viginti dies excedit sexta diei parte, id est, horis 4 et dimidia, quae horae iam ad vicesimumprimum diem pertinent. Triplica numerum dierum septimanae medii circuitus, quem iam saepe esse dixi dierum 6 horarum 17 cum dimidia; videbis summam assurgere ad dies 20 horas 4 cum dimidia.*[255] No word here

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[249] Cf. ibid., 508: *Ego priusquam virum illum bonum consulerem, hunc locum accurate discusseram et sic emendandum iudicabam*: λαμβανέσθω γὰρ τῶν κϛʹ καὶ ἡμίσεως καὶ τρίτου καὶ δωδεκάτου τῆς ἡμέρας τὸ τέταρτον μέρος, ἔσται δὲ χρόνος ἡμερῶν ϛʹ καὶ ἡμίσεως καὶ ἔτι ἕκτου (!) μέρους μιᾶς ἡμέρας καὶ προσέτι μορίων ἄλλων ἐπικειμένων μικροτέρων, ἅπερ ἐστὶ γʹ [= κόππα] πρῶτα ἤτοι δεύτερα ευʹ ἢ σὺν τοῖς ζʹ πρῶτα [read: πρώτοις] δεύτερα ͵απʹ [= 1,080; read: ͵αωʹ = 1,800] τὰ τρία [read: ἤτοι τρίτα] ͵ιηʹ [= 10,008, but he clearly means ͵ιη = 18,000; read: ρηʹ = 108,000], *ut sic Latine dicatur: 'Sumatur enim ex 26 diebus et dimidio, ex tertia item et duodecima diei partibus, pars quarta. Erit vere id tempus dierum sex et dimidij, partis item diei sextae cum adiectis aliquot aliis particulis minoribus, utpote primis seu minuti[i]s 90, seu secundis 5,400, vel cum sexaginta minutis secundis 1,800 vel tertiis 1<0>8,000.'* He then explains, at length, his underlying reasoning and his different mathematical expressions of 90 minutes, reemphasizing his change of the transmitted unit fraction ⅕ to ⅙.

[250] Cf. ibid., 508–509: *Adducor equidem ut credam exactam hanc culculi* (read *calculi*) *discussionem a viro quopiam docto et ad iustum calculum haec revocanti margini primum fuisse adscriptam, deinde a librariis e margine in contextum insertam, ac tum demum excussorum (ut fieri solet) ignorantia tam miris, ut videre est, modis depravatam, ut iam non mihi minus in his erratis emendandis elaborandum fuerit ac illi, qui quondam Augiae stabulum repurgavit.*

[251] Cf. ibid., 508: *ubi paulo post legitur* καὶ ἔτι πρὸς τὸν ἀκριβῆ λόγον ξʹ τε καὶ ρκʹ καὶ σμʹ, *corrigit ille* (that is, Guijon) καὶ ἔτι πρὸς τὸν ἀκριβῆ λόγον ξʹ τε καὶ σμʹ.

[252] Cf. n. 67 above.

[253] Cf. n. 249 above.

[254] Cf. Lalamant p. 509: *Quod autem ad haec verba attinet:* καὶ ἔτι πρὸς τὸν ἀκριβῆ λόγον ξʹ τε καὶ σμʹ, *credidi illa esse adiectitia, quae si tamen retinere placeret, sic emendari posse iudicabam:* καὶ ἔτι πρὸς τὸν ἀκριβῆ λόγον τοῖς λʹ πρῶτα ἤτοι ξʹ δεύτερα ἤτοι ͵γχʹ τὰ τριά [sic, read: τοῖς λʹ πρώτοις ἤτοι ͵αωʹ δευτέροις ἤτοι ͵ρη τρίτοις, i.e., 30 minutes or 1,800 seconds or 108,000 thirds], *id est: 'Et exactiori adhuc calculo minutis seu primis 30 seu 1,800 secundis aut etiam tertiis 3,600* [read: *108,000*]. *Nam horum singula dimidiam horam pariunt.'* Lalamant committed two mathematical lapsus: He confused the results of his multiplications of 30, namely 1,800 and 108,000, with the respective multipliers 60 and 3,600 (in the Greek; in his Latin only 3,600 is wrong), and he corrupted the number 108,000 to 18,000 (hence ͵ιηʹ in his Greek; cf. n. 249).

[255] Lalamant pp. 509–510. He had emphasized already on p. 505 (twice) that three medical weeks equal four and a half hours.

about Guijon, or about the addition of $\frac{1}{48}$ day in **t1**, from which Lalamant had quoted earlier. With this baffling explanation his commentary on Galen's computation ends.

Soon after Lalamant's exchange with Guijon, an Italian scholar wrote a letter which arguably constitutes the best discussion of Galen's computation to that date. It is from the pen of Giovanni Manardi (1462–1536), who had enjoyed a thorough philological education at Ferrara and knew not only Greek and Latin but also Arabic well.[256] At an earlier date, Manardi had written a commentary on Galen's *Ars parva* (Rome 1525, Basel 1536) with the explicit purpose of diverting readers (especially young readers) from the 'barbaric' authors and assisting them in reading the Greek originals.[257] Since Manardi rejected all forms of astrology, he was a sharp critic of Galen's theory of critical days.[258] His letter in question was sent to Pier Nicola Castellano[259] on 29 September 1532.[260] It is the fifth epistle in book XV of Manardi's *Epistolae medicinales* (ed. Basil. 1549, 406–421), a collection of 103 letters highly esteemed by humanists such as Erasmus of Rotterdam and Tommaso Campanella.[261] The long letter 15.5 discusses the falsehood of judicial astrology, the well-balanced climate of the sixth geographical *clima*, sugar, and a particular kind of Indian wood.[262] From p. 409.14 on, Manardi focuses on Galen's *On Critical Days* (*De iudicialibus diebus*), and from p. 411.18 on more specifically on the medical month. Manardi provides a mathematically correct, crystal clear interpretation of the various unit fractions, which he also systematically converts into hours and minutes. Like Guijon before him, he points out (p. 412.35) the need to athetize p. 933.6 K. ἑκατοστὰ εἰκοστὰ (or ρκ′, as various manuscripts and editions have it). In addition, Manardi recognizes the need to supply a fraction after p. 933.9 K. ἕκτου, but his choice of $\frac{1}{60} + \frac{1}{240}$, though mathematically correct, is inferior to the equivalent fraction $\frac{1}{48}$ as transmitted in Ḥunayn's Arabic translation (**h**) and its Latin version (**t1**), which, however, Manardi was unaware of because he used a corrupt reprint of **t1** (ed. 1528).

It is worth quoting pp. 412.3–413.16 of Manardi's letter in full:

[...] *alium* [scil. *mensem*] *ex duorum* [scil. *mensium*] *compositione commentus est* [scil. *Galenus*], *qui minor esset quam is, qui circuitum signiferi de*[412.5]*scribit, maior eo, qui illuminationem metitur. Coniuncto igitur utriusque tempore simul atque eo numero, qui ex commixtione nascebatur, in duas aequas partes resecto, id quod remanebit, quasi in eo utriusque vires complecterentur, idoneum putavit, in quod crisimi dies referrentur. Restabant autem, ut supputanti manifestum fiet, dies integri sex et viginti et praeterea alterius*

[256] Palumbo 2007, 421; cf. Pennuto 2008, 87: "He was praised by many of his contemporaries for being a follower of Erasmus's philological method." Manardi's teacher had been Nicolo Leoniceno (1428–1524), who had been a driving force of the movement for the new translation of Galen's writings before the publication of the Aldine edition (1525); cf. Fortuna 2006, 459–462; Pellegrini 2013; Hirai 2014, 269–270.

[257] Palumbo 2007, 421.

[258] Cf. Pennuto 2008, 87–94.

[259] Cf. Colombero 2006.

[260] For the place of this letter in Manardi's anti-astrological thinking and writings, cf. Zambelli 1965, 234–235 (she also refers to its reception by L. G. Giraldi in *De annis et mensibus*, Basel 1541, 178–179).

[261] Palumbo 2007, 421.

[262] The Latin title is (ed. Basil. 1549, 406.30–33): *Ioannes Manardus P. Nicolao Castellano de falsitate iudiciariae astrologiae, de bona sexti climatis temperatura, de saccharo et ligno Indico Gyarcha.*

diei media et tertia et duodecima pars, quae par[412.10]*tes si ad nostrum horologium reducantur, aequinoctiales horas duas supra viginti constituunt. Media enim diei pars horas duodecim capit, tertia octo, duodecima duas, quae simul iunctae duas et viginti complent, ut sit hic factitius mensis duodecima diei parte, hoc est, horis duabus minor vigintiseptem diebus. Quod tempus si in quatuor secuerimus partes, cuilibet septimanae dabimus dies sex, addita alterius diei media* [412.15] *sexta uigesimaquarta et quadragesimaoctaua parte, quae utique partes simul iunctae horas quot diximus complebunt. Medietas quippe diei horarum est duodecim, sexta pars quatuor, vigesimaquarta unius, quadragesimaoctava semihora perficitur. Iunctae autem hae simul, horas quas diximus septemdecim videlicet semis efficiunt. Et quoniam haec eadem septimae diei fragmenta alio modo signans Galenus dicit esse mediam* [412.20] *et quintam partem adiecta sexagesima, centesima et vigesima, et ducentesima quadragesima diei particulis, videamus quomodo haec in idem recidant cum prioribus, hoc est, quomodo horas septemdecim semis conficiant. Media pars diei est horarum duodecim, quinta pars est horarum quatuor et quatuor ex quinque partibus alterius horae, quae clarioris supputationis gratia sunt iuxta recentiores minuta horae octo et qua*[412.25]*draginta, dando unicuique horae sexaginta talia minuta. Sexagesima porro diei pars quatuor et viginti ex eisdem minutis comprehendit, centesima et vigesima duodecim, ducentesima et quadragesima sex, quae omnia minuta simul iuncta numerum complent nonaginta, quae horam semis constituunt. Quare horis his omnibus simul iunctis redeunt horae priores, decem septem videlicet semis, quibus ad complendam septimam desunt* [412.30] *horae sex semis, quas Galenus dicit esse quartam diei partem additis sexagesima et ducentesima quadragesima diei unius portiunculis, licet in impresso graeco codice*[263] *mendose addatur portio centesima vigesima, tunc enim redundarent minuta horae duodecim. Nam sexagesima, ut diximus, diei portio quatuor et viginti horae minuta constituit et ducentesima quadragesima sex, quae iuncta viginti quatuor triginta partes (medie*[412.35]*tatem videlicet horae) complent. Quare delendum ρκ in graeco codice, atque in hac parte barbara aeditio* [that is **t1**] *non solum novam interpretationem* [that is **t3**[264]]*, sed et graecam ipsam lectionem vincit. Alioqui neque solum in hac parte erratum est in nova interpretatione, sed et eo loco, ubi mensis a Galeno dearticulatur. Referendae enim illae tres portiones, scilicet medietas, tertia et duodecima, ad integram diem, non, ut interpres putasse vide*[412.40]*tur, tertia et duodecima pars medietatis intelligenda.*[265] *Nec velim vel te vel alium forte, qui haec legerit, arbitrari me detrahendi studio in haec divertisse. Tantum enim ab hoc* [413.1] *absum, ut summa laude et praeconio dignum eum interpretem ducam culpandosque potius, si qui culpandi sunt, librorum excusores, qui dum celeritati nimis student, castigationes recognitionesque plerunque praeuertunt et praecipitari aeditiones cogunt, quod de hac ipsa interpretatione interpres ipse queritur. In haec autem diverti, ut Galeni sen*[413.5]*sum, qui prae literae corruptela excerpi non poterat, aperirem. Galenus praeterea, ut ad id redeamus, unde discessimus, ostensurus vigesimam potius quam vigesimamprimam iudicatoriam esse, ait: 'Quod cum haec ita se habeant, ternae*

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[263] For this terminology, cf. n. 241 above.

[264] Apparently Manardi did not know that Niccolò da Reggio's translation (**t2**), which had never been printed, and Guijon (cf. n. 251 above) agreed with **t1** and himself in omitting the fraction $\frac{1}{120}$.

[265] Another legitimate criticism of **t3**; cf. n. 223 above.

septimanae viginti erunt dierum et praeterea alterius diei sextae partis.' In quo loco etiam menda est, tam in graeco quam in barbaro codice, maior tamen in graeco, in quo nihil praeter sextam illam [413.10] *partem legitur,* [266] *quum in barbara aeditione legatur 'sexagesimaquinta pars ex centesima quinquagesimaoctava.'* [267] *Restituendus ergo locus in hunc modum: 'His ita se habentibus tres dierum septimanae viginti dies et praeterea unius diei sextam et sexagesimam et ducentesimam quadragesimam partem continebunt.* [268] *Atque ad hunc modum exacte ratiocinantibus parum excellit viginti dies trium septimanarum numerus et* [413.15] *huius* [scil. *vicesimi diei*] *magis quam vigesimus primus* [sic, read: *vigesimi primi*] *est proprius', quanquam et hic quoque erratum est in graeco codice, pro vigesimo enim die legitur octavus.* [269]

Manardi's critical examination of Galen's iatromathematical positions continues to the end of this letter, but the computation of the medical week is no longer at issue, except for a general criticism: Since the actual synodic period varies, it is nonsensical to devise astrological doctrines based on precise figures for the respective lengths of the month, because these figures will only on rare occasions match the astronomical reality (pp. 413–414). Regrettably, Manardi's discussion of Galen's computation was unknown to Chartier (1679) and Kühn (1825).

Girolamo Cardano (1501–1576)[270] discusses the theory of critical days in his commentary to the Hippocratic *Prognostic* (Basel 1568),[271] where he criticizes Galen sharply[272] but does not specifically address Galen's invention of a medical month and week or even details of its computation. Cardano thus missed the opportunity of examining it from his perspective as a renowned mathematician.[273]

The last early modern scholar who fruitfully dealt with Galen's computation was, to my knowledge, Antonio Magini (1555–1617).[274] He was successor to E. Danti as professor of mathematics at Bologna. His professorship had a traditional orientation toward astronomy and astrology.[275] His *Ephemerides motuum coelestium* (Venice 1582) revealed him as "un calcolatore di primo livello."[276] His *De astrologica ratione ac usu dierum criticorum seu decretoriorum ac praeterea de cognoscendis et medendis morbis ex*

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[266] This is correct; cf. n. 69 above.

[267] This mistake is presumably in the last reprint of **t1** (Venice 1528), which I was unable to check.

[268] This is mathematically correct, but inferior to $\frac{1}{48}$ (**h** and **t1**); cf. n. 69 above and p. 256, Question 5, below.

[269] Cf. n. 72 above, ed. **b**.

[270] Cf. Gliozzi 1976.

[271] Ed. 1568, col. 462–469 (= *Opera omnia* vol. 8, Lyon 1668 [repr. 1966], 741–747). He had been teaching this Hippocratic work at Bologna in 1567–1568 (Siraisi 1997, 123).

[272] Cf. Siraisi 1997, 140–141, and especially 296, n. 124.

[273] On Cardano's mathematical prowess, cf. Gliozzi 1976, 758.

[274] See especially Baldini 2006, but also Thorndike 1923–1958, VI.694 (index s.v. 'Magini'), and Pizzamiglio 2004, 97–103, 134–135, 138–143, 174–177.

[275] Baldini 2006, 413. One of Magini's professional duties was the publication of annual astrological prognostications. While Magini complied with this duty mostly under a pseudonym, he published other astrological works openly, such as his *Introduzioni astrologiche* (Venice 1582) and the *Discorso astrologico della mutationi de' tempi, et de i più notabili accidenti sopra l'anno 1607*, Bologna 1607. It is known that he made part of his living through astrological consultations, and some of the horoscopes that he cast are preserved, yet the major part of his library is lost. I owe these details to Baldini 2006, 413–414 and 416–417.

[276] Ibid., 413.

corporum coelestium cognitione (Venice 1607, repr. Frankfurt 1608) is a synthesis of medical astrology.[277] The first part of this compendium is a commentary to the third book *On Critical Days*.[278] It is organized as a series of quotations from (I suppose) one of the recent Venetian editions of Winter's revised Latin translation,[279] with Magini's sometimes short, sometimes extensive comments interspersed in a smaller font. The section devoted to Galen's computation of medical weeks runs from f. 31v to f. 35v of the *editio princeps* (1607), and the clarity and completeness of Magini's explanations show that he is writing for readers who are not mathematical experts. He admits openly that he has no idea why Galen resorted to "so many superfluous unit fractions," thereby making the whole computation unnecessarily prolix: *Miror autem Galenum tanta verborum prolixitate in re parvi momenti abuti, poterat enim laconice eandem septimanam definire esse dierum sex, horarum 17 et dimidiae; cum quo numero tot supervacaneae fractiones exacte coincidunt* (f. 32r). This remark is valuable for us because Magini knew earlier works on critical days very well.[280] Therefore it is likely that none of those works' authors had understood Galen's motive either. Magini provides a systematic, mathematically impeccable explanation of the equivalents of Galen's fractions in hours and minutes. For example, he shows very clearly, with regard to the length of one medical week, that $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{60}$ $\frac{1}{120}$ $\frac{1}{240}$ day (**S4**) equals 12 h + 4 h 48 m + 24 m + 12 m + 6 m. He is also aware of Manardi's correction of the Greek text of **S6**: *ut bene Manardus admonet, delendum est in Graeco codice ρκ, in Latino 'centesimam vigesimam'* (f. 32v).[281] With regard to **S7**, he supplies the missing fraction $\frac{1}{48}$: *Perperam ergo in textu Galeni dicitur tres septimanas continere dies 20 et sextam unius diei partem; quibus si addideris quadragesimam octavam, errorem sustuleris* (ibid.). He states that each day of Galen's medical month will comprise only 23 hours and about 4 minutes: *Dies autem medicinalis Galeni est horarum tantummodo 23 et minutorum 4 fere, sive horarum 23 cum decimaquinta horae parte*. While this fraction ($\frac{1}{15}$ h) is the exact equivalent of the aforementioned approximate value of four minutes, Magini could have done better by expressing the true fraction—as Niccolò da Reggio had done before him[282]—as $\frac{1}{14}$ h. The reason why he did not may be that $\frac{1}{14}$ h defies conversion into full minutes and might therefore have been difficult to understand for his readers. Be this as it may, Magini must have known that the fraction in question was actually $\frac{1}{14}$, because he concludes his commentary on the medical month with a table (see Figure 1 below) whose rounded data imply that the exact length of the medical day is 23 $\frac{1}{14}$ h. This table specifies after how many natural days, hours and minutes each single medical day of each of the four medical weeks ends. It is, to the best of my knowledge, unique among all early mod-

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[277] Ibid., 415.

[278] Ed. 1607, ff. 1r–38v, = ed. 1608, 1–62. I shall, in the following, quote from the first edition.

