SCIAMVS 20 (2019), 157–201

Tables for Computing Lunar Crescent Visibility in *Adderet Eliyahu*

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Abstract

This article edits and studies the tables for computing lunar crescent visibility in the Karaite Jewish legal compendium *Adderet Eliyahu* (The Mantle of Elijah), which was composed by Elijah Bashyatchi in the 15th century in Istanbul. Some of the most trenchant disputes between Karaites and Rabbanites, their Jewish opponents, centered on the calendar. *Adderet Eliyahu* was the first Karaite legal text to countenance methods of calendar calculation that were due to the Rabbanites. These tables are an important window onto the sources in computational astronomy available to the Karaites and their Rabbanite teachers at the end of the 15th century and a significant indicator of the level of the Karaites' own abilities in mathematical astronomy, as the determination of lunar crescent visibility was a technically significant problem. The tables and text of *Adderet Eliyahu* bespeak an acquaintance with Islamic sources.

I. Introduction

In the history of post-biblical Jewish law, the Rabbanites, those who accepted the authority of the tradition of the rabbinic texts (e.g. the *Talmud*) were the dominant group. Yet, by the middle of the ninth century, the Rabbanites' opponents, the Karaites, emerged, first in Persia and then in Jerusalem (Lasker 2007). The Karaites rejected the Rabbanite doctrine of the revelation of the oral law along with the written law at Sinai and the authority of the *Talmud*, the Rabbanite code of post-biblical Jewish law. As the Karaites developed their own codes, and because the Karaites often disagreed amongst themselves, Marina Rustow has described them as a school of thought, akin to a Muslim *madhhab*, within post-biblical Jewish law (Rustow 2008, xxviii). Much other scholarship has described them as a sect, and a significant legal debate between the Karaites and the Rabbanites was over the Jewish calendar (Lasker 2007).[1] Some of the ways in which the Karaites diverged from Rabbanite legal thought with regard to the calendar were in disallowing exceptions that led to the postponement of Rosh ha-Shanah, which was done to avoid two successive holidays with restrictions on work or to prevent Hoshannah Rabbah (the end of Sukkot, the festival of

[1] For more on the place of astronomy in the debates between Rabbanites and Karaites, see (Goldstein 2001, 31-45). See also (Shamuel 2003, 591-629) for an earlier article on calendar determination.

booths), with its celebrations, from falling on the Sabbath; in how Passover and Sukkot are only seven days, even in the Diaspora; and in the methodology for counting the Omer, the days between Passover and the festival of Shavuot (the Feast of Weeks). The most pressing topic of the dispute was how the Karaites traditionally determined the new Moon only through observation whereas the Rabbanites calculated it.

This paper studies the astronomical tables in Elijah Bashyatchi's (d. 1490) *Adderet Eliyahu* (The Mantle of Elijah; *AE* hereafter) a significant Karaite text, written in Hebrew, on Jewish law that includes tables that facilitate calculations of the new Moon with methods, involving mean and true conjunctions of the Sun and Moon, that had been previously used only by Rabbanites. Karaites had been interested only in determining the Moon's first visibility. The text was completed by Bashyatchi's student and brother-in-law Caleb Afendopolo (d. 1525) (Lasker 2008, 96).[2] Bashyatchi and Afendopolo studied with Mordek̲ai K̲umaṭiano (sometimes written Comtino; d. <1487), a noted rabbi (i.e. a Rabbanite), and *AE* cited and recommended Rabbanite texts, among them Moses Maimonides' (d. 1204) treatise on the calculation of the new Moon. Maimonides' treatise went beyond determining true conjunctions to calculating the Moon's first visibility, a subject that would have been of interest mostly to Karaites.

Bashyatchi came from a Karaite family of Edirne (Adrianople) that relocated to Istanbul in 1455 (Anonymous 2007).[3] The family's move to Istanbul may have been a result of the *sürgün*, Sultan Mehmed II's (d. 1481) policy of forced relocation to Istanbul, a policy intended to repopulate the capital after the ravages of the Ottoman conquest (Murphey 2005). Elijah's grandfather Menaḥem was a noted Karaite scholar who ruled that the Karaite schedule of Torah readings should begin in Tishrei (not Nisan), as was the Rabbanite custom. Menaḥem Bashyatchi also determined that Karaites should kindle lights for the Sabbath, a longstanding bone of contention between Rabbanites and Karaites and a practice that did not become fully accepted among Karaites until the nineteenth century. *AE* was a massive Karaite code of law and the willingness to compute the calendar indicated an accommodation with Rabbanites, though the Karaites' calendar remained distinct from the Rabbanites'.