[279] If so, Magini must have adopted the important marginal note quoted in n. 230 above and rejected the erroneous marginal note quoted in n. 233 above yet overlooked the necessary addendum contained in the marginal note quoted in n. 234 above (that is, the fraction $\frac{1}{48}$ is missing from his translation but present in his commentary).

[280] He provides a useful list of printed works on the critical days ibid. (ed. 1607), f. c<1>r–v. I did not personally see all these works.

[281] See Manardi p. 412.35, quoted on p. 232 above.

[282] Cf. the Latin quotation in n. 176 above and the mathematical comment in n. 434 below.

ern publications on Galen's medical month. Magini does not say why he lists the entire series of all 28 medical days within a medical month. In all likelihood he deemed it useful to practicing physicians. All that one needed to make practical use of this table was the precise date of the last new moon before a patient's *decubitus*, which could easily be obtained from printed ephemerides and almanacs. The physician would then be able to determine critical days by adding the respective number of natural days, hours, and minutes to the new moon's date.

Prima hebdomada (D. H. M.)			Tertia hebdomada (D. H. M.)		
0.	23.	4	14.	10.	4
1.	22.	8	15.	9.	8
2.	21.	13	16.	8.	13
3.	20.	17	17.	7.	17
4.	19.	21	18.	6.	21
5.	18.	26	19.	5.	26
6.	17.	30	20.	4.	30

Secunda hebdomada (D. H. M.)			Quarta hebdomada (D. H. M.)		
7.	16.	34	21.	3.	34
8.	15.	39	22.	2.	39
9.	14.	43	23.	1.	43
10.	13.	47	24.	0.	47
11.	12.	51	24.	23.	51
12.	11.	56	25.	22.	56
13.	11.	0	26.	22.	0

Fig. 1: End points of all medical days of one medical month,
reckoned from new moon in natural days,
hours, and minutes (Magini 1607, f. 33r)

When René Chartier (1572–1654) embarked on the gigantic project of editing the first bilingual (Greco-Latin) edition of the vast Galenic corpus in unprecedented completeness (167 works, five of which were previously unpublished, plus fragments),[283] he carefully examined the earlier editions. Among Latin editions, he considered the Venetian *Giuntines* the best, and among these the first four editions (1541–1565) prepared by Agostino Gadaldini.[284] Moreover, Chartier searched Greek manuscripts widely, especially in what is now the Parisian *Bibliothèque Nationale de France*, and made use of his collations for the constitution of the original Greek texts (and, to a lesser extent, for the Latin translations).[285] Regrettably, his Greek and Latin versions of *On Critical Days*[286] did not benefit from this meritorious effort, at least not in the passage we are interested in: The sometimes important variant readings of MS Paris, BnF gr. 2272 (**P**)[287] left no trace in his edition, except (may-

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[283] Fortuna 2012b, 315.
[284] Fortuna 2012b, 309 and 311.
[285] Fortuna 2012b, 311.
[286] Chartier 1679, 450–507.
[287] Cf. notes 47–74 above.

be) one minor detail.[288] Even more regrettably, Gadaldini's marginal notes were equally disregarded by Chartier. Since his edition of Galen's computation is basically a reprint of the heavily corrupt Greek text of the 16th-century editions (more precisely, of the *Basileensis* of 1538, which, however, depends almost entirely on the Aldine edition of 1525)[289] accompanied by the Latin translation of Winter (**t3**) in the revised yet still mathematically muddled version of Gadaldini's *Giuntine*,[290] which often does not even match Chartier's Greek text, the importance of this 1679 edition for the present survey is virtually null. The same is true of its reprint by Karl Gottlob Kühn (1825, 769–941).[291] One gets the impression that in this late period, when the general attitude towards astrology grew increasingly sceptical, if not utterly hostile, these two otherwise meritorious editors were not willing to spend their time on an astrologically motivated, mathematically challenging, and medically useless passage.

V. Rhetorical Examination

Galen employs a rhetoric of accuracy that culminates in the concluding sentence of his computation (**S7**) with the rare compound ἀκριβολογουμένοις (p. 933.9–10 K.). While the entire corpus of Greek literature before Galen features 57 instances of the various nominal and verbal compounds derived from ἀκριβολογ-, Galen himself employs such compounds

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[288] Cf. note 53 above. Pietrobelli 2012, 105–109, provides a list of 22 Greek manuscripts of Galen of the BnF consulted by Chartier. No. 2272 is not among them. However, Pietrobelli emphasizes (p. 104) that his list is by no means complete.

[289] Cf. Perilli 2005, 428, and especially Gundert 2006, 98. In our passage, Chartier's dependence on the edition of Basel is confirmed by one instance in the text of Galen's computation where **p** agrees with **b** against **v** (cf. n. 62 above). On Chartier's generally low appreciation of the *Aldine*, cf. Fortuna 2012b, 310.

[290] Chartier made three changes: Two of them are substitutions of single words by synonymous words (see notes 221 and 229 above), while the third change is a failed attempt at correcting a textual corruption (see note 231 above). Chartier does not explicitly mention the source of his Latin translation. In his *Concisae notae ac variae lectiones* he writes (VIII.921), with regard to *On Crises* (sic, this work precedes *On Critical Days* in his edition): *Libros de crisibus ac de diebus iudiciariis Hippocratis sequuntur tres Galeni libri De crisibus ac totidem de diebus iudicatoriis* [...]. After a brief summary of the content of Galen's *On Crises* the entry ends thus: *Interpretationem Nicolai Leoniceni ab Augustino* [i.e., Agostino Gadaldini] *emendatam emendatiorem dedimus. Cum manuscriptis Regiis collata sunt haec opera*. In his next entry Chartier gives a much shorter summary of *On Critical Days*, which is followed by brief notes on eight manuscript variants regarding this work in "M.R." (*manuscripti Regii*). None of them concerns the third book. The translator of *On Critical Days* is not mentioned. Chartier's preface to the first volume contains a general statement to the effect that he preferred reprinting good existing translations, especially those made by Janus Cornarius, Johannes Winter, Thomas Linacre and Giovanni Battista Rasario, instead of making new translations (which he did in some cases). Cf. Fortuna 2012b, 313–314, who quotes the respective passage from the preface, and ibid., 321, where she correctly identifies Chartier's Latin translation of *On Critical Days* as that of Johannes Winter of Andernach in Gadaldini's revised version. On the whole, Chartier reproduced 27 of over 40 Galenic translations made by Winter (he took seven of them from the *Giuntines*) and expressed great admiration for Winter, as shown by Fortuna 2012b, 316, who adds that one reason for Chartier's preference for Winter was probably his desire to emphasize the French or more specifically the Parisian tradition of Galenic physicians.

[291] On each page, Kühn refers to the respective pages of both Chartier's edition and the edition Basel 1538. Kühn's Greek text of Galen's computation differs from Chartier's only in a slightly modified punctuation (without consequences for the interpretation), in the typographically motivated replacement of the number letter ϛʹ (6) by στʹ, and (as far as our passage is concerned), in the addition of one typo (p. 932.16 K. τέταρτου). In his long preface to all twenty volumes (vol. I, Leipzig 1821, VII–CCLXV), Kühn does not claim to have translated our text in question himself. He reviews the (at his time) available editions, manuscripts, translations, and commentaries ibid., CVII–CVIII.

25 times, that is, more frequently than any Greek author before him.[292] His more general use of ἀκριβ- amounts to 2,923 hits, that of all previous authors combined 2,409. Hence, it is not surprising that our passage, in which computational accuracy forms the basis of successful argumentation, contains three forms of ἀκριβ- (pp. 932.14, 933.6, 933.8 K.), and that there are four more in the immediate context (pp. 929.16, 932.2, 933.15, 934.1 K.).[293]

It seems to have gone unnoticed by previous scholars that Galen is here alluding to a passage in Plato's *Republic* on medicine, mathematics, and accuracy (!). Thrasymachus asks Socrates:

> Do you describe someone who makes mistakes about his patients as a doctor by virtue of the fact that he makes mistakes? Or do you describe someone who makes mistakes in his calculations as a mathematician, at precisely the time when he is making a mistake, and in virtue of the mistake that he is making? It's true that the expression is in our language: We say that a doctor or a mathematician or a teacher makes mistakes; but in fact, in my opinion, to the extent that each of them is what we call him, he never makes mistakes. And the consequence of this is that, strictly speaking—and you're the stickler for verbal precision—no professional makes mistakes (ὥστε κατὰ τὸν ἀκριβῆ λόγον, ἐπειδὴ καὶ σὺ ἀκριβολογῇ, οὐδεὶς τῶν δημιουργῶν ἁμαρτάνει): A mistake is due to a failure of knowledge, and for as long as that lasts he is not a professional.[294]

It is not only likely that this well-known passage, with its relevance for the judgment of medical mistakes,[295] was on Galen's mind, but certain: in *De placitis Hippocratis et Platonis* 9.5.1–8[296] (written between 169 and 176 CE)[297] Galen refers to an immediately following passage in the first book of Plato's *State* (341C–342E), and especially to 343B (9.5.1) where Thrasymachus scornfully advances an argument based on shepherds and cowherds which is refuted by Socrates in 345B–D. In our allusion to Plat. *resp*. 340D–E, Galen even imitates the subordination with ὥστε.[298] Hence, the Aldine's and Kühn's τρόπον (p. 933.9 K.) may have to be replaced with the variant reading λόγον of **BP** (supported by Niccolò da Reggio's translation *rationem*). Note, however, that the imitation of Plato's κατὰ τὸν ἀκριβῆ λόγον may occur instead in Galen's preceding words πρὸς τὸν ἀκριβῆ λόγον (p.

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[292] Source: TLG. This assertion is true in absolute terms, but not in relative terms because the Aristotelian corpus, which is significantly smaller than the gigantic Galenic corpus, features twelve attestations. Among Galen's other predecessors, one finds five or fewer attestations in each individual author.

[293] While the present focus is on mathematical exactness denoted by ἀκρίβεια, the semantic range of this word is broader, including philosophical, rhetorical, sociological, historiographical, medical, and other notions (cf. Kurz 1970).

[294] Plat. *resp*. 340D–E, trans. Waterfield 1993, 21–22.

[295] On another Platonic allusion in *On Critical Days*, which equally has to do with medical mistakes, cf. Cooper 2011a, 73–74 (with reference to *Apology* 21E–22E). The present allusion to the *Republic* escaped Cooper's notice.

[296] De Lacy 1978–1984, II.564–565, with English translation. On Galen's high esteem for Plato, whom he considered second only to Hippocrates, see De Lacy 1972, 27.

[297] De Lacy 1978–1984, I.46–47.

[298] Moreover, Plat. *resp*. 340E is the only parallel for ἀκριβολογ- in combination with κατὰ τοῦτον τὸν τρόπον / λόγον (see the next note) in all of ancient Greek literature (TLG).

933.6 K.), in which case he may have preferred κατὰ τοῦτον τὸν τρόπον in p. 933.9 K. for the sake of variation.[299]

This allusion to the Platonic passage suggests a parallel between Galen and wise Socrates refuting the sophist Thrasymachus, as Galen found himself for years in conflict with the so-called iatrosophists who enjoyed great popularity among educated Romans.[300] Especially in *On Critical Days*, polemical remarks against 'sophists' are frequent,[301] and the two that one finds in the third book are perfectly in line with the allusion to Socrates prevailing over Thrasymachus (p. 933.9–10 K.): In ch. 3.6 Galen points out that the astrological influence of square and opposite lunar aspects to the sun is admitted by experts in the astral sciences and obvious to anyone willing to make his own observations; whoever is willing neither to believe those experts nor to verify the truth by means of personal observations must be "one of those sophists who are now omnipresent and fashionable in their insolence."[302] The other reference is in the last chapter (3.13) where Galen claims to have spoken the truth and demands that the sophists' nonsense "must now have an end."[303]

There is also another rhetorical elaboration. Our computation is framed by two remarks that roughly correspond to each other in tone and function. In the first instance (a), Galen modestly admits that only God knows the exact nature of the mixing of both months' effects;[304] in the latter (b) he says that whoever finds his pursuit for accuracy too difficult to follow is, of course, not obliged to learn those things but may rather wish to read only the first book or at most both the first and the second; he is perfectly aware that the third book addresses a very limited readership, and he did not even write it of his own accord but, as the gods know and witness, because certain unnamed colleagues (ἑταῖροι, literally 'comrades') had vehemently urged him to write these things for them.[305] Thus, he shows respect for the gods by distancing himself from hybris and for his readers by apologizing for the difficulty of his writing. Both remarks are stylistically elevated above the dry, technical

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[299] Whether Ḥunayn ibn Isḥāq found τρόπον or λόγον in his manuscript(s), is hard to tell. Cooper 2011a, 374, translates his Arabic rendering with "in this manner." In his entire corpus, Galen uses the expression κατὰ τοῦτον τὸν τρόπον 23 times, and κατὰ τοῦτον τὸν λόγον 42 times (in both cases partly with intermediate particles).

[300] Cf. Schlange-Schöningen 2003, 165; cf. ibid., 149: "Galen mußte sich von der Gruppe der Sophisten abheben, die Vorträge über theoretische Grundlagen der Medizin oder über Begriffsfragen hielten, ohne über praktische Fähigkeiten zu verfügen" (with reference to Debru 1995, 76–77).

[301] The whole treatise contains 21 attestations of σοφιστ- (TLG).

[302] P. 913.9–15 K.: καὶ τοῦτο πρὸς τῷ τοῖς ἀστρονόμοις ὁμολογεῖσθαι πάρεστιν, εἰ βούλει, καί σοι παραφυλάξασθαι. εἰ δὲ μήτ᾽ αὐτὸς παρατηρεῖν ἐθέλεις τὰ τοιαῦτα μήτε τοῖς τηρήσασι πιστεύεις, τῶν νῦν ἐπιπολαζόντων τις εἶ σοφιστῶν· οἳ λόγῳ κατασκευάζειν ἡμᾶς ἀξιοῦσι τὰ σαφῶς φαινόμενα, δέον αὐτὸ τοὐναντίον ἐκ τῶν ἐναργῶς φαινομένων ὁρμωμένους ὑπὲρ τῶν ἀδήλων συλλογίζεσθαι. On the ambivalent meaning of ἀστρονόμοις, which denotes both astronomers and astrologers in the modern sense, cf. the seminal study by Hübner 1989.

[303] P. 939.6–8 K.: τὸ μὲν ἀληθὲς ἅμα καὶ χρήσιμον εἴρηταί μοι, τὸ δὲ τοῦ σοφιστικοῦ λήρου περαινέσθω, μετὰ ταῦτα δεικνύντων ἡμῶν κἀνταῦθα, πόσον ἀγνοοῦσι τοῦ προσήκοντος οἱ πλεῖστοι τῶν ἰατρῶν.

[304] P. 932.2–5 K.: τὸ μὲν οὖν ἀκριβὲς ἢ τῆς μίξεως ἢ τῆς ἀφαιρέσεως ὁ θεὸς οἶδεν· ὅσον δὲ ἀνθρωπίνῳ στοχασμῷ συμβάλλειν ἐγχωρεῖ, πιθανώτερόν ἐστι μέσον ἀμφοῖν ποιῆσαι τῶν περιόδων τὸν ἐξ ἀμφοῖν κοινόν.

[305] P. 934.1–9 K.: Ὅστις δὲ τῇ τῶν λόγων ἀκριβείᾳ δυσχεραίνει, τοῦτον οὐδεὶς ἀναγκάζει μανθάνειν αὐτούς, ἀλλ᾽ ἀρκεῖ τὸ πρῶτον αὐτῷ τῆσδε τῆς πραγματείας ὑπόμνημα. εἰ δὲ μὴ πάνυ τις ἀργὸς εἴη, προστιθέτω μὲν καὶ τὸ δεύτερον, ἀπεχέσθω δὲ τοῦ τρίτου. ἡμεῖς γὰρ ταῦτα καὶ αὐτοὶ γινώσκομεν, ὀλίγοις παντελῶς γεγραφότες, οὐδὲ τούτοις γε ἑκόντες. ἀλλ᾽ ἴσασι γὰρ θεοὶ καὶ καλείσθωσαν μάρτυρες, ὅτι πολλὰ βιασθέντες ὑπό τινων ἑταίρων ἠναγκάσθημεν αὐτοῖς γράψαι ταῦτα.

language of the enclosed computation and the larger context through their references to the gods,[306] and seem to serve the purpose of a *captatio benevolentiae*. However, Galen would not be Galen, a notoriously ambitious, complacent, and quarrelsome individual,[307] if he did not include a disparaging barb: After having stated in a conciliatory tone that a reading of his first book is enough, he literally phrases the remainder of (b) thus: "if someone is not entirely lethargic [!], he may wish to read the second, too, but not the third".[308]

In sum, Galen is clearly aware of the challenging nature of his computation while he proudly emphasizes its accuracy. On this point, the results of my rhetorical examination appear to square with those reached by Singer, an expert on Galen's philosophy, who based his work on an examination of various passages from other (especially psychological) Galenic writings. Singer's final paragraph, where he points out two reasons why it suited Galen to appear «ideologically» (rather than strictly philosophically) as a Platonist, is worth quoting:[309]

> One is the standard of rigour, the importance of mathematical and geometrical discipline for the aspiring scientist or philosopher, which is a feature of Plato and with which Galen obviously feels sympathy, not just, presumably, because he himself had had some amount of this kind of training,[310] but also because this gave him a position of authority from which to despise his opponents who had no such rigorous approach. Another reason, I believe, lies in the Platonic ideas of a small elite as the only possible possessors of knowledge, and the close connection between knowledge of the highest kind and virtue. Both these features of Platonic rhetoric, in Galen's hands, serve to fuel his polemics against rivals: Presenting himself as a disciple of Plato, he sets up a moral divide between himself and his opponents, so that in attacking them he seems to be undermining their characters as well as their opinions;[311] the strength of Galen's well-known rhetorical attacks would not seem equally justified in the context of a mere disagreement on facts. Frequently he talks in terms of moral outrage at the tricks to which his opponents are capable of descending, accusing them not only of fallacy but also of ἀναισχυντία or

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[306] There is only one other occurrence of the word θεός in this work, ch. 1.3 p. 778.6 K. (ὦ πρὸς θεῶν, 'in the name of the gods').

[307] Cf. Kollesch 1965. On Rome as a highly competitive environment for Galen during his first years there (that is, from 162 CE onwards), cf. Schlange-Schöningen 2003, 148. Galen himself used to blame others for what he perceived as a mostly difficult professional relationship, for example, at the end of his introduction to *On Critical Days*, where he "sarcastically emphasizes the stupidity of his rival physicians by suggesting that not even a god would or could cure them of their ignorance and desire for conflict" (Cooper 2012, 23–24). The original reads thus (ch. 1.1 p. 773.16–18 K.): ἀλλὰ τὴν μὲν τούτων ἀμαθίαν τε καὶ φιλονεικίαν οὐδ' ἂν αὐτὸς ὁ Ζεὺς ἐξιάσαιτο, καί μοι καὶ ταῦτα πλείω τοῦ δέοντος εἴρηται πρὸς αὐτούς.

[308] I disagree with Sudhoff 1929, 8–9, who overlooks the rhetorical elaboration of this passage and thinks that Galen started feeling uneasy about his future readers' reaction to his astrological 'subtleties and heresies': "Übrigens wurde es dem Galenos selber bange wegen der Beurteilung dieser Spitzfindigkeiten und Ketzereien, und er erklärt am Schlusse des 12. Kapitels, wem's nicht gefalle, der möge das dritte Buch beiseite lassen; er selbst habe es nur ungern zu Papier gebracht auf dringenden Wunsch einiger Freunde. Doch das Verhängnis nahm seinen Lauf. Das dritte Buch von den kritischen Tagen hat der astrologischen Schwärmerei in der Heilkunde Tür und Tor geöffnet."

[309] Singer 1991, 55 (the guillemets in «ideologically» are his). It was on the very day of my article's submission to *SCIAMVS* that I happened to find Singer's article. He does not mention the work *On Critical Days*, nor does he speak of calculations or ἀκρίβεια. Apart from this addendum, my Section V remained unchanged.

[310] More on this below, pp. 242–243.

[311] Cf. p. 238 above on Galen's polemic against the iatrosophists.

ἀγροικία. He thus makes his polemic all the more devastating. He has right, he seems to say, as well as truth on his side: woe betide his rival.

VI. Mathematical Examination

In modern notation, the seven steps of Galen's calculation are as follows (^{d} = days):

S1 $26 \frac{1}{2}^d + 27 \frac{1}{3}^d = 53 \frac{1}{2} \frac{1}{3}^d$

S2 $53 \frac{1}{2} \frac{1}{3}^d \div 2 = 26 \frac{1}{2} \frac{1}{3} \frac{1}{12}^d$

that is, $(26 \frac{1}{2} + 27 \frac{1}{3})^d \div 2 = 26 \frac{1}{2} \frac{1}{3} \frac{1}{12}^d$

S3 $27^d - 26 \frac{1}{2} \frac{1}{3} \frac{1}{12}^d = \frac{1}{12}^d$

S4 $26 \frac{1}{2} \frac{1}{3} \frac{1}{12}^d \div 4 = 6 \frac{1}{2} \frac{1}{5} \frac{1}{60} \frac{1}{120} \frac{1}{240}^d$

S5 $26 \frac{1}{2} \frac{1}{3} \frac{1}{12}^d \div 4 = 6 \frac{1}{2} \frac{1}{6} \frac{1}{24} \frac{1}{48}^d$

S6 $7^d - 6 \frac{1}{2} \frac{1}{5} \frac{1}{60} \frac{1}{120} \frac{1}{240}^d = \frac{1}{4} \frac{1}{60} \frac{1}{240}^d$
          or: $7^d - 6 \frac{1}{2} \frac{1}{6} \frac{1}{24} \frac{1}{48}^d = \frac{1}{4} \frac{1}{60} \frac{1}{240}^d$

S7 $3 \times 6 \frac{1}{2} \frac{1}{5} \frac{1}{60} \frac{1}{120} \frac{1}{240}^d = 20 \frac{1}{6} <\frac{1}{48}>^d$

It is now time to ask several questions which previous scholars have either not been able to answer (2, 3, 8, 9) or not asked at all (1, 4, 5, 6, 7, 10):[312]

1. Why did Galen modify the results of **S1**–**S2** and **S3** with μάλιστα ('about'), even though they are mathematically exact?
2. Why did Galen calculate with days and fractions of days instead of days and hours?[313]
3. Why did he express all fractions as sums of unit fractions instead of single common fractions?
4. How did he calculate steps **S1** to **S6**?
5. Which missing unit fraction(s) ought one to expect in the result of **S7**?
6. How can its/their omission in the manuscripts be explained?
7. Why did Galen bother to calculate the seemingly superfluous steps **S3** and **S6**?
8. Why did he provide an alternative sum of unit fractions (**S5**), why exactly that one, and not others?

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[312] I thank Nathan Sidoli, the chief-editor of *SCIAMVS*, for precious advice on this section (recent personal communications). Any remaining mistakes are mine.
[313] Cf. Magini above, p. 234.

9. What is the exact meaning of the final words καὶ ταύτης ἔσται πολὺ μᾶλλον ἢ τῆς εἰκοστῆς πρώτης οἰκεῖος?

10. Was Galen's computational precision necessary to achieve his goal?

Question 1

The sum of 26 ½ and 27 ⅓ days divided by 2 equals *exactly* 26 ½ ⅓ $\frac{1}{12}$ (= 26 $\frac{11}{12}$) days (**S1**–**S2**), and this result falls short of 27 full days by *exactly* $\frac{1}{12}$ day (**S3**). So why twice μάλιστα (pp. 932.9 and 932.13 K.)?[314] The only plausible explanation that I can think of is that these modifiers are *caveats* of a conscientious writer who is aware that his two initial parameters (26 ½ and 27 ⅓), on which the entire, in itself accurate, calculation is based, are handy but not accurate, especially the former which results from the subtraction of a very rough value for the period of lunar invisibility (three days) from a value for the mean synodic month (29 ½ days) which is in itself not accurate. If so, we may expect that Galen signaled his awareness of the inaccuracy of the two initial parameters before our passage in question, and this is indeed what he did in earlier chapters of the same third book of *On Critical Days*:

On the period of lunar invisibility, cf. ch. 3.4 p. 907.1–7 K.: οὐ γὰρ ἴσον ἀεὶ κινεῖται (scil. ἡ σελήνη; I skip Galen's subsequent reference to three astronomical reasons), ὥστ' οὐκ ἀεὶ μὲν εἷς (!) ὁ χρόνος ἐστίν, ἐν ᾧ φαίνεται σαφῶς ἡ σελήνη. τοὐπίπαν (!) δὲ τρεῖς ἡμέρας τὰς περὶ σύνοδον [ἢ][315] ἀόρατος γίνεται τελέως, ἐν αἷς οὔπω τὰ παρ' ἡμῖν ἀλλοιοῦν ἱκανή.

On the length of the synodic month, cf. ibid. p. 907.14–16 K. (with reference to Hipparchus): ὅτι δ' ὁ μηνιαῖος χρόνος οὐ τελέως τριάκοντά ἐστιν ἡμερῶν, ἀλλ' ἥμισύ που (!) καὶ τούτῳ προσδεῖ μιᾶς ἡμέρας, Ἱππάρχῳ μὲν ἀποδέδεικται δι' ἑνὸς ὅλου βιβλίου. On the same value and the subtraction of the period of invisibility from it, cf. ch. 3.9 p. 931.10–14 K.: ἐὰν γὰρ ἀπὸ μὲν τοῦ μηνιαίου χρόνου τοῦ ἀκριβοῦς (!), ὃς ἀπολείπεσθαι τῶν τριάκονθ' ἡμερῶν ἡμίσει μάλιστα (!) μιᾶς ἡμέρας ἐλέγετο (this refers back to p. 907.14–16 K.), τὰς τρεῖς ἀφέλῃς ἡμέρας τὰς περὶ σύνοδον, αἱ λοιπαὶ δηλονότι τῶν ἑπτὰ καὶ εἴκοσι ἡμερῶν ἔσονται μείους ἥμισυ τῆς μιᾶς ἡμέρας.

On the length of the sidereal month, cf. ch. 3.5 p. 911.7–8 K.: [...] ἑπτὰ καὶ εἴκοσιν ἡμέραις [...] μετά τινος ἐπικειμένου μορίου, τρίτου μάλιστα (!) μιᾶς ἡμέρας.[316]

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[314] The Arabic translation by Ḥunayn ibn Isḥāq omits these modifiers (Cooper 2011a, 372–373); **t1** renders the first μάλιστα as *quasi* (on this surprising fact, see n. 125 above); **t2** renders both instances of μάλιστα as *maxime*, and so does **t3** (omitting the second one), while **t4** renders only the second one (as *potissimum*).

[315] ἢ *delevi* (*recte om. cod.* **B**).

[316] On this passage, too, cf. n. 125 above.

Question 2

The answer to this question lies in the difference between ancient seasonal hours and astronomical equal (that is, equinoctial) hours, which later became the modern hours. It will be convenient to treat the latter first. Equal hours became a widespread phenomenon in the late Middle Ages with the invention and use of mechanical clocks, which began quite early in Italy.[317] The first firmly attested public chiming clock was installed in Milan in 1336, thus providing the *terminus ante quem* for the invention of this technically sensational device.[318] The Milanese clock signaled each of the 24 equal hours with the corresponding number of strokes. Other cities soon followed, thus making, for the first time ever, large numbers of people familiar with equal hours. This technical development and the increasing presence of mechanical time measurement explains why all late medieval and early modern scholars in our survey above replaced Galen's sums of unit fractions of full days, which they considered unwieldy and odd, with their equivalents in equal hours and minutes.

In ancient everyday life under the Roman Empire, time measurement had been based on seasonal hours as indicated by sundials. Seasonal hours vary in length during the year because they are based on a subdivision of the time between sunrise and sunset by twelve. Hence, in the Mediterranean world one seasonal hour at the summer solstice would, depending on the geographical latitude, be roughly 40–60% longer than at the winter solstice, and every day a slight change in length between these solstitial extremes would occur. Only very few ancient people, namely those with an astronomical education, were familiar with the concept of equinoctial hours, which are, regardless of season and geographical latitude, of equal length and identical with our modern hours. Thus neither concept of hours was suitable for Galen's computation: the seasonal ones because their flexibility makes exact assertions independent of the day of the solar year impossible, while the equinoctial hours would not have been understood by his non-mathematically educated readers. Galen himself, however, would undoubtedly have been able to phrase his computation in equinoctial hours.

This answer to Question 2 finds ample confirmation in Galen's writings. He reports that his mathematical and astronomical education had been excellent.[319] When he was about fourteen years old, he started learning under various renowned teachers, and his father, whose "training lay chiefly in the sciences of geometry, arithmetic, architecture, and

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[317] The classical study of this topic is that of Bilfinger 1892. The authoritative study is now that of Dohrn-van Rossum 1992.

[318] Dohrn-van Rossum 1992, 106–110. Cf. ibid., 107, on the fact that already before 1336 there had been municipal tower clocks and municipal hour signals whose existence, however, is certain only for Italy, and we know nothing about how they indicated the hours. Moreover, it is important not to confuse the equal hours as chimed by the mechanical clock in Milan (1336) with the older monastic *horae canonicae* (prayer hours) which were fewer in number (namely seven from around sunrise to around sunset: *matutina, prima, tertia, sexta, nona, vespera, completorium*), did not coincide with equal hours (except for *sexta* = midday) but rather (if at all) with seasonal hours, and allowed for some flexibility. For details, cf. ibid., 35 and 37 (on the etymology of 'noon' from the *nona hora* which shifted over time), and 41. These canonical hours were publicly chimed from many municipal towers of the 13th century (ibid., 45).

[319] On Galen's mathematical education, cf. Sidoli 2015, 395–396.

astronomy," went along with him to hear them, after having, for the boy's sake, "made a close investigation of the lives and doctrines of all these men."[320] Mathematics had been cultivated for generations in Galen's family, including by his grandfather and great-grandfather. This distinguishes Galen from many other ancient physicians who were born into so-called 'medical families.'[321] At one time in his life, when the conflicting opinions of various philosophers nearly precipitated Galen into intellectual crisis, the certainty of mathematics with its incontestable proofs saved him.[322]

Bolstered by such an education, Galen refers to astronomical issues on many occasions, especially in his commentaries on the Hippocratic treatises *On Airs, Waters and Places* and *Epidemics* I and III.[323] Of particular interest is his treatise *On Seven-Month Children*,[324] where he quotes Hipparchus' figuring of a half year as 182 days, 15 hours, "and a little fraction of about one 24th of an hour," with the clarification that he is, of course, referring to the astronomically defined equal hours, which he explains as the average of the longest and shortest seasonal hours on a given geographical latitude.[325] Earlier in the same treatise, he had shown his awareness that the figure of 29 ½ days is but a handy approximation of the exact value of the mean synodic month, which he specifies with reference to Hipparchus as 29 ½ $\frac{1}{60}$ $\frac{1}{72}$ $\frac{1}{27,000}$ days "plus a very small part that is not worth mentioning here" (namely $\frac{1}{648,000}$ of a day, which is about one tenth of a second).[326] Galen's use of a

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[320] *De propriorum animi cuiuslibet affectuum dignotione et curatione* 8.4–5 p. 28.17–20 De Boer (= vol. V p. 42.3–6 Kühn = p. 49 Magnaldi; Engl. trans.: Harkins 1963, 58): ὧν ἁπάντων [*that is, Galen's teachers*] ὁ πατὴρ δι' ἐμὲ τοῦ τε βίου καὶ τῶν δογμάτων ἐξέτασιν ἐποιεῖτο σὺν ἐμοὶ πρὸς αὐτοὺς ἀφικνούμενος. ἐγεγύμναστο δ' ἐπὶ πλεῖστον ἐν γεωμετρίᾳ καὶ ἀριθμητικῇ <καὶ> ἀρχιτεκτονίᾳ καὶ ἀστρονομίᾳ. In *De bonis malisque sucis* 1.15, too, Galen emphasizes his father's excellence in geometry, architecture, practical calculations, number science, and astronomy: ἐμοὶ μὲν γὰρ πατὴρ ἐγένετο γεωμετρίας μὲν καὶ ἀρχιτεκτονικῆς καὶ λογιστικῆς ἀριθμητικῆς τε καὶ ἀστρονομίας εἰς ἄκρον ἥκων, ὑπὸ πάντων δὲ τῶν γνόντων αὐτὸν ἐπὶ δικαιοσύνῃ καὶ χρηστότητι καὶ σωφροσύνῃ θαυμασθεὶς ὡς οὐδεὶς τῶν φιλοσόφων (p. 392.21–24 Helmreich, = vol. VI p. 755.12–16 K.). Cf. Schlange-Schöningen 2003, 40–41, who refers (n. 28) to al-Bīrūnī's report of an astronomical conversation between Galen and his father which goes back to a remark in Galen's commentary on Hippocrates *De aere aquis locis* (extant in Arabic; see below, n. 332). The question for whose correct solution Galen's father praised his son concerned heliacal risings, more specifically, "why Sirius appears in Pergamum two days later than in Alexandria, a phenomenon that most astronomers in Asia were unable to explain" (Strohmaier 2002, 115).

[321] Cf. Nutton 1995, 18.

[322] Cf. *De libris propriis* 11 p. 116.21–26 Müller (= vol. XIX p. 40.5–9 K.): εἰς τὴν τῶν Πυρρωνείων ἀπορίαν ἐνεπεπτώκειν ἂν καὶ αὐτός, εἰ μὴ καὶ τὰ κατὰ γεωμετρίαν ἀριθμητικήν τε καὶ λογιστικὴν κατεῖχον, ἐν αἷς ἐπὶ πλεῖστον ὑπὸ τῷ πατρὶ παιδευόμενος ἐξ ἀρχῆς προεληλύθειν ἀπὸ πάππου τε καὶ προπάππου διαδεδεγμένῳ τὴν θεωρίαν.

[323] Garofalo 2003, 47.

[324] We have a substantial fragment of the Greek original of this work, which is from Galen's maturity (Garofalo 2003, 47), and a complete Arabic translation which we shall quote according to their editions by Walzer 1935. Note that Walzer, while improving on the Greek text of the *editio princeps* by Schöne 1933 (this treatise is missing from Kühn's *Opera omnia*), preserves Schöne's line count.

[325] Gal. *de septim. partu* p. 355, lin. 32–39 W./Sch.: δῆλον γὰρ ὅτι τὰς ἰσημερίνας [read: -ρινὰς] ὥρας λέγομεν ἐν πᾶσι τοῖς τοιούτοις, ὧν ἐστι καὶ τὸ νυχθήμερον ὀνομαζόμενον ὑπὸ τῶν ἀστρονομικῶν εἴκοσι καὶ τεττάρων ὡς ἁπάντων αὐτῶν ἴσων ὄντων, ἐπειδὴ τὸ παραλλάττον ἐλάχιστόν ἐστι, ὥς τινας νομίζειν ὅλως αὐτὸ μηδ' εἶναι. μέσαι τινὲς γὰρ πώς εἰσι κατὰ τὸ μέγεθος αἱ κατὰ τὴν ἰσημερίαν ὧραι τῶν τε μεγίστων ἐν ἑκάστῳ χωρίῳ καὶ τῶν ἐλαχίστων. Cf. ibid., 350, lin. 212–224 (German trans. of the Arab. trans.) and the comments by Neugebauer 1949, 92.

[326] Gal. *de septim. partu* p. 354, lin. 16–21 W./Sch. [...] καὶ τὸν μηνιαῖον χρόνον εἴκοσι ἐννέα καὶ ἡμίσεως. ὅτι δὲ μὴν ὁ ἀπὸ συνόδου σελήνης πρὸς ἥλιον ἄχρι τῆς ἑξῆς συνόδου χρόνος ὀνομάζεται, [ἢ] γινώσκεται πᾶσιν· καὶ προσέρχεσθαι ἄλλο τι μόριον Ἵππαρχος ἀπέδειξεν τῆς ὅλης ἡμέρας *ἑξηκοστὸν καὶ ἑβδομηκοστὸν*

sum of unit fractions implies that he has converted Hipparchus' original sexagesimal value "into popular language."[327]

Whenever fractions of days must be specified in this treatise, Galen switches randomly between equinoctial hours and (sums of) unit fractions. In one case he explicitly converts 182 ½ ⅛ days into 182 days and 15 hours.[328] Clearly, Galen could have calculated in days and equinoctial hours in *On Critical Days* as well.[329]

His reason for not doing so, namely the pursuit of a language that his readers would understand, has to do with Galen's low estimation of most physicians' knowledge of astronomy and geometry (he criticizes their ignorance of these disciplines on various occasions),[330] and especially of the Romans among them. His commentary on Hippocrates' *Airs, Waters and Places* (written during the late reign of Commodus, that is, around 190 CE),[331] chapter 3.11, is especially revealing for his view of the Roman physicians and physician-astrologers he had met.[332] To be sure, these Romans were members of a socially elevated intellectual class that was able to converse with Galen in Greek.[333] He reports that some of his Roman interlocutors thought there was only one equinox per year (§ 16), and when Galen told them the exact day when the summer solstice takes place, "they supposed or believed that I was telling them about white ravens, because of the dimness of their intellects and the distance they were from understanding" (§ 12).[334] Galen chastises their "obvious

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δεύτερον καὶ δισμυριεπτακισχιλιοστὸν ἄλλο τέ τι πρὸς τούτῳ πάνυ σμικρόν, οὗ περιττὸν εἰς τὰ παρόντα μεμνῆσθαι. Cf. ibid., 349–350, lin. 195–198 (German trans. of the Arabic trans.; see also ibid., 347, lin. 107–113, which is missing from the Greek fragment). Note that the Greek and Arabic versions disagree on the first two fractions, which are discussed by Neugebauer 1949, whose mathematical emendation ("1/60 1/72," 93) I have here tentatively translated into Greek (italicized) as a 'diagnostic conjecture' (Paul Maas).