Bashyatchi's concluding remarks about the tables reflected Maimonides' argument that the mathematical techniques for calendar calculation were first revealed to the prophets of Israel but were subsequently lost:

> From among that which you need to know is that all of the calculations and equations (*tiqqunim*) that we mentioned in calculating the arc of visibility are explained in the science of astronomy in the books of the Greeks and Arabs, through demonstrations and proofs which have nothing strange or contradictory[4] in them, that they stole from books composed by the sages of Israel who lived in the time of the prophets, and particularly from the sons of Issachar.

[2] On Afendopolo's death date, see (Zobel 2007).

[3] For Bashyatchi's philosophy more broadly, see (Lasker 2008, 96-122).

[4] (C, 32b): *u-b̲p^{e}raqim*; I: *kep̲irah*.

> From among the sons of Issachar are those who have knowledge of the understanding for the times [of the festivals] which arrived in their hands in the time of the dispersion. They copied them into their language and after that they hid them and they have not arrived in our hands. Indeed seeing that they are by means of proofs and demonstrations, it is not appropriate for us to refute them. Instead, we depend upon them and we do not care whether they were composed by scholars of Greece or Israel (I, 21a).[5]

AE was first printed in 1531 and reprinted subsequently as recently as 1987 with the printed editions from the nineteenth century onward expanding on both the text and tables of the 1531 edition. The tables in *AE* are interesting for their subtle departures from the sources that Bashyatchi acknowledged. Bashyatchi wrote that his new Moon computations relied on al-Battānī's (d. 923) tables (presumably from *al-Zīj al-ṣābi'*) and on Rabbi Emmanuel b. Jacob's (d. 1377) *Six Wings* (*Sheish K^e^napim*) (I, 13a).[6] In Chapter 29, Bashyatchi referred to Maimonides and Abraham Ibn Ezra (d. 1167), though Ibn Ezra's *Seiper ha-'ibbur* (The Book of Intercalation) did not provide lunar and solar motion tables (I, 19a).[7] Bashyatchi was aware of Ulugh Beg's (d. 1474) tables, but did not seem to draw on them directly in *AE* (Steinschneider 1964, 196). Still, even with these sources at his disposal, Bashyatchi might have had to recompute some of these tables for his latitude of Constantinople. The tables in *AE* are also the only known tables to be produced by Romaniot Jews after the Ottoman conquest of Constantinople in 1453. Thus, they are an important window onto the sources in computational astronomy available to the Karaites and their Rabbanite teachers and a significant indication of the level of the Karaites' own abilities in mathematical astronomy, as the determination of lunar crescent visibility was a technically significant problem. For all of these reasons, this publication focuses on *AE*'s relationship to its sources.

The variations between the early MSS and the 1531 printed edition, the handwritten updates in the 1531 printed edition, the expansions found in the later printed editions of *AE*, and the composition of commentaries on the section of *AE* about calendrical calculations are all evidence for how Karaites continued to use the tables in *AE* even if some errors endured. I examined several MSS and three printed editions of *Adderet Eliyahu* and the types of tables, as well as their accuracy, varied from MS to MS. For that reason, I made the 1531 printed edition my base text, and compared it with other early MSS even if they were

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5 Cf. (Gandz 1956, 73): "As regards the logic for all these calculations—why we have to add a particular figure or deduct it, how all these rules originated, and how they were discovered and proved—all this is part of the science of astronomy and mathematics, about which many books have been composed by Greek sages—books that are still available to the scholars of our time. But the books which had been composed by the Sages of Israel, of the tribe of Issachar, who lived in the time of the Prophets, have not come down to us. But since all these rules have been established by sound and clear proofs, free from any flaw and irrefutable, we need not be concerned about the identity of their authors, whether they were Hebrew Prophets or gentile sages." See also (Lasker 2008, 106) for how one needed to have learned science and philosophy to understand the Torah.

6 See also (Goldstein 1979, 33-34). The tables of Abraham Bar Ḥiyya (d. 1136), the earliest tables to be produced in Hebrew, relied on al-Battānī's *zīj* (an astronomical handbook with tables). Bar Ḥiyya's tables were also the basis for *Six Wings*. Battānī's *zīj* was never translated into Hebrew.