[327] Neugebauer 1949, 94 (this will be elucidated below, in the answer to Question 2). The traditional sexagesimal value used by Hipparchus and Ptolemy as well as their Babylonian predecessors was 29;31,50,8,20 (cf. ibid., 93, and Neugebauer 1975, 69).

[328] Gal. *de septim. partu* p. 357, lin. 109–111 W./Sch.: [...] γένοιτ' ἂν ὁ τοῦ ἡμίσεως ἐνιαυτοῦ χρόνος ἡμερῶν ἑκατὸν ὀγδοήκοντα δύο καὶ ἡμίσεως καὶ ὀγδόου μιᾶς ἡμέρας, ὅπερ ἔμπροσθεν ἔφην ὥρας εἶναι πεντεκαίδεκα πάσας; cf. ibid., 353, lin. 310–312 (German trans. of the Arabic trans.). Galen's self-reference is to a passage lost in the Greek fragment but extant in Arabic (p. 346, lin. 91 of its Engl. trans.); the figure given there is repeated also at p. 352, lin. 273, and p. 353, lin. 312, which correspond to the Greek text at pp. 355–356, lines 32–33 and 79–80 W./Sch.

[329] Incidentally, Galen quotes Hipparchus once in *On Critical Days* (p. 907.16 K., quoted on p. 241 above, Question 1) with regard to the length of the synodic month.

[330] Cf. Toomer 1985, 202 (comm. on §§ 16–22).

[331] Strohmaier 1993, 157.

[332] The Greek original of this work is lost; a facsimile of the *codex unicus* of its Arabic translation was edited by Sezgin 2001. Toomer 1985 has edited, translated and commented on chapter 3.11 of this Arabic translation. He mentions (p. 194, n. 6) that an edition of the complete Arabic translation is being prepared by G. Strohmaier for the *Corpus Medicorum Graecorum* (http://cmg.bbaw.de/publikationen/corpus-medicorum-graecorum-1), which, however, is still not available. Toomer (p. 194) admits that "it is not clear to me whether his [that is, Galen's] remarks in §§ 16–22 on the ignorance of the 'people of Rome' concerning mathematics and astronomy refer to the Roman astrologers or the Roman doctors." Since both arts were often practiced by one and the same person, Toomer's attempt at distinguishing between physicians and astrologers is probably pointless.

[333] Strohmaier 1993, 162. Galen's encounters with such Romans are likely to have occurred, at least partly, in the context of his public lectures in Rome, which Galen delivered in Greek. Cf. Schlange-Schöningen 2003, 157 n. 80: "Galen hat seine Vorträge in Rom offensichtlich in griechischer Sprache gehalten."

[334] The text does not explicitly confirm Garofalo's statement (2003, 46) that Galen feels the need to repeat several times the meaning of 'equinoctial hour' to his interlocutors ("Galeno sente il bisogno di ripetere più volte il significato di 'ora equinoziale'"). However, Garofalo's words are a rather plausible inference from what the text actually says.

poverty" in geometry, of which they (with a few laudable exceptions) had only a basic knowledge for the instruction of youths and boys (§ 16). They were largely unfamiliar with various Greek treatises on advanced geometrical problems mentioned by Galen in §§ 17–21.[335] At their insistent request, Galen composed for them a book on geometry and called it *The Stars of Hippocrates and the Geometry Useful in the Science of Medicine* (§ 30).

Thus it is not surprising that he avoided the concept (comprehensible only to readers with an astronomical education) of equal hours in most of his writings,[336] and especially in his discussion of the medical week in *On Critical Days*, because that would have required specification of fractions of hours. Instead, he resorted to fractions of days, surely more readily intelligible, though he was aware that many readers would still find his computation too difficult to grasp.[337]

Question 3

Sums of unit fractions are a Greek inheritance from Egyptian mathematics. It is indicative of the poor medieval, early modern, and even modern understanding of Galen's math here that the keyword 'Egypt(ian)' is never mentioned by the scholars who have tackled this passage. Moreover, there is no positive evidence that Galen was familiar with our modern concept of common or general fractions whose numerators can be any positive integer. It will be useful briefly to summarize a recent debate among historians of mathematics regarding the status of fractions and the techniques of their treatment in Greek mathematics. Wilbur Knorr argued in a seminal article (1982) that the Greek computational tradition consisted of two largely independent branches, which he dubbed the 'general mode' and the 'unit-fractional mode.'[338] Knorr emphasized the "interplay of the two modes."[339] Some computers, he thought, re-expressed a given sum of unit-fractional parts in the general fractional mode, carried out the arithmetic operations as one now would do in modern school arithmetic, and then stated the solution after conversion back to the unit-fractional mode;[340] therefore Knorr emphasized the importance of distinguishing between notational and computational significance of each of the two modes. Applied to Galen, whose writings neither Knorr nor any other historian of mathematics active in our debate took into account, Knorr's position means that the fact that Galen gives his results as sums of unit

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[335] Strohmaier 1993, 162, remarks that this picture is in keeping with the historical fact that no Roman ever translated a work of either Hipparchus or Ptolemy.

[336] Apart from the special case of *On Seven-Month Children* (see above), Galen speaks only seven more times in his whole corpus of ἰσημεριναὶ ὧραι (source: TLG), and none of these attestations involves fractions: *De sanitate tuenda* vol. VI pp. 405.12, 406.10, 412.5 K., *De differentiis febrium* vol. VII p. 339.11 K., *De methodo medendi* vol. X pp. 479.10 and 568.11 K. and *De morborum temporibus* p. 16.11–12 Wille. The first of these passages (vol. VI p. 405.12 K.) distinguishes between the longest and shortest hours at Rome and those at Alexandria.

[337] Cf. p. 934.1–9 K., quoted in n. 305 above.

[338] Knorr 1982, 158. As he rightly observes (ibid., 165, n. 41), a third mode, namely sexagesimal computation, was in Greco-Roman antiquity strictly limited to the astronomical field.

[339] Ibid., 153.

[340] Ibid., 150.

fractions, thereby complying with "the preferred notational mode for expressing results,"[341] does not necessarily mean that he performed all arithmetic operations in the same mode: He may, in between, have tacitly resorted to the more serviceable general mode of fractions.

In the following years David Fowler expressed his sharp disagreement with Knorr, based on a re-examination of Knorr's as well as many other ancient sources. "Nowhere do I find any convincing evidence for the proposal that 'the Greeks' used anything like our notations for common fractions and our ways of performing fractional arithmetic."[342] At best, one finds ambiguous cases.[343] "Just one example of some operation such as the addition, subtraction, multiplication, or division of two fractional quantities, expressed directly as something like, 'the *n*th of *m* multiplied by the *q*th of *p* gives the *nq*th of *mp*' and *clearly unrelated, by context, to any conception in terms of simple and compound parts*, could be fatal to my thesis that we have no good evidence for the Greek use or conception of common fractions. I know of no such example."[344]

In 2004 Jean Christianidis sided basically with Fowler but pointed out that he had in fact discovered one isolated example of the kind of operation described by Fowler, namely in Diophantus' *Arithmetica* 4.36.[345] This eventually leads Reviel Netz (2017) to conclude that common fractions were "simply something which is very rare. So why do Greeks use unit fractions so often, general fractions so rarely? Because unit fractions were the standard form: a convention of shared practice, not the constraint of a given concept."[346]

Since there appears to be only one known Greek example of computation with common fractions, namely in Diophantus, but countless Greek examples of unit fractions, it seems very unlikely that Galen should have made any use of common fractions. We shall therefore not speculate (as Knorr would have done) whether Galen performed any unexpressed intermediate steps of his computation in the 'general fractional mode.' Instead, we

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[341] Ibid., 153. Incidentally, "sometimes a result is expressed [*by mathematical authors*] in both modes, one immediately after the other" (ibid., 154).

[342] Fowler 1992, 134. Vitrac 1992 (in the same volume) equally concludes that the Greek logistic tradition did not use a concept of 'general fraction.'

[343] It will be useful to adduce one example of these many ambiguous cases. Knorr had paid special attention to the *Metrica* of Hero of Alexandria (late first century CE), "in which the unit-mode is of minimal importance" (Knorr 1982, 151). At close inspection, however, none of Hero's alleged common fractions withstands close scrutiny. Sidoli (cf. n. 312 above) gives the following example: "In *Metrica* III 2, we find $51/52$ in the translation by Acerbi and Vitrac 2014, 313 ("Ce sera de 12 unités et $51/52$"; cf. the Greek ibid., 312: ἔσται μονάδων ιβ να νβ′), but we could translate the fractional remainder more precisely as 'fifty-one of the one-fifty-second parts.' When we write $51/52$, we are compelled to see this as a common fraction, or as a division, but this is not so clear from the text, which rather seems to mean a number of equal proper parts, that is, a multiplication ($51 \times \overline{52}$)." Sidoli analyzes Hero's computation, which leads to the result quoted above, step by step and concludes: "I do not see how this involves any computations with common fractions. The same can be said for the rest of the *Metrica*."

[344] Fowler 1999, 264–265 (= 1st ed. 1987, 263; italics are his).

[345] Cf. the brief discussion of Knorr's and Fowler's positions given by Christianidis 2004, part 5 (*Did the Greeks have the notion of common fraction? Did they use it?*), 331–332, which is followed by Christianidis' reference to and analysis of the case in Diophantus (ibid., 332–336) who probably lived around 250 CE.

[346] Netz 2017, 45. It is not my intention to discuss the inferences to be drawn from the ancient evidence regarding the nature of mathematical concepts (Netz' main concern). Nothing on unit fractions among the Greeks in Heiberg 1925, Gericke 1984, and Herrmann 2014.

shall provide possible unit-fractional explanations of the single steps of Galen's computation.

Question 4

It would go beyond the scope of the present article to provide an introduction to the Egyptian technique of calculating with unit fractions, especially since clear and recommendable introductions by experts are available.[347] It will here be assumed that the reader has a basic acquaintance with this technique, which harks back to Egyptian practice of the Middle Kingdom and can be traced all the way from hieratic (especially P.Rhind) through Demotic mathematical papyri to Greek mathematical papyri of late and very late antiquity such as P.Mich. 145 (2nd century CE), P.Mich. 146 (4th century CE), and P.Akhmīm (7th/8th century CE?).[348]

    Nevertheless, two preliminary remarks may be useful:

a) It would be anachronistic to apply the modern distinction between common fractions and unit fractions to ancient texts (with the exception of Diophantus' *Arithmetica* 4.36; see above, n. 345). Greek fractions were μέρη, proper parts, and we should present ancient calculations as their authors engaged with them. We shall therefore comply with the recent notational habit of historians of mathematics who, when dealing with ancient parts, avoid the conceptually misleading modern notation with a separating bar (for example, $\frac{1}{5}$) and prefer, instead, notation with a typically Egyptian overbar (in our example, $\bar{5}$) to stress Greek continuity with the Egyptian tradition.[349]

b) Moreover, we shall follow these historians in rejecting the disqualification of ancient sums of proper parts as "unwieldy," "cumbersome," or even "utterly artificial."[350] Such judgments are prejudiced by modern mathematical education. Although sums of proper parts take more space than one arithmetically equivalent common fraction, the use of proper parts has its own advantages which tend to be overlooked; otherwise that system would

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[347] See especially Gillings 1972, 74–119; Imhausen 2007; Imhausen 2016a, 89–99 and 189–192 (on fraction reckoning in the Pharaonic and Greco-Roman periods); Imhausen 2016b, 300–302. For an older yet still useful German summary, see van der Waerden 1956, 28–47. For the traditional Egyptian use of two non-unit-fractions, $\frac{2}{3}$ and $\frac{5}{6}$, see Hughes and Jasnow 1997, 85. In Demotic texts from the 3rd century BCE to the Roman period one finds complex expressions that seem to be common fractions (cf. Parker 1972, 8–10), but Fowler 1999, 258–262, especially p. 262 (= 1st ed. 1987, 259–263, especially p. 263), more appropriately calls them "incomplete divisions." Fowler is followed by Imhausen 2016a, 190–191, who states (p. 191): "The evidence from the demotic mathematical papyri clearly proves that the concept of fractions did not change."

[348] In this respect, Knorr (1982, 147) rightly spoke of a "uniform computational tradition." Cf. Sidoli 2015, 390 (on elementary mathematical education): "It appears that this stage of mathematics education was fairly constant through much of Greco-Roman antiquity. For example, fractional parts were always handled using tables of Egyptian fractions—what we call unit fractions—and there is no indication that the more sophisticated sexagesimal fractions that Greek mathematicians adopted from Mesopotamia for use in the astral sciences were ever taught in elementary education."

[349] One could also use prime marks to be closer to the Greek sources, but overbars are now the rule.

[350] These are quotations from Knorr 1982, 151, 135, and 150.

simply never have survived thousands of years. Sidoli elucidates these advantages with a good example:[351]

> Many modern writers do not fully appreciate the virtue of proper parts, because we have become so used to our own system that we cannot see its weaknesses. For example, it is almost impossible for me to form a feeling of how much, say, $\frac{3}{17}$ involves, without converting it to 0.17645. On the other hand, the amount involved in the equivalent expression $\overline{6}\ \overline{102}$ is clearer to me: more than a sixth, but less than two sixths, or a fifth. That is, whereas I find it extremely annoying to compute in proper parts, I can appreciate that they give us a better sense of the actual magnitudes involved. But this only works if the largest proper part has in fact been found and expressed.

The seven steps mentioned above will then, in appropriate mathematical notation, read thus:

S1 $\quad 26\ \overline{2}\ ^\mathrm{d} + 27\ \overline{3}\ ^\mathrm{d} = 53\ \overline{2}\ \overline{3}\ ^\mathrm{d}$

S2 $\quad 53\ \overline{2}\ \overline{3}\ ^\mathrm{d} \div 2 = 26\ \overline{2}\ \overline{3}\ \overline{12}\ ^\mathrm{d}$

that is, $(26\ \overline{2} + 27\ \overline{3})\ ^\mathrm{d} \div 2 = 26\ \overline{2}\ \overline{3}\ \overline{12}\ ^\mathrm{d}$

S3 $\quad 27\ ^\mathrm{d} - 26\ \overline{2}\ \overline{3}\ \overline{12}\ ^\mathrm{d} = \overline{12}\ ^\mathrm{d}$

S4 $\quad 26\ \overline{2}\ \overline{3}\ \overline{12}\ ^\mathrm{d} \div 4 = 6\ \overline{2}\ \overline{5}\ \overline{60}\ \overline{120}\ \overline{240}\ ^\mathrm{d}$

S5 $\quad 26\ \overline{2}\ \overline{3}\ \overline{12}\ ^\mathrm{d} \div 4 = 6\ \overline{2}\ \overline{6}\ \overline{24}\ \overline{48}\ ^\mathrm{d}$

S6 $\quad 7\ ^\mathrm{d} - 6\ \overline{2}\ \overline{5}\ \overline{60}\ \overline{120}\ \overline{240}\ ^\mathrm{d} = \overline{4}\ \overline{60}\ \overline{240}\ ^\mathrm{d}$
$\quad\quad$ or: $7\ ^\mathrm{d} - 6\ \overline{2}\ \overline{6}\ \overline{24}\ \overline{48}\ ^\mathrm{d} = \overline{4}\ \overline{60}\ \overline{240}\ ^\mathrm{d}$

S7 $\quad 3 \times 6\ \overline{2}\ \overline{5}\ \overline{60}\ \overline{120}\ \overline{240}\ ^\mathrm{d} = 20\ \overline{6}\ {<}\overline{48}{>}\ ^\mathrm{d}$

We shall now examine how Galen may have performed these steps. This examination will necessarily be speculative in many details, because it is impossible to ascertain intermediate computational steps Galen did not put into writing. Nonetheless such an examination is not superfluous because it elucidates the techniques Galen could employ and contextualizes his reasoning within the mathematical tradition. References to Egyptian sources of far earlier epochs, especially the famous papyrus Rhind (written about 1550 BCE) and the Egyptian Mathematical Leather Roll (henceforth, EMLR, assumed to be contemporary with P.Rhind) are, of course, not meant to say that Galen used Egyptian sources but that

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[351] Cf. n. 312 above.

the mathematical tenets and techniques of these sources were long-established elements of the mathematical tradition at Galen's time.[352] The unit day ($^{\mathrm{d}}$) will henceforth be omitted.

The implicit result of **S1**, namely 53 $\overline{2}\,\overline{3}$, is obvious.

S2 Since the division of 53 by 2 is trivial, yielding 26 $\overline{2}$, the remaining problem is the division of $\overline{2}\,\overline{3}$ by 2, which is $\overline{4}\,\overline{6}$. This result is not satisfactory because the aim of ancient mathematicians was to approximate the fractional result as closely as possible with the first part of a sum of parts because the result is easier to understand if we express the values starting with the largest part possible.[353] Since in the present case the result is obviously smaller than $\overline{2}$ and larger than $\overline{3}$, the sum of parts must start with $\overline{3}$. For someone computing in the Egyptian tradition, the conversion of $\overline{4}\,\overline{6}$ into a sum of parts starting with $\overline{3}$ was easy, because some basic equations among proper parts had been standard knowledge of people with a mathematical education from very early times, one of these equations being $\overline{4}\,\overline{12} = \overline{3}$.[354] Hence, $\overline{4}\,\overline{6} = \overline{4}\,\overline{12}\,\overline{12} = \overline{3}\,\overline{12}$, the best possible solution based on a sum of parts. Taken together with the previous result of the division of 53 by 2 (26 $\overline{2}$), the final total of **S2** is 26 $\overline{2}\,\overline{3}\,\overline{12}$.[355] The same sum of parts (equivalent to our modern common fraction $^{11}/_{12}$) occurs in a Greek papyrus of the 4th century CE in a fraction table as the result of the division of 11 by 12.[356] I am not aware of any ancient attestation of the alternative, mathematically equivalent sum of parts, $\overline{2}\,\overline{4}\,\overline{6}$.

S3 can be reduced to the task of complementing $\overline{2}\,\overline{3}\,\overline{12}$ up to 1. This kind of calculation has methodological precedents in P.Rhind[357] and is, as far as we know, the typical Egyptian method of subtraction of fractions. We shall proceed analogously. Hence, the numbers in the first row below are the proper parts to be complemented. The auxiliary numbers ('aux. numb.') in the second row are the respective integers by which the numbers in the first row must be multiplied in order to convert them to their least common multiple,[358] that is, in the present case, to 12:

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[352] For P.Rhind (British Museum 10057 and 10058), cf. Peet 1923 and Robins and Shute 1987. For the EMLR (BM 10250), cf. the English translation in Imhausen 2007, 21 (my source for quotations); for more details and photographs, cf. Gillings 1972, 89–103, and Imhausen 2016a, 96–99.

[353] Cf. Knorr 1982, 142 (in the context of the papyri mentioned above, p. 247): "In both the Egyptian and Greek cases the computer tends to seek the largest such unit fraction possible (that is, the smallest possible denominator in the sequence) to start off, although exceptional cases are to be found."

[354] This equation is the third entry in col. I (= col. III) of the EMLR. "In the interest of semantic precision, I speak of 'equations,' although it has become customary to speak either of 'equalities' (for example, Gillings 1972) or 'identities' (for example, Knorr 1982 and Imhausen 2016a). While the former is a semantical shift from a quality to an assertion of a quality, the latter may seem philosophically objectionable.

[355] In other words, the immediate result of **S2**, which Galen skips, is 26 $\overline{2}\,\overline{4}\,\overline{6}$. The following step, whose result Galen actually reports, is but a simple conversion into the equivalent sum 26 $\overline{2}\,\overline{3}\,\overline{12}$ to make the previously computed value 26 $\overline{2}\,\overline{4}\,\overline{6}$ more intuitively clear, similar to our expression of a common fraction in least terms.

[356] P.Mich. 146, col. xi (Robbins 1936, 56).

[357] Cf. problems nos. 21–22 of that papyrus in Peet 1923, 58–59. They are also explained by van der Waerden 1956, 43–45, as well as by Robins and Shute 1987, 20.

[358] This modern term is employed in the context of P.Rhind, though in quotation marks, by Gillings 1972, 78.

$$\overline{2} \qquad \overline{3} \qquad \overline{12}$$

aux. numb.:   6   4   1   sum: 11, remainder (up to 12): 1

Divide the remainder 1 by 12: the result is $\overline{12}$.

Another case of complementation up to the next higher integer occurs in **S6**. We shall ask later why Galen calculated these values.[359]

S4 and **S5** provide alternative results to the task of dividing 26 $\overline{2}\,\overline{3}\,\overline{12}$ by 4. Since the division of 26 by 4 is rather trivial, yielding 6 $\overline{2}$, the problem can be reduced to dividing $\overline{2}\,\overline{3}\,\overline{12}$ by 4. The typical Egyptian solution would be to multiply each part's numeral (not each part)[360] twice by 2. This leads to $\overline{8}\,\overline{12}\,\overline{48}$. Both solutions that Galen gives in **S4** and **S5** are different, not surprisingly: $\overline{8}\,\overline{12}\,\overline{48}$ is disadvantagous from the ancient mathematical perspective because it does not satisfactorily help us understand the quantity in question. As mentioned above on the occasion of **S2**, the aim of the ancient computers was to approximate the result as closely as possible with the first (and possibly also with the second or even third) part.[361] There are various ways Galen may have calculated, one of them being rather complicated further manipulations of $\overline{8}\,\overline{12}\,\overline{48}$ based on standard equations such as those in the EMLR.[362] I shall limit my discussion of **S4** and **S5** to proposing a different, in my view simpler approach to the task of dividing $\overline{2}\,\overline{3}\,\overline{12}$ by 4. Let us employ once more the ancient method used above (**S3**):

$$\overline{2} \qquad \overline{3} \qquad \overline{12}$$

aux. numb.:   6   4   1   sum: 11.

Hence, divide 11 by (4 × 12).

What method is this actually? It is a multiplication of all terms of the problem $\overline{2}\,\overline{3}\,\overline{12} \div 4$ with an integer n (in this case 12) which converts the task of dividing a sum of parts to the more handy task of dividing a single integer.[363] In other words, it is an application of the basic insight that $a \div b = na \div nb$ for all integers a, b, and n.[364]

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[359] See Question 7, p. 258 below.

[360] That is, in modern expression, to multiply each denominator (not each unit fraction).

[361] The frequently occurring parts down to $\overline{12}$ (especially $\overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{8}, \overline{10}, \overline{12}$) were clearly more meaningful to people than smaller parts. Knorr 1982, 144, calls $\overline{2}, \overline{3}, \overline{4}, \overline{10}$ and $\tfrac{2}{3}$ the Egyptian "standard parts" (cf. the similar though not identical selection of van der Waerden 1956, 32). Other fractions required sums of parts.

[362] See above, n. 352. Sidoli (cf. n. 312 above) kindly provided me with one such example which I deem too complicated to be quoted here.

[363] In modern notation: $(\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{12}) \div 4 = (6 + 4 + 1) \div 48 = \tfrac{11}{48}$.

[364] Cf. Gillings 1972, 203–204, on problem no. 36 of P.Rhind, where the scribe shifts from the problem of dividing 1 by 3 $\overline{3}\,\overline{5}$ to dividing 30 by 106: "Expressed in algebraic terms, it is equivalent to a : b = ax : bx." Similar insights appear to underly the same papyrus' famous $2 \div n$ Table, on which cf. Gillings 1972, 45–80, and Imhausen 2016a, 93–96, namely that if $a \div b = \tfrac{1}{c} + \tfrac{1}{d}$, then $a \div nb = \tfrac{1}{nc} + \tfrac{1}{nd}$ for all integers a, b, c, d and n. See

If Galen arrived thus (or in another way) at the reformulation of his original problem $\overline{2}\,\overline{3}\,\overline{12} \div 4$ as $11 \div 48$, he is likely to have noticed the following points: (a) since 11 is a prime number, the ratio $11 \div 48$ cannot be mathematically reduced; (b) compared to 48, the next larger multiple of 11 is 55 (5×11); therefore the final result will be a sum of parts beginning with $\overline{5}$; (c) 5 is not among the positive integer divisors of 48; therefore it is convenient to expand both terms once more, namely by 5, to ease calculation.[365] I am inclined to believe that this is the reason why a two-hundred-fortieth part comes into play in Galen's final result.

Note that the two steps of this consecutive expansion of the original problem (first by 12 and then by 5, that is, altogether by 60) are not arbitrarily proposed here: They both fulfill the respective minimum requirements to eliminate two difficulties at issue, namely to obtain a dividend free of parts (here: 11) and to obtain a multiple of 48 that allows for division by 5 (here: 240). The importance of this second expansion becomes clear when we consider the Egyptian manner of dividing a smaller integer a by a larger integer b. For $55 \div 240$, various solutions are possible, for example[366]

	I		II		III		IV	
	1	240	1	240	1	240	1	240
	$\overline{4}$	60	$\overline{4}$	60	$\overline{4}$	60	$\overline{4}$	60
	$/\,\overline{5}$	48	$/\,\overline{5}$	48	$/\,\overline{5}$	48	$/\,\overline{5}$	48
	$/\,\overline{60}$	4	$/\,\overline{40}$	6	$/\,\overline{48}$	5	$/\,\overline{60}$	4
	$/\,\overline{120}$	2	$/\,\overline{240}$	1	$/\,\overline{120}$	2	$/\,\overline{80}$	3
	$/\,\overline{240}$	1	sum:	55	sum:	55	sum:	55
	sum:	55						

This typically Egyptian notation is to be understood thus (for example, column I): The full value in question is 240; the fourth part of 240 is 60, the fifth part is 48, and so forth. Select the terms of the left sub-column in such a way that the corresponding numbers in the right sub-column sum up to the required total, in this case: 55. The selected left terms are here, as in Egyptian papyri, marked with checkmarks $/$.[367] They form, read from top to bottom, one possible sum of parts that solves the mathematical problem.

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the examples quoted by Gillings 1972, 74–77. Cf. also the EMLR, which appears to imply the insight that if $a + b = c$, then $na + nb = nc$ for all integers a, b, c and n. Cf. Gillings 1972, 39–44 on the 'G-Rule.'

[365] Similarly, the scribe of P.Rhind no. 36 (cf. n. 364 above) does not scale his problem $(1 \div 3\,\overline{3}\,\overline{5})$ by the least number needed to eliminate all fractional parts, namely 15 (that would reformulate the problem as $15 \div 53$), but by 30, probably because 53 is a prime number.

[366] Note that the entries in the respectively right sub-columns are in principle allowed to (but do not here) contain fractional parts, especially (but not limited to) 'natural' fractions (van der Waerden, see above, n. 361). For examples from P.Rhind, cf. Knorr 1982, 136–139.

[367] That is, terms without a checkmark (one finds them in many computations of P.Rhind) ease computation but have not been used as summands. Terms without a checkmark can be more or less numerous depending on the computer's preferences and the number of mathematically available divisors; that is, the second row of examples I to IV could be missing without any damage to the four computations.

Hence, the results of our examples are either (I) $\overline{5}\ \overline{60}\ \overline{120}\ \overline{240}$ or (II) $\overline{5}\ \overline{40}\ \overline{240}$ or (III) $\overline{5}$ $\overline{48}\ \overline{120}$ or (IV) $\overline{5}\ \overline{60}\ \overline{80}$. The first example (col. I), which leads to the same result Galen actually gives in **S4**, has the advantage of matching the ancient preference for proceeding from line to line (whenever possible) multiplying the previous term in the respective left sub-column by 2 (or 3).[368] The second solution (col. II) looks quite 'natural', too, while those in cols. III and IV seem less likely to be adopted by an ancient computer. More solutions are possible if one is willing to accept results starting with parts smaller than $\overline{5}$, for example

V		VI		VII	
1	240	1	240	1	240
$/\,\overline{6}$	40	$/\,\overline{8}$	30	$/\,\overline{10}$	24
$/\,\overline{24}$	10	$/\,\overline{16}$	15	$/\,\overline{12}$	20
$/\,\overline{48}$	5	$/\,\overline{24}$	10	$/\,\overline{24}$	10
sum:	55	sum:	55	$/\,\overline{240}$	1
				sum:	55

These results would be either (V) $\overline{6}\ \overline{24}\ \overline{48}$ or (VI) $\overline{8}\ \overline{16}\ \overline{24}$ or (VII) $\overline{10}\ \overline{12}\ \overline{24}\ \overline{240}$. Example V leads to the same result that Galen actually gives as an alternative (**S5**). The reason why he adopted this solution (V) while dismissing other possible alternatives will be discussed below (Question 8).

At the present point it is important to emphasize that Galen had the option, and may actually have used it, to proceed systematically to ensure he did not overlook suitable solutions. By 'systematically' I mean taking down *all* positive integer divisors of 240 in the right sub-column, thus taking full advantage of the fact that 240 is a highly composite number (see example VIII below); 'suitable solutions' can, depending on the computer's goal, mean different things, for example, that sum of parts whose terms are selected in such a way that each one of them, reading from left to right, approximates the overall fraction in question (here, in modern notation: $^{11}\!/_{48}$) as closely as possible 'from below' (I should like to call this 'optimized approximation through sums of parts'). Even if I am not aware of any extant Greek source that engages *explicitly* in such systematic decomposition of the largest admissible number (in modern terminology: denominator) in the context of the arithmetic of parts, such decompositions must have been made, as is clear through the indirect testimony of various ancient sources.

Suffice it to adduce one example, P.Mich. 145 (2nd century CE), whose col. I i–ii contains the beginning of a table of twenty-thirds (extant: $1 \div 23$, $2 \div 23$, $3 \div 23$, $4 \div 23$, $5 \div 23$) and part of a table of twenty-ninths (extant: $12 \div 29$, $13 \div 29$, $14 \div 29$, $15 \div 29$, $16 \div 29$, $17 \div 29$).[369] According to the editor,[370] it is probable that the original list proceeded by

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[368] Cf. Neugebauer 1930, 327, who speaks, in the context of Egyptian division, of 'dyadic procedure' (cf. ibid., 374–376 "Dyadik und Stammbruchpostulat"), as well as Knorr 1982, 136–137.
[369] Robbins 1936, 36 (Greek text) and 43 (Engl. trans.).
[370] Ibid., 48.

prime divisors, with dividends covering all integers from 1 to the respective divisor. The better preserved fragment (col. I ii) reads thus:[371]

(*The 29th part* ...)	of 12: $\overline{4}\ \overline{8}\ \overline{29}\ \overline{232}$
	[of] 13: $\overline{3}\ \overline{15}\ [\overline{29}\ \overline{8}]\overline{7}\ \overline{4[35]}$
	[of] 14: $\overline{4}\ \overline{5}\ [\overline{5}]\overline{8}\ \overline{116}\ \overline{145}$
	[of] 15: $\overline{2}\ \overline{58}$
	[of 1]6: $[\overline{2}\ \overline{2}]\overline{9}\ \overline{58}$
	[of 17: $\overline{2}\ \overline{12}]\ \overline{348}$

One easily notes that the numbers of all parts smaller than $\overline{29}$ are multiples of 29. This strongly indicates that the computer systematically decomposed a sufficiently large multiple of 29 in order to ease the task of dividing twenty-nine different numbers (namely all integers from 1 to 29) by the prime number 29. If so, he must have chosen the expansion factor in such a way as to make the most elementary and meaningful parts possible, that is, $\overline{2}, \overline{3}, \overline{4}, \overline{5}$. Hence, the expansion factor must be at least 60 (= 3 × 4 × 5), which means that the number that he decomposed was either 1740 (60 × 29) or a multiple of 1740. Since the integer divisors of 1740 do not include 8 (see $\overline{8}$ in the first row above), I conjecture that he decomposed 3480 which contains all integer divisors required by the extant lines of this table, namely 2, 3, 4, 5, 8, 12, 15, 29, 58, 87, 116, 145, 232, 348, 435. The number 3480 has, of course, many more integer divisors, including—within the range from 1 to 15—the numbers 6 and 10 which may well have occurred as parts ($\overline{6}$ and $\overline{10}$) in the lost lines of the papyrus. The parts $\overline{6}$ and $\overline{10}$ do occur in the extant lines of the less well preserved col. I i whose analysis leads *mutatis mutandis* to the same result, that is, that the computer decomposed a suitably chosen multiple of 23.[372]

Now back to Galen. The following example gives the complete list of integer divisors of 240 needed for a systematic approach. Since a Greek computer would, of course, not have used Arabic ciphers but Greek number-letters with prime marks for parts, column VIIIb shows how such a list might have looked in Greek.[373]

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[371] I employ the same notation adopted here throughout. Robbins employed modern unit-fractional notation. Lacunae in the papyrus are, as usual, indicated with square brackets. The supplements are Robbins'.

[372] Since the original list of fraction tables may well have gone beyond the extant highest prime number 29 (the list may well have covered all prime numbers within the range from 1 to 100), the advantage of systematic decomposition grows larger with each higher prime number. For similar, partly contemporary mathematical documents, cf. Harrauer and Sijpesteijn 1985, 151–164, on the Viennese papyri MPER N.S. 15.158–170, which contain fraction tables, especially no. 167 (2nd century CE), which contains fragments of tables of sevenths, eighths, and ninths (cf. the catalogue of ancient division tables in Fowler 1999, 269–274, here: 273, no. 36; = 1st ed. 1987, 271–276, here: 275, no. 36). Of particular interest is col. I (a table of sevenths), where the last (that is, smallest) part's number (in modern terminology: the unit fraction's denominator) is always a multiple of 7 (up to 42), while the other parts that precede this last part in the respective sum of parts are not multiples of 7 (they are 2, 3, 4, 11). This table seems to be based on a systematic decomposition of a multiple of 7, for example, of 924 (= 3×4×11×7).

[373] Cf., for example, the Greek notation of proper parts with prime marks in the aforementioned P.Mich. 145 (Robbins 1936, 36) and the analysis of that notation by the editor (ibid., 46). It seems possible, though unlikely, that a Greek computer arranged the data of a decomposition horizontally rather than (as usual in the Egyptian tradition) vertically.

VIIIa		VIIIb	
1	240	α	σμ
$\overline{2}$	120	β′	ρκ
$\overline{3}$	80	γ′	π
$\overline{4}$	60	δ′	ξ
$\overline{5}$	48	ε′	μη
$\overline{6}$	40	ς′	μ
$\overline{8}$	30	η′	λ
$\overline{10}$	24	ι′	κδ
$\overline{12}$	20	ιβ′	κ
$\overline{15}$	16	ιε′	ις
$\overline{16}$	15	ις′	ιε
$\overline{20}$	12	κ′	ιβ
$\overline{24}$	10	κδ′	ι
$\overline{30}$	8	λ′	η
$\overline{40}$	6	μ′	ς
$\overline{48}$	5	μη′	ε
$\overline{60}$	4	ξ′	δ
$\overline{80}$	3	π′	γ
$\overline{120}$	2	ρκ′	β
$\overline{240}$	1	σμ′	α

A computer searching the best possible approximation for 55 ÷ 240 would start from the top of the right sub-column of VIIIa or VIIIb and move down searching for the first number equal to or smaller than 55 (here: 48), mark this row, remember the remainder to be allocated (here: 7), continue searching downwards for the first number equal to or smaller than that remainder (here: 6), mark that row, too, remember the new remainder (here: 1), move down to the respective row and mark that, too. In this manner, one would find that if the smallest admissible part is $\overline{240}$, the best possible sum of parts (in the above defined sense of optimized approximation) to express the result (modern: $^{11}/_{48}$) is $\overline{5}\ \overline{40}\ \overline{240}$. Since this is not the result Galen reports in **S4**, it is worth noting that there are three more solutions beginning with $\overline{5}$, namely $\overline{5}\ \overline{48}\ \overline{120}$ (this, in ancient notation, is the same as Guijon's proposal),[374] $\overline{5}\ \overline{60}\ \overline{80}$ (not mentioned by anyone), and $\overline{5}\ \overline{60}\ \overline{120}\ \overline{240}$ (Galen's solution, the least desirable among these four in terms of optimized approximation), while other existing solutions begin with $\overline{6}$ or even smaller parts, thus being further removed from the best possible approximation. Hence, it is unclear whether Galen bothered to decompose the number 240 systematically. We can only speculate as to why he reports $\overline{5}\ \overline{60}\ \overline{120}\ \overline{240}$: One possible explanation, of course, is that he did not decompose the number 240 systematically

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[374] Cf. p. 228 (n. 240) and p. 229 with notes 247 and 248 above. Note that Guijon's proposal of replacing $^{1}/_{60}$ + $^{1}/_{240}$ in **S4** with $^{1}/_{48}$ does simplify and 'improve' Galen's sum of parts, but it falls short of 'optimizing' it.

and that this sum of parts is just what he arrived at by some other method;[375] it is equally possible that he had enough with optimizing the first part; still another possibility is that he cared for optimized approximation in the first part and suitability for sexagesimal conversion in the subsequent parts: as a matter of fact, $\overline{5}\ \overline{60}\ \overline{120}\ \overline{240}$ is the only one among all four sums of parts starting with $\overline{5}$ that allows for easy conversion of the following parts into minutes, that is, into sexagesimal fractions of hours, namely 24, 12, and 6 minutes. It must, however, be admitted that Galen says nothing of hours in this context.