7 For Abraham Ibn Ezra's *Seiper ha-'ibbur*, see (Goodman 2011).

incomplete. I used MSS from the 16th century and thereafter to ascertain whether the discrepancies between the early MSS and the printed edition endured and to see whether errors in the printed edition were corrected in later MSS. The Jewish National and University Library (JNUL) catalogue lists these MSS of *Adderet Eliyahu*:

1. Saint Petersburg, Institute of Oriental Studies, C42. This MS is only 29 folios and does not contain the tables.
2. Saint Petersburg, The National Library of Russia, Evr. 633. This fifteenth-century MS does not contain tables.
3. Columbia University Library MS X 893 Im 6 is a 15th or 16th century MS that contains *AE*'s table for setting times and a calculated example for the arc of visibility on folios 55-58. I did consult this MS, and labeled it "X" in the footnotes and apparatus.
4. Saint Petersburg, Institute of Oriental Studies, C 132. This is a 16th century copy (or earlier) of 288 folios. As it is the most complete early MS of *AE*, it is the earliest, most complete witness to the tables. This MS is labeled "C" in the footnotes and apparatus.
5. Saint Petersburg, Institute of Oriental Studies, B58 is a 16th to 17th century copy of 31 folios which does not contain tables.
6. Saint Petersburg, The National Library of Russia, Evr. 635 is a 1502 copy of only eight folios. This MS does not contain tables.
7. Saint Petersburg, Institute of Oriental Studies, A96 is a 17th century copy with only twelve folios, but it does contain tables. I consulted this MS and it is labeled "A" in the footnotes and apparatus.
8. Leiden University Library MS Or. 1099K has 32 folios and is a 17th century copy.
9. JTS MS 3409, fols. 72-135, contains *AE*. This is a 17th or 18th century copy.
10. Saint Petersburg, Institute of Oriental Studies, MS B186, fols. 36a-45a, contains the tables from *AE* along with instructions on how to use them. This MS is from the 18th or 19th centuries. I consulted this MS and it is labeled "B" in the footnotes and apparatus.
11. Oxford Bodleian Oriental 404 (Neubauer 811) is a MS of 26 folios with a portion of *AE*, containing the laws of the Sabbath. This MS is from the 18th or 19th centuries and does not contain tables.
12. Oxford Bodleian MS Mich. 506 (Neubauer 894) is a MS of *AE* of 140 folios with the end missing. The JNUL catalogue suggests that it is an 18th century MS.
13. Saint Petersburg, Institute of Oriental Studies MS B 228, folios 140a-147a contains tables from *AE*. It is an eighteenth century MS and has the tables for the cycles of the conjunctions. I consulted this MS, but did not record its variants in the footnotes and apparatus.
14. Saint Petersburg, Institute of Oriental Studies D19, pages 203-318, contains part of *AE*. The JNUL catalogue indicates that 203a-204b are about the sanctification of the new Moon, but there are no tables.

OCLC/WorldCat lists, though Google Books differs, printed editions of *AE* that can be resolved into four groups:

15. 1530-1531 from Constantinople, published by Gershom Soncino. In the table of the correction for the hours of the distance, most of the bottom two lines are written in by hand (I, 17b). The handwritten parts could have been added by a reader, but that would mean that there had been blank spaces in the table. MS C 132 had blank spaces in the same places. This edition is labeled "I" in the footnotes and apparatus.
16. 1833-1835 from Eupatoria (Gozlov) in the Crimea. One of the 1835 editions is available on Google Books, though the publisher is not named. I could find no differences between the tables in this edition and those in the 1870/1966 edition. Although the 19th century printed edition added tables, I have not found new parameters in tables that are preserved from one printed table to the next.
17. Y. Beim published the 1870 Odessa edition.

17a. 1966 from Israel (Ramleh), by the society for the success of Karaite Judaism. The JNUL catalogue explains that this edition is a photo-reproduction of the 1870 Odessa edition. There is a 1987 reprint of this 1966 edition. This edition is listed as "OR" in the footnotes and apparatus.

The 1531 printed edition (I) is the base text for the edition of the tables. I have noted MS witnesses that diverge from the base text after a slash. When I prefer a reading from another manuscript, I note those MSS in parentheses before the slash, and the reading from I follows the slash.