Last, it may be useful to note that a computation with common fractions, which we have ruled out on principal grounds, would almost certainly have led to a different result, even after conversion of its fractional remainder back to a sum of parts: $26\ ^{11}/_{12} \div 4 = 6\ ^{35}/_{48}$,[376] and it would only be natural to split up $^{35}/_{48}$ into a sum of fractions which all allow for reduction of terms, that is, into $^{24}/_{48} + {}^{8}/_{48} + {}^{3}/_{48} = {}^{1}/_{2} + {}^{1}/_{6} + {}^{1}/_{16} = \overline{2}\ \overline{6}\ \overline{16}$. It is hard to imagine how Galen's first part following $\overline{2}$, namely $\overline{5}$, could have come into play. This confirms our *a priori* assumption that Galen (and his contemporaries) did not make use of common fractions for tacit intermediate steps of their calculations. Moreover, this line of thought nicely shows that the result of a hypothetical calculation with common fractions whose fractional remainder is eventually converted back to a sum of parts is inferior (in terms of optimized approximation) to a systematic decomposition within the arithmetic of parts.

S6 works in principle like **S3** insofar as **S6** can be reduced to the task of complementing either $\overline{2}\ \overline{5}\ \overline{60}\ \overline{120}\ \overline{240}$ (**S4**) or $\overline{2}\ \overline{6}\ \overline{24}\ \overline{48}$ (**S5**) up to 1.

	$\overline{2}$	$\overline{5}$	$\overline{60}$	$\overline{120}$	$\overline{240}$	(**S4**)
aux. numb.:	120	48	4	2	1	sum: 175, remainder: 65

	$\overline{2}$	$\overline{6}$	$\overline{24}$	$\overline{48}$	(**S5**)
aux. numb.:	120	40	10	5	sum: 175, remainder: 65

Divide 65 by 240. Again, a wide variety of solutions is possible (see again col. VIIIa on p. 254 above), but only three of them start with $\overline{4}$ (the largest possible part), namely (I) $\overline{4}\ \overline{48}$, (II) $\overline{4}\ \overline{80}\ \overline{120}$ and (III) $\overline{4}\ \overline{60}\ \overline{240}$.[377] One may wonder why Galen chose (III) and not (I), which is more compact and a better approximation (the latter advantage is true of II, too). Again, one possible answer is that he had not decomposed 240 systematically, but this does not convince me. Even a non-systematic approach (as typical in Egyptian papyri) would, in a first step, lead to the incomplete data in column IX below: since the computer knows that he must arrive at the sum of 65 in the right sub-column, his next question (after

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[375] Cf. n. 362 above.

[376] The fraction $^{35}/_{48}$ was correctly computed by Garofalo 2003, 53, and (independently) by Cooper 2004, 55 (repeated by Cooper 2011a, 372, n. 1127; 2011b, 129; 2012, 619; 2014, 84 and 88).

[377] Sidoli agrees with my derivation (cf. n. 312 above).

having selected the summand 60) is most likely to be 'what is the fifth part of 240?' (in order to obtain the second summand 5), and this would lead him to complete the calculation as indicated in col. X.

IX		X	
1	240	1	240
$\overline{4}$	60	/ $\overline{4}$	60
	....?	/ $\overline{48}$	5
		sum:	65

Moreover, even a partial decomposition as hypothesized in cols. I to IV above implies through the choice of $\overline{5}$ (which Galen did choose) that he was aware of the corresponding summand 48. Since each such row can obviously be reversed (for example, $\overline{5}$ ... 48 to $\overline{48}$ ... 5, because the part's number multiplied with the respective summand will always yield the same product, here: 240), it is hard to deny that Galen knew that he could have replaced $\overline{60}$ $\overline{240}$ (corresponding to summands 4 and 1) with $\overline{48}$ (corresponding to summand 5). For these various reasons, I prefer the assumption that Galen selected $\overline{4}$, $\overline{60}$, and $\overline{240}$ from his systematically decomposed list in order to establish continuity with the result given in **S4**.[378]

Question 5

The task in **S7** is triplication of a sum of parts, namely of the previously computed length of one medical week. The simplest old Egyptian method in such a case was to separate this task in two steps, first duplication of the sum in question and then addition of the same initial sum to the result.[379] Duplication is easy when the reciprocals of all parts are even integers, because then the computer only needs to halve each single integer.[380] Therefore it would be wise, in the present case, not to start from the result of **S4** (6 $\overline{2}$ $\overline{5}$ $\overline{60}$ $\overline{120}$ $\overline{240}$), which contains one odd part ($\overline{5}$), but from the alternative sum given in **S5** (6 $\overline{2}$ $\overline{6}$ $\overline{24}$ $\overline{48}$), so as to obtain (a) 13 $\overline{3}$ $\overline{12}$ $\overline{24}$ + (b) 6 $\overline{2}$ $\overline{6}$ $\overline{24}$ $\overline{48}$. Since both (a) and (b) contain $\overline{24}$, this sums up to $\overline{12}$ which, taken together with $\overline{12}$ in (a), sums up to $\overline{6}$. This leaves the computer with (a) 13 $\overline{3}$ $\overline{6}$ + (b) 6 $\overline{2}$ $\overline{6}$ $\overline{48}$. Since $\overline{3}$ $\overline{6}$ = $\overline{2}$,[381] he eventually obtains 20 $\overline{6}$ $\overline{48}$.[382]

In order to triplicate 6 $\overline{2}$ $\overline{5}$ $\overline{60}$ $\overline{120}$ $\overline{240}$ (**S4**), which contains the aforementioned odd part $\overline{5}$, an Egyptian computer would have to split this task into three steps: (a) triplication of 6 $\overline{2}$,

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[378] On the question what motivated his selection in **S4** see above, p. 254.

[379] Cf. van der Waerden 1956, 37. See also ibid., 31, on the survival of the *duplicatio* into the Middle Ages.

[380] Cf. ibid., 37, and Imhausen 2016a, 93.

[381] For this important equation cf. Gillings 1972, 95. It is not explicitly stated in the EMLR (cf. n. 352 above), but it is an obvious consequence of the EMLR's consecutive equations $\overline{6}$ $\overline{6}$ = $\overline{3}$ and $\overline{6}$ $\overline{6}$ $\overline{6}$ = $\overline{2}$ in col. I (= col. III), lin. 5–6.

[382] This is, of course, equivalent to the result of a modern computation with common fractions: $3 \times 6\,^{35}\!/_{48}\,^{d}$ (cf. n. 376 above) = 20 $^{9}\!/_{48}\,^{d}$ (= 20 $^{3}\!/_{16}\,^{d}$) = 20 $^{1}\!/_{6}\,^{1}\!/_{48}\,^{d}$.

which is trivial (= 19 $\overline{2}$), (b) triplication of $\overline{5}$ through duplication of $\overline{5}$ by means of the so-called 2 ÷ n Table,[383] which yields $\overline{3}\ \overline{15}$, and subsequent addition of another $\overline{5}$ (hence, 3 × $\overline{5}$ = $\overline{3}\ \overline{5}\ \overline{15}$), and (c) triplication of $\overline{60}\ \overline{120}\ \overline{240}$ with the same method applied in the previous paragraph to **S5**, which yields $(\overline{30}\ \overline{60})\ (\overline{60}\ \overline{120})\ (\overline{120}\ \overline{240})$. The triplicate of 6 $\overline{2}\ \overline{5}\ \overline{60}\ \overline{120}$ $\overline{240}$ (**S4**) would then be 19 $\overline{2}\ \overline{3}\ \overline{5}\ \overline{15}\ \overline{30}\ \overline{60}\ \overline{60}\ \overline{120}\ \overline{120}\ \overline{240}$ = 19 $\overline{2}\ \overline{3}\ \overline{5}\ \overline{15}\ \overline{15}\ \overline{60}\ \overline{240}$. The sum of parts that follows the integer 19 is clearly larger than 1 (because $\overline{3}\ \overline{6} = \overline{2}$ and $\overline{5} > \overline{6}$), therefore we shall make use of the equation $\overline{6}\ \overline{30} = \overline{5}$ in order to obtain 19 $(\overline{2}\ \overline{3}\ \overline{6})\ \overline{15}\ \overline{15}\ \overline{30}$ $\overline{60}\ \overline{240}$ = 20 $\overline{15}\ \overline{15}\ \overline{30}\ \overline{60}\ \overline{240}$. Since twice $\overline{15}$ cannot stand, we shall further replace $\overline{15}\ \overline{30}$ with $\overline{10}$[384] and then $\overline{15}\ \overline{10}$ with $\overline{6}$.[385] Eventually, someone versed in the decomposition of 240 (as I see Galen) should know that $\overline{60}\ \overline{240} = \overline{48}$,[386] which leaves us with the result 20 $\overline{6}$ $\overline{48}$. For two reasons, however, I doubt that Galen pursued this method: He had earlier in **S4** not taken the opportunity to replace $\overline{60}\ \overline{240}$ with $\overline{48}$ (so why should he here?) and, more importantly, the previously explained triplication of 6 $\overline{2}\ \overline{6}\ \overline{24}\ \overline{48}$ (**S5**) is far less complicated and leads directly to 20 $\overline{6}\ \overline{48}$.

A noteworthy third method to execute the required triplication, which again yields 20 $\overline{6}$ $\overline{48}$ days, will be explained below.[387]

Question 6

It is now clear that the most plausible expectation regarding the missing fraction in **S7** is a single part, namely $\overline{48}$. Since the Greek manuscript tradition mentions only 20 $\overline{6}$ days, it remains to ask what most likely happened: that Galen made a mathematical mistake, did not bother to specify the part $\overline{48}$, or that he did specify it but it was lost over the course of textual transmission. In view of Galen's mathematical competence, his extreme precision on other occasions,[388] and his (on the present occasion) emphasis on having calculated accurately (κατὰ τοῦτον τὸν τρόπον ἀκριβολογουμένοις) which immediately follows, it seems extremely unlikely that he should have miscalculated the present value or intentionally omitted the part $\overline{48}$.

Maybe someone would argue that the particle τινὸς attached to μορίου (p. 933.8 K.) indicates an approximation instead of a precise figure,[389] but this would seem strained because Galen had employed a similar modifier for numerical values (μάλιστα) twice before

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[383] Cf. n. 364 above.
[384] EMLR col. II, lin. 7 = col. IV, lin. 5.
[385] I have no attestation for this readily available.
[386] See my explanation above, p. 256 (Question 4).
[387] See Question 7, p. 258 below.
[388] See above on Hipparchus (n. 326).
[389] Cf. LSJ s.v. τις, τι A.II.8 "with numerals and adjectives expressing number, size, or the like [...], sometimes the τις softens the definiteness of the numeral [...], so without an actual numeral [...]," one example being "ἐνιαυτόν τινα a year or so Thuc. 3.68." It is difficult to find parallels with τις attached to fractions in Galen's work. A *prima vista* similar though ultimately different case is in Gal. *in Hipp. progn. comm.* 3 vol. XVIIIb p. 240.4–6 K.: ὁ μὲν γὰρ ἐνιαυτὸς οὐ τριακοσίων ἑξήκοντα πέντε μόνον ἡμερῶν ἐστιν, ἀλλὰ καὶ τετάρτου μέρους ἡμέρας, ἔτι δὲ πρὸς αὐτῷ μορίου τινὸς ἐγγύς πως ἑκατοστοῦ ("For the year does not comprise only 365 days but also a fourth of a day plus a certain part that is somehow near a one-hundredth").

in our passage (pp. 932.9 and 932.13 K.), and the explanation given for those cases also applies here.[390]

Another strained justification of the transmitted Greek text would be to emphasize Galen's interest in minimizing the excess of three medical weeks over twenty full days. His implicit goal is to arrive at a figure that is clearly smaller than $20\,\overline{2}$ days.[391] The invention of a medical week was, in his view, necessary for the very reason that the sidereal month alone, with its mean length of $27\,\overline{3}$ days, leads to weeks of '$7 - \overline{6}$' (that is, $6\,\frac{5}{6}$) days each[392] and to a three-week-span of $20\,\overline{2}$ days, an ambiguous value that is equally close to twenty and twenty one full days.[393] $20\,\overline{6}\,\overline{48}$ days, however, is a thoroughly satisfactory value.

If, then, the part $\overline{48}$ was in Galen's original text (**S7**) and went missing in the course of transmission, one may hypothesize with all due caution that this happened during the copying of a manuscript in which this part was expressed as καὶ μη′: The letter η is often misread as καὶ in Greek manuscripts, and vice versa;[394] hence, a *saut du même au même* (that is, from καὶ to η) would lead to ἕκτου καὶ ὥστε. Note that the καὶ after ὥστε does not make sense in the context: Galen has developed only one mathematically accurate line of thought, not several, and it is only now, with **S7**, that this line of thought is complete. It is therefore not surprising that most translators omit the καὶ.[395] Hence, a later scribe who came across the words ἕκτου καὶ ὥστε κατὰ τοῦτον τὸν τρόπον may have inverted the sequence of καὶ and ὥστε. I do not, of course, claim that this must have happened (there are many cases of abundant καὶ in Greek manuscripts), but it seems a plausible conjecture, and since the καὶ after ὥστε is suspicious anyway, it is here athetized.

Question 7

Nathan Sidoli raised the pertinent question of why Galen bothered to calculate the seemingly superfluous steps **S3** and **S6**. In the case of **S6**, he was able to provide a convincing answer:[396] It is possible that Galen made this step so that he could apply the method of

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[390] Cf. p. 241 above (Question 1).

[391] Yet it must have a fraction, because any other result would contradict Hippocrates, whose true teaching Galen claims to have confirmed with his computation (p. 933.12–17 K.): ἐμοὶ μὲν δέδεικται κατά τε τὸ ἀληθὲς αὐτὸ καὶ κατὰ τὴν Ἱπποκράτους γνώμην, ὅτι μηδὲν τούτων ὅλησιν ἡμέρησιν ἀτρεκέως ἀριθμεῖσθαι δύναται, τουτέστιν ἀπηκριβωμένως τε καὶ ἀπηρτισμένως, οὔθ' ἡ ἑβδομὰς οὔθ' ἡ τετρὰς οὔθ' ὁ μὴν οὔθ' ὁ ἐνιαυτὸς οὔτ' ἄλλο τῶν ἁπάντων οὐδέν. Galen's reference is to Hippocr. *progn*. 20 vol. II pp. 168.16–170.2 Littré: Οὐ δύναται δὲ ὅλησιν ἡμέρησιν οὐδὲν τουτέων ἀριθμέεσθαι ἀτρεκέως· οὐδὲ γὰρ ὁ ἐνιαυτός τε καὶ οἱ μῆνες ὅλησιν ἡμέρησι πεφύκασιν ἀριθμέεσθαι.

[392] See p. 929.14–15 K. (the Greek is quoted in n. 57 above).

[393] Galen clearly says that ibid. p. 929.8–12 K.: συμβαίνει τοιγαροῦν οὐ μιᾶς καὶ εἴκοσιν ἡμερῶν γίνεσθαι τὰς τρεῖς ἑβδομάδας ὁλοκλήρων, ἀλλὰ ἀπολείπεσθαι σχεδὸν ὅλῳ τῷ ἡμίσει μέρει τῆς μιᾶς ἡμέρας καὶ διὰ τοῦτο ἐπαμφοτερίζειν τὸν ἀριθμόν, οὐ μᾶλλόν τι τῶν εἴκοσιν ἐγγὺς ἢ τῶν ἑνὸς καὶ εἴκοσιν ὑπάρχοντα.

[394] The most detailed description of this phenomenon is, to my knowledge, in Bast's paleographical commentary in Schaefer 1811, 815 (cf. ibid. plate II, n° 5).

[395] Thus Winter (**t3**) as well as his epigones Lalamant (**t4**), Chartier (**p**), and Kühn (**l**). As expected, Niccolò da Reggio (**t2**) translates verbatim (*et*). The Arabic translation (**h**) changes the syntax by starting a new period ("If we calculate ...," Cooper 2011a, 374); its offspring **t1** has *Et cum nos etc.* Cf. n. 70 above.

[396] I paraphrase his personal communication.

complementing up to 1 explained above (Question 4, on **S3**). This indirect method would be simpler, in his opinion, than the direct methods of triplication envisaged by me above (Question 5). Galen would then have $3 \times 7 = 21$, from which we take away $3 \times (\overline{4}\ \overline{60}\ \overline{240})$ $= \overline{2}\ \overline{4}\ \overline{30}\ \overline{60}\ \overline{120}\ \overline{240}$. Then, noting that $\overline{30}\ \overline{60} = \overline{20}$,[397] we have to find the complement between $\overline{2}\ \overline{4}\ \overline{20}\ \overline{120}\ \overline{240}$ and 1. We have

$\overline{2}$	$\overline{4}$	$\overline{20}$	$\overline{120}$	$\overline{240}$
aux. numb.: 120	60	12	2	1

which sums to 195, with a remainder (up to 240) of 45. So, we divide 45 by 240 as follows[398]

1	240
/ $\overline{6}$	40
/ $\overline{48}$	5
sum:	45

summing 40 and 5 to give $\overline{6}\ \overline{48}$ as the answer. While Sidoli's explanation is certainly illuminating, I wonder if it really simplifies the computation. I should therefore like to propose another possible *raison d'être* of **S6** in Galen's line of thought: It may serve the purpose of verification of the directly obtained result of **S7** by way of an indirect computation as described by Sidoli. As a matter of fact, they both lead to the same sum of parts that quantifies the excess over 20 full days, namely $\overline{6}\ \overline{48}$.[399]

In the case of **S3**, I should like to propose two different explanations: One is again verification (of either **S4** or **S5** or both **S4** and **S5**), the other clarification, and they may both be relevant here.

Clarification would make sense because the reader who wishes to get an idea of the length of Galen's new medical month will find it easier to imagine one twelfth missing from 27 full days than to imagine $26\ \overline{2}\ \overline{3}\ \overline{12}$ days.