The tables are found in the first section of *AE*, entitled *Qiddush ha-ḥodesh* (sanctification of the month). For an overview of the article, I list the chapters in the order that they appear in the sixteenth century SP IOS C132 MS, i.e. MS C, and the printed editions and name the table, if any, that appears in the chapter:

22. Table for the mean conjunctions and the *argumentum* for the Sun, Moon, and nodes
23. More information on using the tables in Chapter 22
24. Tables for the equation and hourly velocity of the Sun and the Moon
25. Table for the elongation (the distance between the Sun and the Moon) and its twelfth
26. Table for determining the time between mean and true conjunctions
27. Table for the time between sunrise and mid-day
28. Table for the duration of twilight
29. Older methods of calculating the new Moon
30. Table for the lunar latitude
31. Lunar parallax, the deviation of the circle of the Moon, and setting times
32. Determining whether or not the Moon will be seen

With the exception of Chapter 24 which contains two tables, each lettered sub-heading of the article treats a separate chapter of *AE*. An explanation of how a given table helps one determine lunar visibility accompanies the description and analysis of each table. Sometimes the instructions for using a table are not found in the same chapter in which the table appears. The tables in *AE* facilitated not the computation of the precise moment of lunar crescent visibility, but rather the determination of whether the crescent would be visible at the beginning of a given evening. If so, then a new Jewish month should begin. If not, then the new Jewish month would begin the following evening. At the end of the article, just before the conclusion, there is a sample calculation for whether the lunar crescent will be visible at nightfall at the beginning of Iyyar, 5311. Each lettered step of the calculation corresponds to the table involved, as well as to the lettered section of the article.

A. Chapter 22 and Table A. This table, found in two sections, provides the time of mean conjunction, with location and the *argumentum* (*ḥoq*) of the Sun (its motion on its eccentric as an angle from the center of the eccentric; see α_m in Figure One), the *argumentum* of the Moon (its motion on its epicycle; see α in Figure Two), and the *argumentum* of the nodes (the uniform motion of the intersection of the lunar deferent and the ecliptic in the opposite direction of the signs) for a.) certain years and b.) months.[8] The columns for the days of the new Moon for the new year indicate on which day of the week, hour, minute, and second the new Moon falls. In regular years, one adds 4d 8h 48m, modulo seven days, from one year to the next. In embolismic years, one adds 5d 21h 32m (Gandz 1956, 29 and 115).[9] For the location of the two luminaries, one adds 18; 23, 25° when moving from a simple year to an embolismic year. When moving from one simple year to another, subtract 10; 43, 0°. The difference in the Sun's *argumentum* for a simple year is a subtraction of 10; 44, 0°; from a simple year to an embolismic year add 18; 22, 20°. For the Moon's *argumentum* from a simple year to an embolismic year, subtract 24; 23, 0°; from a simple year to a simple year, subtract 1s 20; 12, 0°. The position of the nodes advances 8; 5, 0° in a simple year and 1s 8; 45, 25° in an embolismic year. These intervals hold for both the tables from the 1531 printed edition (I) and from MS C, but the printed edition (5292 to 5309, with an additional table running from 5315 to 5334) and the MS (5240 to 5244) provide information for different years. Thus, I could not edit this table, though I did ascertain that the intervals between years and months were uniform throughout the exemplars of *AE*. The years given in the MS C table yield a *terminus ante quem* for the composition of that part of the text. 5240 starts in 1479. 5244 starts in 1483 (C, 24a; I, 13a-14b).[10] In the modern printed editions, the tables had many more sexagesimal places. Neither Abraham Bar

[8] On the definition of the Sun's motion on the eccentric as the *argumentum*, see (Pedersen 2011, 139). See (Pedersen 2011, 169) for the *argumentum* of the Moon. On the use of the Latin *argumentum* as the translation for this parameter, see (Chábas and Goldstein 2015, 241).
[9] Maimonides reported a standard rabbinic value of an interval of 4d 8h 876 parts between regular years. *AE*'s 48 minutes equal 864 parts. Between a regular and embolismic year, Maimonides, again following earlier rabbinic sources, found an interval of 5d 21h 589 parts. *AE*'s 32 minutes equal 576 parts. As one minute is 18 parts, these divergences are on the order of less than a minute. For the history of the rabbinic calendar, see (Stern 2001).
[10] See (I, 18a) for the additional table. This table is absent in the later printed editions; see (OR, 58). The text also mentioned (Anonymous 2007) the year 1457.

Ḥiyya's *Luḥot ha-Nasi* nor Maimonides' *Sanctification* had a table like this one, but Emmanuel b. Jacob's *Six Wings* did, with the same parameters (F, 4a).[11] Battāni provided tables to convert between the Islamic (*sanat al-'arab*) and solar (*sanat al-rūm*) calendars (N, II. 9-18).