As to verification, Galen may have proceeded thus: $27 \div 4 = 6\ \overline{2}\ \overline{4}$ (this step is rather trivial). From this interim result we must subtract one fourth of the difference between 27 and $26\ \overline{2}\ \overline{3}\ \overline{12}$, that is, one fourth of $\overline{12}$, that is, $\overline{48}$, in order to find the length of one medical week. In order to compute $\overline{2}\ \overline{4} - \overline{48}$, we shall once more employ the Egyptian method of subtraction of fractions (Question 4, on **S3**), with one additional step: We shall first complement $\overline{48}$ up to 1 in such a way as to obtain a sum of parts comprising $\overline{4}$ and then subtract $\overline{4}$ by eliminating this part from the resulting sum of parts (because what we actually need is only complementation of $\overline{48}$ up to $\overline{2}\ \overline{4}$).[400] For the complementation, we have

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[397] EMLR, col. II, lin. 6 (see above, n. 352).

[398] Or selecting the same lines from col. VIIIa above (p. 254).

[399] Verifications occur already in P.Rhind, cf. Imhausen 2016a, 94.

[400] Cf. a similar problem in P.Rhind no. 23, namely to complement $\overline{4}\ \overline{8}\ \overline{10}\ \overline{30}\ \overline{45}$ up to two thirds (Peet 1923, 59–60; cf. Gillings 1972, 85–86). Robins and Shute 1987, 21, presume that this problem was created from a

$\overline{48}$

aux. numb.:   1   sum: 1, remainder: 47

Hence, we divide 47 by 48 based on systematic decomposition of 48:

1	48
/ $\overline{2}$	24
$\overline{3}$	16
/ $\overline{4}$	12
/ $\overline{6}$	8
$\overline{8}$	6
$\overline{12}$	4
$\overline{16}$	3
/ $\overline{24}$	2
/ $\overline{48}$	1
sum:	47

In the course of moving down through this list from top to bottom we shall, in the usual interest of optimized approximation, always select the largest possible part, with one exception: We shall purposely skip $\overline{3}$ because we keep in mind that we wish to subtract $\overline{4}$ in the next step; that is, our sum of parts must contain $\overline{4}$. This procedure leads to $\overline{2}\,\overline{4}\,\overline{6}\,\overline{24}\,\overline{48}$. Now we subtract $\overline{4}$; the result is $\overline{2}\,\overline{6}\,\overline{24}\,\overline{48}$, and it is exactly what Galen gives us in **S5**. Obviously, this could serve as a successful verification of **S5** only if the same sum of parts had, in a previous computation, been obtained, as hypothesized above (Question 4, on **S5**).

In order to verify the result of the previous step (**S4**), whose smallest part is $\overline{240}$, we return to the above complementation, but this time we will not divide 47 by 48 but 5×47 by 5×48, that is, 235 by 240.[401] This is easy thanks to our already existing systematic decomposition of 240 (col. VIIIa, p. 254 above), where we shall comfortably check whether it is possible to reach the sum 235 in the right sub-column by selecting $\overline{2}$, $\overline{4}$ (!), $\overline{5}$, $\overline{60}$, $\overline{120}$, and $\overline{240}$ in the left sub-column (we must include $\overline{4}$ because this part will, as before, be subtracted in the last step). It is possible.

Hence, since the results of both **S4** and **S5** can without too much trouble be verified by means of the result of **S3** (which has the additional merit of clarifying the length of the medical month), and since **S7** can be verified by means of **S6**, we have a satisfactory explanation for why Galen engaged in **S3** and **S6**.

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precursor in which the task was complementation of $\overline{3}\,\overline{4}\,\overline{8}\,\overline{10}\,\overline{30}\,\overline{45}$ up to 1, and that $\overline{3}$ was subtracted from both values so as to obtain problem no. 23 mentioned above. In our case the procedure would be the opposite, that is, from complementation up to a (sum of) part(s) to complementation up to 1.

[401] Cf. p. 250 above on ancient uses of the equation $a \div b = na \div nb$.

Question 8

The question of why Galen provided an alternative sum of parts (**S5**) was raised in passing by Cooper, but without a satifactory answer.[402] I shall focus on those parts which actually differ between **S4** ($\overline{6}\,\overline{24}\,\overline{48}$) and **S5** ($\overline{5}\,\overline{60}\,\overline{120}\,\overline{240}$), those which I have earlier interpreted as Galen's solutions to the problem of dividing 11 by 48.[403] We have seen that various alternative sums of parts were possible.[404] However, those were just a very few examples. The truth is that if one allows $\overline{240}$ to be the smallest part, systematic exploration of all possible combinations of the numerous divisors of the highly composite number 240 leads to the insight that there are no less than 173 (!) different sums of parts which all equal the fraction in question (in modern notation: $^{11}/_{48}$), ranging from the 'best' possible sum ($\overline{5}\,\overline{40}\,\overline{240}$) to the 'worst' ($\overline{16}\,\overline{20}\,\overline{30}\,\overline{40}\,\overline{48}\,\overline{60}\,\overline{80}\,\overline{120}$).[405] Even if Galen was probably unaware of the existence of *so many* alternatives, he must have sensed that there were dozens of alternatives, that is, many more than the one which he actually gives ($\overline{6}\,\overline{24}\,\overline{48}$). Is this just a randomly chosen alternative or the chance product of this or that method of computation? This question will be easier to answer if we first ask whether the alternative sum of parts $\overline{6}\,\overline{24}\,\overline{48}$ has anything special to it, so special as to compensate for the obvious disadvantage that already its first part ($\overline{6}$) thwarts the typical goal of approximating the overall fraction in question (in modern notation: $^{11}/_{48}$) as closely as possible.[406] In order to satisfy that goal, the first part ought to have been $\overline{5}$, a prerequisite that allows for four different solutions, as shown above.[407] Since Galen did not mention one of the three alternative solutions beginning with $\overline{5}$, but one of the fourteen solutions beginning with $\overline{6}$,[408] it is all the more legitimate to ask again: why just one alternative, and why precisely this one, $\overline{6}\,\overline{24}\,\overline{48}$? I think its value lies in the fact that its parts are respectively the double, one half, and one fourth of $\overline{12}$. This fact makes Galen's alternative solution particularly suitable for the cultural environment in which he was writing *On Critical Days*: This work originated in 175 or 176

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[402] Cf. Cooper 2011, 372, n. 1130: "Galen feels the need to give another form of this number, perhaps because the first is not as convenient to deal with as a slightly simpler form. I doubt that he is merely showing off his facility with arithmetic."

[403] See p. 250 above.

[404] See pp. 250–254, solutions II, III, IV, VI, VII and especially VIII (regarding **S4** and **S5**).

[405] The interested reader can verify this assertion with nothing but a pen and paper if he or she is willing to spend a few leisurely hours on the list of divisors given on p. 254 (example VIIIa). It is important to keep in mind that each term of the left sub-column can be selected only once (that is, duplicates of one and the same part in one sum of parts are inadmissible). My qualifiers 'best' and 'worst' refer to the above definition of 'optimized approximation.' There are four solutions starting with $\overline{5}$, fourteen starting with $\overline{6}$, thirty-five starting with $\overline{8}$, forty-one starting with $\overline{10}$, forty-five starting with $\overline{12}$, twenty-six starting with $\overline{15}$, and eight starting with $\overline{16}$. I should like to add that my initial thought that the present mathematical problem might belong to the field of enumerative combinatorics was kindly corrected by Dr. Martin J. Sauer (University of Münster): Combinatorics has no techniques for solving this kind of problem. As to modern mathematical research on sums of unit fractions which is ultimately inspired by, but mostly not related to ancient Egyptian texts, cf. Guy 1981, 87–93 (section D 11: *Egyptian fractions*; with extensive bibliography), as well as Goldstein 1992, 381.

[406] This disadvantage seems to account for the relegation of this possibility to the second position in Galen's line of thought.

[407] Cf. p. 254 with note 374.

[408] Cf. n. 405 above.

CE[409] when Galen had permanently settled in Rome[410] and was personal physician to emperor Marcus Aurelius (emperor 161–180 CE).[411]

Roman fractions, like their Greek counterparts, are conceived of as proper parts.[412] More importantly, they are typically duodecimal and have survived thus through the end of antiquity.[413] Hoppe thinks this preference for duodecimal fractions, whose most obvious witness is the Roman term *uncia* for one twelfth of any twelve-part whole, is based not on any number theory, which the Romans never developed, but on the natural division of the year into twelve months.[414] The paramount importance of *uncia* as a unit of measurement (length, area, capacity, time, weight) and coinage in Roman life is illustrated by the vast spectrum of its dependent units: its multiples are *sextans* (the sixth part of an *as* or two *unciae*), *quadrans* (three *unciae*), *triens* (four *unciae*), *quincunx* (five *unciae*), *semis* (six *unciae*), *septunx* (seven *unciae*), *bes* (eight *unciae*), *dodrans* (nine *unciae*), *dextans* (ten *unciae*) and *deunx* (eleven *unciae*), its fractions are *semuncia* (half an *uncia*), *sicilicus* (the fourth part of an *uncia*), *sextula* (the sixth part of an *uncia*), *scripulum* (the twenty-fourth part of an *uncia*) and *siliqua* (the one-hundred-forty-fourth part of an *uncia*).[415] More importantly, "the Roman *abacus* was based on an uncial division of whole numbers."[416]

As a unit of time, we find *uncia* applied to the hour, but almost exclusively in astronomical contexts. Most of the attestations are in Pliny's *Natural history* who twice emphasizes that he is not talking about the seasonal hours of everyday life.[417] He mentions the following duodecimal fractions of one equinoctial hour (note that in each case there is one fraction that he could have converted to smaller terms but did not, obviously motivated by his typically Roman preference for *unciae*): Plin. nat. 2.58 *dodrantes semuncias horarum* (= $9 \times \overline{12} + \overline{24}$ h);[418] 2.186 *octo partes unius horae* (= $8 \times \overline{12}$ h);[419] 18.324–325 *horae noctis unius dextante sicilico ... primae horae nocturnae dextantem sicilicum* (both expressions mean $10 \times \overline{12} + \overline{48}$ h).[420] In these passages, Pliny appears to have adapted fractional

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[409] Bardong 1942, 636–637; repeated by Garofalo 2003, 49.

[410] He lived there from 162 to 166 CE and from 169 CE until his death about 214 CE.

[411] Cf. Schlange-Schöningen 2003, 206–207, who also quotes passages from *On Critical Days* in which Galen reports medical experiences he had in Rome.

[412] Maher and Makowski 2001 argue, in their analysis of the Roman literary evidence (and without knowledge of the Greek-centered debate reported on pp. 245–246 above), that truly general (that is, common) fractions can be found in the work of one Latin author, namely Frontinus, but on closer inspection those passages do not fulfill the criteria established by Fowler (cf. p. 246 with n. 344 above). They rather turn out to be of the same kind described by Sidoli in n. 343 above.

[413] Cf. Hoppe 1911, 361: "Die Brüche sind duodezimal und haben sich so bis in die späteste Zeit erhalten. Ein Zwölftel und die Hälfte eines Zwölftels sind hier die normalen Werte."

[414] Ibid., 361–362. Hoppe does not speak of the respectively twelve hours of day and night, maybe because these became an important part of Roman everyday life much later than the twelve months.

[415] Stumpf 2006; cf. the more copious data in the table of Bouché-Leclercq 1886, 567. One *uncia* is, in its turn, the twelfth part of one Roman *libra*.

[416] Stumpf ibid., who adds: "Θ was the symbol for an *uncia*. Parts of an estate, of debts, of prizes or of property were expressed in *unciae*. *Unciarius heres* denoted an heir with an $^1/_{12}$ entitlement."

[417] Plin. nat. 2.213 and 18.220.

[418] In modern notation: $^9/_{12} \, ^1/_{24}$ h = $^{19}/_{24}$ h. Cf. the explanation given by Neugebauer 1975, 830–831.

[419] In modern notation: $^8/_{12}$ h = $^2/_3$ h.

[420] In modern notation: $^{10}/_{12} \, ^1/_{48}$ h = $^{41}/_{48}$ h. On this last, astronomically wrong figure, cf. Neugebauer 1975, 830. Pliny further mentions a few non-duodecimal fractions: 6.213 *accedente bis quinta parte unius horae* (= $2 \times \overline{5}$ h, that is, $^2/_5$ h); 6.214 *dimidiae* (scil. *horae*) *cum tricesima* (scil. *parte*) *unius horae* (= $\overline{2} \, \overline{30}$ h, that is, $^1/_2 \, ^1/_{30}$ h =

values found in his Greek sources to the metrical habits of Roman readers, and thereby to have preceded Galen in his explanatory method in **S5**.

Outside the field of astronomical literature, we find *uncia* as a unit of time on an interesting gravestone indicating the life span of a child as 6 years, 8 months and 7 $\overline{12}$ hours.[421]

Hence, Roman readers would probably interpret Galen's $\overline{6}\,\overline{24}\,\overline{48}$ days as one *sextans* plus one *semuncia* plus one *sicilicus* of a full day (that is, in the arithmetic of parts: 2 $\overline{2}\,\overline{4}$ *unciae*). And Galen himself was, of course, aware of the importance of *unciae* in his host culture. His writings abound with indications of pharmaceutical quantities measured in οὐγγίαι, the Greek transliteration of *unciae*,[422] and he also refers about ten times to half *unciae* (ἡμίσεια οὐγγία or οὐγγίας ἥμισυ), that is, twenty-fourths.[423]

Not only does Galen's alternative $\overline{6}\,\overline{24}\,\overline{48}$ (that is, the last three parts in the result of **S5**) perfectly match the Roman preference for duodecimal fractions; it is also the *only* solution with these qualities among all fourteen mathematically possible sums of parts beginning with $\overline{6}$,[424] which are—from 'best' to 'worst' in terms of optimized approximation—$\overline{6}\,\overline{16}$, $\overline{6}\,\overline{20}\,\overline{80}$, $\overline{6}\,\overline{20}\,\overline{120}\,\overline{240}$, $\overline{6}\,\overline{24}\,\overline{48}$ (Galen's choice), $\overline{6}\,\overline{24}\,\overline{60}\,\overline{240}$, $\overline{6}\,\overline{24}\,\overline{80}\,\overline{120}$, $\overline{6}\,\overline{30}\,\overline{40}\,\overline{240}$, $\overline{6}\,\overline{30}\,\overline{48}\,\overline{120}$, $\overline{6}\,\overline{30}\,\overline{60}\,\overline{80}$, $\overline{6}\,\overline{30}\,\overline{60}\,\overline{120}\,\overline{240}$, $\overline{6}\,\overline{40}\,\overline{48}\,\overline{60}$, $\overline{6}\,\overline{40}\,\overline{48}\,\overline{80}\,\overline{240}$, $\overline{6}\,\overline{40}\,\overline{60}\,\overline{80}\,\overline{120}$, and $\overline{6}\,\overline{48}\,\overline{60}\,\overline{80}\,\overline{120}\,\overline{240}$. If one tried to improve on Galen's choice by starting with $\overline{12}$ (one *uncia*) instead of $\overline{6}$ and continuing with $\overline{24}$, the only two possible solutions would be $\overline{12}\,\overline{24}\,\overline{30}\,\overline{40}\,\overline{48}\,\overline{60}\,\overline{120}$ and $\overline{12}\,\overline{24}\,\overline{30}\,\overline{40}\,\overline{48}\,\overline{80}\,\overline{240}$, both of them obviously unsatisfactory from a duodecimal point of view. Thus it appears that Galen's alternative is, among all 173 mathematically possible sums of parts based on the list of divisors of 240,[425] the only one that neatly complies with the Roman preference for duodecimal fractions. This suggests that Galen's alternative in **S5** is not a randomly discovered solution (that would be a serendipitous find!) but the result of a systematic examination of that list of divisors of 240 with the goal of finding the best possible 'duodecimal' sum of parts.

Question 9

The last words of the transmitted Greek text (καὶ ταύτης – οἰκεῖος) do not make satisfactory sense because it is not, strictly speaking, the three weeks' time span (ὁ τῶν τριῶν ἑβδομάδων ἀριθμός) which had just been calculated, namely 20 $\overline{6}$ <$\overline{48}$> days, that belongs to either the 20th or the 21st day. Cooper interprets Galen's ἀριθμός as meaning not the time span itself but its end point, but this approach does not solve the problem: "Still, 20 $\frac{9}{48}$ days technically falls on the twenty-first day. One is left to wonder how Galen

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[16]/₃₀ h = ⁸⁄₁₅ h); 6.215 *tertias duas unius horae* (= 2 x $\overline{3}$ h, that is, ²⁄₃ h); 6.217 *addita VIIII parte unius horae aut, ut Nigidio placuit, quinta* (= $\overline{9}$ h or $\overline{5}$ h, that is, ⅑ h or ⅕ h); 6.218 *quintarum partium horae trium* (= 3 × $\overline{5}$ h, that is, ³⁄₅ h).

[421] Chantraine 1961, 663.13–18.

[422] There are 355 attestations of οὐγγία, five of the variant spelling οὐγκία, and 1,008 of the abbreviation γο in the Galenic corpus (TLG).

[423] Cf. Hoppe's words quoted in n. 413 above.

[424] Cf. p. 261 with n. 405 above.

[425] Cf. p. 254 above (example VIII) and, for the figure of 173 solutions, p. 261 above.

would explain that."[426] No scholar to the present seems to have thought of the following conjecture: καὶ ταύτης <ἡ κρίσις> ἔσται πολὺ μᾶλλον ἢ τῆς εἰκοστῆς πρώτης οἰκεία. Several arguments speak in its favor: (a) Ḥunayn ibn Isḥāq translated thus,[427] which means that he either found ἡ κρίσις in his Greek exemplar or made the same conjecture based on his excellent knowledge of Galen's thinking;[428] (b) there is no Galenic parallel that could support the transmitted text;[429] (c) there are Galenic parallels that support the present conjecture, especially in *On Critical Days*, namely ch. 1.8 p. 810.9–11 K.: ταῖς μὲν περιτταῖς ἡμέραις μᾶλλον πονοῦντος τοῦ κάμνοντος οἰκειοτέρα τῆς περιττῆς ἡμέρας ἡ κρίσις, ὥσπερ εἰ καὶ ταῖς ἀρτίαις, ἐκείνης ἴδιος ('if the patient suffers more on the uneven days, the crisis belongs rather to the uneven day, just as if he suffers more on the even days, it belongs to the even day'), and ch. 2.2 p. 843.3–5 K.: ἐπειδὰν εἰς πλείονας ἡμέρας ἐφεξῆς ἀλλήλων οὔσας ἡ κρίσις μερισθῇ, τίνος ἐστὶν οἰκεία δύνασθαι διαγινώσκειν ('to be able to discern, whenever the crisis is distributed over several consecutive days, to which day it belongs'); (d) if the words ἡ κρίσις followed ταύτης, their loss may have been caused by *saut du même au même* (that is, from -ης to -ις, pronounced the same by Byzantine scribes);[430] a later copyist would then have 'corrected' οἰκεία to οἰκεῖος in order to restore proper grammar.

On the other hand, Galen has developed his computation without reference to a crisis (the last occurrence of the term κρίσις had been four pages earlier, at p. 930.1 K.). Hence, if the term had been mentioned here with the definite article, it would have come quite abruptly. Therefore it seems better to keep the transmitted text, to assume that Galen phrased this thought somewhat loosely, and that it was Ḥunayn ibn Isḥāq who sharpened it by introducing the technical term for 'crisis.'

But even if we concede that Galen used a somewhat vague expression, it is legitimate to expect the man who claimed to practice 'acribology'[431] to have had a precise and mathematically correct concept in mind, one he may not have expressed because he considered it too challenging for his average reader.[432] This concept is easy to detect: Since he defines an artificial 'medical month' and divides this into four equally artificial 'medical weeks,' each such week must consist of seven equally artificial 'medical days.' We thus obtain, besides the unit of the natural 24-h-day mentioned several times by Galen in the text, a somewhat shorter 'medical day.' This insight is attested in several texts surveyed above, starting with the anonymous author of the *Aggregationes*[433] and Niccolò da Reggio, the first to correctly specify the medical day's length as 23 $\frac{1}{14}$ hours (that is, natural equinoc-

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[426] Cooper 2011a, 496.

[427] Cf. Cooper 2011a, 374: "the crisis is much more suited to the twentieth day" (cf. **t1** *et fit crisis dignior die .xx.*).

[428] Cf. Strohmaier 1993, 158: "Ḥunayn verrät in der Regel ein perfektes Verständnis der Galenischen Denkweise, was ihn auch zur Heilung von Korruptelen befähigt hat."

[429] The only parallel for ἀριθμ- near οἰκεῖ- (maximum distance 15 words) in the whole Galenic corpus is *De usu partium* vol. IV p. 44.8–9 K., a passage that has nothing to do with medical crises.

[430] For the word order, cf. also *Ad Glauconem de medendi methodo* vol. XI p. 66.9 K. σκοπεῖν, ὁποία τις ἡ κρίσις ἔσται ('to examine of what kind the crisis will be').

[431] Cf. Section V above.

[432] Cf. his words at p. 934.1–9 K., quoted in n. 305 above.

[433] See n. 149 above (the exact length of the medical day is not specified there).

tial hours),[434] and ending with Magini who first tabularizes the end points of all 28 medical days within a medical month.[435]

It now becomes clear what Galen would have written if he had expressed himself more precisely: that the last medical day of the third medical week ends after $20\,\overline{6}\,\overline{48}$ natural days and belongs, therefore, mostly to the twentieth natural day. This is true: Since $\frac{1}{6}$ plus $\frac{1}{48}$ of a natural day equal $4\,\frac{1}{2}$ natural hours, we must subtract this figure from the medical day's length ($23\,\frac{1}{14}$ h, cf. n. 434) in order to find out which part of the latter overlaps with the twentieth natural day, and we find that this part comprises $18\,\frac{4}{7}$ natural hours ($23\,\frac{1}{14} - 4\,\frac{1}{2} = 18\,\frac{8}{14}$).[436]

The superiority of this explanation of the transmitted text compared to the conjectural addition of <ἡ κρίσις> is evident in another respect, too: While a crisis on the 21st medical day can, of course, extend over parts of both the 20th and the 21st natural days,[437] it does not have to. If the crisis sets in and comes to its end within the first $18\,\frac{4}{7}$ natural hours of the 21st medical day, it belongs entirely to the twentieth natural day. In such a case the crisis would not belong 'much more' (πολὺ μᾶλλον, p. 933.11 K.) but *entirely* to the 20th day. It is only with respect to the extension of the whole 21st medical day, not to the potential crisis within this time span, that Galen's assertion is true without any restriction. In sum, we find that the vagueness in his expression results from a metonymy: He has replaced the last medical day of three medical weeks with the entire time span of three medical weeks whose length he had just computed.

Question 10

Our last question is whether Galen's computational precision was necessary to achieve his goal. His goal was to provide a rational explanation for why three medical weeks are less than $20\,\overline{2}$ yet more than 20 natural days.[438] Hence, would it not have been sufficient simply to argue that the result of **S1**–**S2** is 'about 27' days (instead of $26\,\overline{2}\,\overline{3}\,\overline{12}$, that is $26\,\frac{11}{12}$ days), especially since Galen openly admits the inaccuracy of his initial parameters regarding the respective lengths of the synodic month, the sidereal month, and the period of lunar invisibility?[439] The result of **S4** would then have been $6\,\overline{2}\,\overline{4}$ days (= $6\,\frac{3}{4} = 6\,\frac{36}{48}$, instead of $6\,\overline{2}\,\overline{5}\,\overline{60}\,\overline{120}\,\overline{240} = 6\,\frac{35}{48}$; that is, the imprecision would be limited to $\frac{1}{48}$ d = 30

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[434] Cf. the Latin quotation in n. 176 above. Niccolò probably calculated thus: 6d $17\frac{1}{2}$h (this is value H1b in Table 1 above) $\div 7 = 161\frac{1}{2}$h $\div 7 = 23$h ($7\times23 = 161$) plus the seventh part of half an hour. The result is, of course, identical with what Galen would have calculated, namely $6\,\overline{2}\,\overline{5}\,\overline{60}\,\overline{120}\,\overline{240}$ (**S4**) $\div 7$ or $6\,\overline{2}\,\overline{6}\,\overline{24}\,\overline{48}$ (**S5**) $\div 7$ or $26\,\overline{2}\,\overline{3}\,\overline{12}$ (**S2**) $\div 28$. It goes without saying that every medical day has, in its turn, 24 medical hours which are each a bit shorter than one natural equinoctial hour.

[435] See Fig. 1 above.

[436] I use modern notation of fractions with a separating bar (both unit fractions and common fractions) to express mathematical reasonings by late medieval, early modern, and modern authors (that is, including myself) to avoid anachronisms, while continuing the Egyptianizing notation of fractions as parts when reporting Galen's or other ancient authors' math.

[437] Cf. Galen's last passage just quoted from p. 843.3–5 K.

[438] Cf. notes 391 and 393 above. The reader may also wish to remember that any figure smaller than 20 days would have had the consequence that the 21st day is *never* critical, but that is not what Galen argues.

[439] Cf. the passages quoted on p. 241 above (Question 1).

min), and the result of **S7** would have been $20\,\overline{4}$ days. This alternative computation would have required no more than two extremely simple fractional parts ($\overline{2}$ and $\overline{4}$) to express the indispensable results of **S4** and **S7**, and the final result (**S7**: $20\,\overline{4}$ days) would have been thoroughly satisfactory because it falls right in the middle between 20 and $20\,\overline{2}$ days. Admittedly, Galen's actual result of **S7**, $20\,\overline{6}\,\langle\overline{48}\rangle$ days, is slightly preferable because it is one sixteenth day (that is, 90 min.) smaller than $20\,\overline{4}$ days. But was this small advantage worth the effort of computing with sums of up to five parts of which the tiniest ones had no place in everyday life and were therefore likely to cause problems to the average reader? Common sense suggests the answer ought to be 'no' because it makes little sense to pursue a negligible advantage at significant cost. Hence, Galen must have had a different motivation. I am inclined to believe that he saw this as an opportunity to display his mathematical prowess, with an implicit claim for scholarly superiority over the majority of contemporary (mostly rival) physicians, whom he could thus tacitly put in their place. Moreover, he may well have pursued a second goal, namely that his indisputable mathematical precision and accuracy, which he emphasized with a strong dose of rhetoric (cf. Section V above), would more easily convince his readers of the appropriateness and truth of his entire new explanation of the critical days, which is actually highly speculative.[440]

VII. Conclusions

Galen's computation of medical weeks is an interesting example of a mathematically educated author who is not himself a professional mathematician, doing some real mathematics of the sort that dominates in the papyri, but of which there is relatively little evidence in the texts of philosophically trained mathematicians.[441]

In view of Galen's solid mathematical and astronomical education, his demonstration of such knowledge in various writings, and his specific Platonizing rhetoric of accuracy[442] in the passage that has been the object of our scrutiny (ch. 3.9 pp. 932.5–933.12 K.), his original text must have contained a flawless computation. The omission of $\overline{48}$ ($\frac{1}{48}$) in **S7** in the Greek manuscripts[443] must be a loss that occurred in the course of the textual transmission, which also led to corruptions of other numerical values in some (not all) manuscripts. The first critical edition of *On Critical Days*, which is still a *desideratum*, ought to take two important witnesses into account, which appear to have been overlooked by all scholars who wrote on this Galenic work over the last two centuries: MS Vat. Barb. gr. 221 (**B**, 15th/16th century) and the verbatim translation by Niccolò da Reggio (14th century), which faithfully reflects the Greek text of an old, apparently lost manuscript of high quality.

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[440] Cf. Pico della Mirandola's devastating philosophical criticism (cf. p. 224 above), for example, his appropriate argument that even if it were taken for granted that the moon exercises astrological influences at certain moments of its synodic month and at certain other moments of its sidereal month, it would be absurd to infer from this premise that the moon exercises influences also at those times which are but the mathematical averages of the aforementioned moments.

[441] I owe this judgment to Sidoli (cf. n. 312 above).

[442] See Section V above.

[443] We must wait for one of the two critical editions that have been announced by Alexanderson and Cooper to be sure that this fraction is really missing from all extant manuscripts.

Galen employed a range of strategies to present his computation in a reader-friendly manner: (1) he framed it with rhetorical elements of *captatio benevolentiae*; (2) he expressed the single numerical values neither in equinoctial hours (though he undoubtedly mastered them)[444] nor in common fractions of days,[445] because knowledge of the former was mostly limited to astronomers and use of the latter alien to ancient mathematicians,[446] but in sums of proper parts of days, that is, in a computational and notational mode of Egyptian origin that was standard practice in the Greek-speaking world and almost certainly the only mode comprehensible to his contemporary readers; (3) he inserted, with special regard to his Roman readers, a duodecimal variant (**S5**) of the sum of parts (**S4**) expressing the most important single parameter, the length of his new medical week; to deny this cogent interpretation of **S5** would leave us without a plausible explanation for why he mentioned just one alternative sum of parts from the more than one hundred mathematically possible alternative sums of parts,[447] this one being the only one that suited the Roman preference for duodecimal fractions.

Galen aptly chose the highly composite number 240 for his Egyptian-style computation. It was either a systematic decomposition of this number[448] or (less likely, in my opinion) an extremely serendipitous chance find that allowed him to identify the duodecimal sum of parts which he expressed in **S5**. His previous main result (**S4**) complies, in principle, with the goal of optimized approximation,[449] but it is not the 'very best' among the four 'best' solutions (that is, those starting with $6\ \overline{2}\ \overline{5}$), which would be $6\ \overline{2}\ \overline{5}\ \overline{40}\ \overline{240}$ days; various explanations are possible.[450]

The final words of **S7**, causing problems for some readers, mean that the last medical day of the third medical week belongs much more to the twentieth natural day than to the twenty-first.[451] On the whole, Galen's computational precision was unnecessary to achieve the goal of his argument but it suits his interests in advertising himself and impressing the reader.[452]

Our historical survey shows that Galen's treatise *On Critical Days* in general and his computation of medical weeks in particular have been the subject of assiduous study over one and a half millennia. A milestone in the correct understanding of Galen's computation, Ḥunayn ibn Isḥāq's Arabic translation, became the main source through which Western scholars from the 13th to the late 15th centuries accessed the Galenic treatise. In this period, only a few scholars such as Pietro d'Abano and Niccolò da Reggio read the orginal Greek text. Gerard of Cremona may have belonged to this small group, because his Latin translation of Ḥunayn's Arabic translation contains two details that appear inexplicable

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[444] See above, p. 244.
[445] That is, whose numerator can be any positive integer.
[446] With only one established exception: Diophantus, *Arithmetica* 4.36. Cf. n. 345 above.
[447] See above, p. 261.
[448] Cf. col. VIIIa on p. 254 above.
[449] See my definition on p. 252 above.
[450] See pp. 254–255 above.
[451] Cf. pp. 264–265 above.
[452] Cf. pp. 265–266 above (Question 10).

without access to a Greek manuscript.[453] In the early 16th century, a 'change of paradigm'
occurred, namely the rejection of the Arabic tradition and the humanistic attempt to recov-
er the original Greek text. The new contempt for the Arabic ('barbaric') sources implied
the loss of valuable insights, both generally and with specific regard to our computation:
Unlike Ḥunayn's Arabic translation, none of the early modern editions or translations of
the entire text of *On Critical Days* presents our passage in question without mathematical
blunders. Regrettably, the very few early modern scholars who were able to emend those
blunders left rather faint traces of their insights (Jean Guijon in a quotation adduced by
Jean Lalamant, and Giovanni Manardi in a letter to Pier Nicola Castellano). It is telling that
even Gadaldini's revised version[454] of the most successful Latin translation, that by Johan-
nes Winter of Andernach, still contained mathematically nonsensical details in the main
text. For whatever reason (lack of time or competence?), no one ever established a mathe-
matically sound text by selecting the correct readings among the variants which Gadaldini
had printed on the margins.[455] Nonetheless, this translation eclipsed all others thanks to its
numerous reprints. While those by Gerard of Cremona (**t1**) and Jean Lalamant (**t4**) had at
least a modest reception, the more important earlier translation by Niccolò da Reggio (**t2**)
seems to have been unknown to all humanists.

The most typical feature of the late medieval and early modern reception of our pas-
sage is that Galen's sums of parts of days are regularly converted into equinoctial hours
and minutes, yet without asking *why* the great ancient physician had employed those sums
of parts which were by those later readers perceived as unwieldy. Only our latest author,
Magini (1607), raises this question, but he has no answer. Moreover, none of these early
modern authors provides explicit evidence that he noticed the fact that all fractions in Ga-
len's computation are proper parts or, in modern terminology, unit fractions. While it is
fair to assume that someone like Magini, being a professor of mathematics, had noticed
this, Lalamant probably had not, because otherwise he would not have tried mathematical
emendations that involved non-unit-fractions.[456] The helplessness of these early modern
scholars in dealing with Galen's math is no surprise in view of the fact that the modern
disciplines of Egyptology and history of science, on whose insights the present contribu-
tion builds, did not yet exist. It took four centuries after Magini's commentary (1607) be-
fore scholarly interest in the text of Galen's unique computation came to life again with
Garofalo (2003), whose emendations,[457] here given philologically stronger foundations,[458]
formed the starting point for further textual corrections and for the mathematical elucida-
tions given in Section VI above.

Various questions for future research remain: Are there any detailed Greek or Latin in-
struction texts for the mathematical manipulations of sums of parts as Galen performed? Is

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[453] Cf. n. 125 above.
[454] Cf. p. 226 above.
[455] Cf. the edition above, pp. 226–227, with notes 230 and 234 (correct variants) and n. 233 (wrong variant).
[456] See notes 249 and 254 above.
[457] Cf. notes 61, 63, 67, 69 above.
[458] Garofalo 2003, 52, rightly restores "$\frac{1}{48}$" in the result of **S7**, but he does so on the authority of Ḥunayn's
Arabic translation without examining either the value of Ḥunayn as a witness (cf. pp. 214–215 above) or Ga-
len's rhetoric of accuracy (cf. Section V above).

there any documentary evidence, possibly on papyrus, of the actual computational steps of such operations (especially identification of suitable divisors) as opposed to reports that simply give their final results? Does Greco-Roman literature contain any closely comparable computation? Is there any evidence of the ancient Roman reception of Galen's *On Critical Days*, as opposed to its obvious importance in the late antique curriculum at Alexandria? What was the specific fortune of Galen's computation of medical weeks in the Arabic world after Ḥunayn ibn Isḥāq, especially among scholars such as Ibn Sīnā (Avicenna)? And did any modern scholar in the seemingly 'dark age' of the 18th, 19th and 20th centuries examine Galen's computation?

Acknowledgments

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1. Abbreviations

CMG   *Corpus Medicorum Graecorum*, ed. by the *Berlin-Brandenburgische Akademie der Wissenschaften*, Berlin 1908–.

CTC   *Catalogus Translationum et Commentariorum: Mediaeval and Renaissance Latin Translations and Commentaries. Annotated Lists and Guides*, ed. P.O. Kristeller et al., Washington D.C. 1960–.

DBI   *Dizionario Biografico degli Italiani*, ed. by the *Istituto dell'Enciclopedia Italiana*, Rome 1960–.

DNP   Cancik, H., Schneider, H., eds., *Der neue Pauly. Enzyklopädie der Antike*, 16 vols., Stuttgart 1996–2003.

EMLR   See below, Original Sources.

GW   *Gesamtkatalog der Wiegendrucke*, ed. by the *Staatsbibliothek zu Berlin*, Leipzig 1925–.

ISTC   *Incunabula Short Title Catalogue*, ed. by the British Library, http://www.bl.uk/catalogues/istc/.

ITER ITALICVM   See below, Bibliography s.v. Kristeller 1963–1997.

LLN   *Lexicon Latinitatis Nederlandicae Medii Aevi*, 9 vols., Amsterdam and Leiden 1970–2005.

LSJ   *A Greek-English Lexicon*, compiled by H.G. Liddell and R. Scott, revised and augmented throughout by Sir H.S. Jones, with a Revised Supplement ed. by P.G.W. Glare, Oxford 1996.

MLW   *Mittellateinisches   Wörterbuch*,   ed.   by   the   *Bayerische   Akademie   der Wissenschaften*, Munich 1999–.

ThLL   *Thesaurus Linguae Latinae editus auctoritate et consilio academiarum quinque Germanicarum*, Leipzig et al. 1900–.

TLG   *Thesaurus Linguae Graecae*, http://stephanus.tlg.uci.edu/.

2. Original Sources

Egyptian documents:

P.Rhind (British Museum 10057 and 10058); Egyptian Mathematical Leather Roll (EMLR, = BM 10250)

Greek papyri:

P.Akhmīm; P.Mich. 145; P.Mich. 146; MPER N.S. 15.158–170

Medieval and early modern manuscripts:

London, British Library, Arund. or. 17; Harley 6305

London, Wellcome Collection, 285; 286 (here: **W**)

Madrid, Biblioteca Nacional, 1978 (here: **M**)

Madrid, El Escorial, arab. 797

Oxford, Bodleian Library, Laud. gr. 58

Paris, Bibliothèque National de France, gr. 2272 (here: **P**)

Vatican, Biblioteca Apostolica Vaticana, lat. 2375 (here: **C**); Barb. gr. 221 (here: **B**); Pal. lat. 1092 (here: **G**)

Venice, Biblioteca Marciana, gr. 282; gr. app. V 4; gr. app. V 8 (here: **V**); gr. app. XI 5

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Bilingual (Greek-Latin):

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