Both MS C and the 1531 print edition tabulated the time of the new Moon, place of the luminaries, and anomalies of the Sun, Moon, and nodes for the mean lunar months. Each month, the time of mean conjunction advances by 1d 12h 44min 3sec, modulo seven days.[12] Each month, the position of the luminaries at the time of mean conjunction advances by 0s 29; 6, 25°. The values in the minutes column for the nodes' *argumentum* are corrected by hand in the 1531 printed edition. Each month, the Sun's *argumentum* increases by 0s 29; 6, 20°. Each month, the Moon's *argumentum,* its motion on its epicycle, increases by 12s 25; 49°. Each month, the nodes' *argumentum* increases by 1s 0; 40, 25°. In any case, the manual corrections, like the handwritten tables, found in the printed edition (I) suggest the use of the tables for calendrical calculations. These parameters resemble those found in *Six Wings*, in the first wing (F, 3b). The similarities are precise for the hours. Though *AE* included a column for seconds and *Six Wings* did not, the intervals between the entries in *Six Wings* reflected the values found in that additional column. For instance, the mean position of the luminaries advanced every month by 0s 29; 6, 25° in *AE*. In *Six Wings*, there were cases where the luminaries advanced by seven minutes instead of six, suggesting that the number of seconds topped 60 in those instances. There were other cases in which the columns in *Six Wings* took account of seconds without listing them. For example, with regard to the Sun's *argumentum*, *AE* added 18; 22, 20° for an embolismic year. *Six Wings* added between 18; 22° and 18;23° suggesting that minutes were being tracked even if not listed.

By the 1835 printed version of *AE*, one can find the time of the mean conjunction, the mean position of the luminaries, the *argumentum* of each luminary, and the *argumentum* of the nodes for any day of any year down to the minute (OR, 38-48). One would determine in which 19 year cycle the particular year falls and, then, the position of the given year within that 19 year cycle. Once one determines the time of the mean conjunction for that year, one uses the table for months and adds the entries corresponding to the appropriate number of months to the entries for the chosen year. Then, when one adds the *ḥoq* of the Sun to the location of the apogee, one has the Sun's mean motion. The motion of the Sun's apogee in one year is 52 seconds and 57 in an embolismic year.[13]

[11] For the motion of the nodes, *Six Wings* had added 8; 3, 0° for a simple year and between 1s 8; 43° and 1s 8; 44° for an embolismic year.

[12] Cf. (Goodman 2011, 147). In Ibn Ezra's analogous table, the time of conjunction advanced from month to month by 1d 12h 793p. Maimonides (Gandz 1956, 27) had the same parameter. There are 1080 parts in an hour, so 44min 3 sec yields 866 parts.

[13] For AE's parameters, see (I, 13a). See (Gandz 1956, 125) for Maimonides' parameter (1°/70 years or 51.4 seconds per year) which Neugebauer, in his commentary, understood to be an approximation from Battānī's parameter of 1°/66 years.

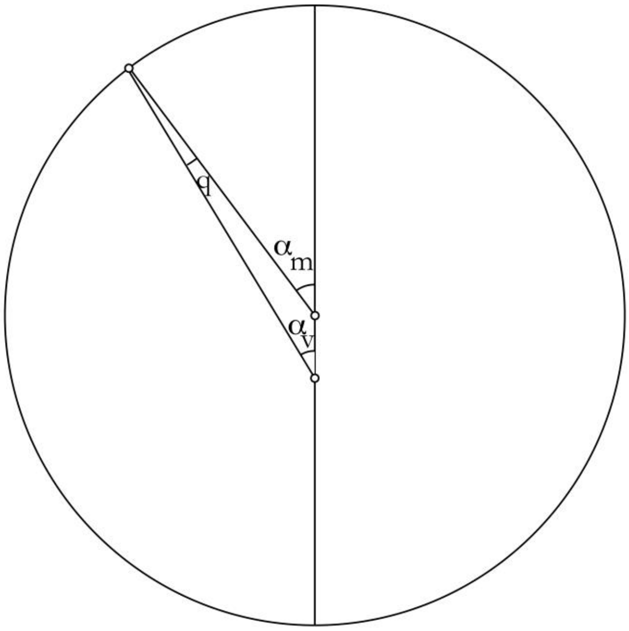

Figure One: The Mean Solar Argument

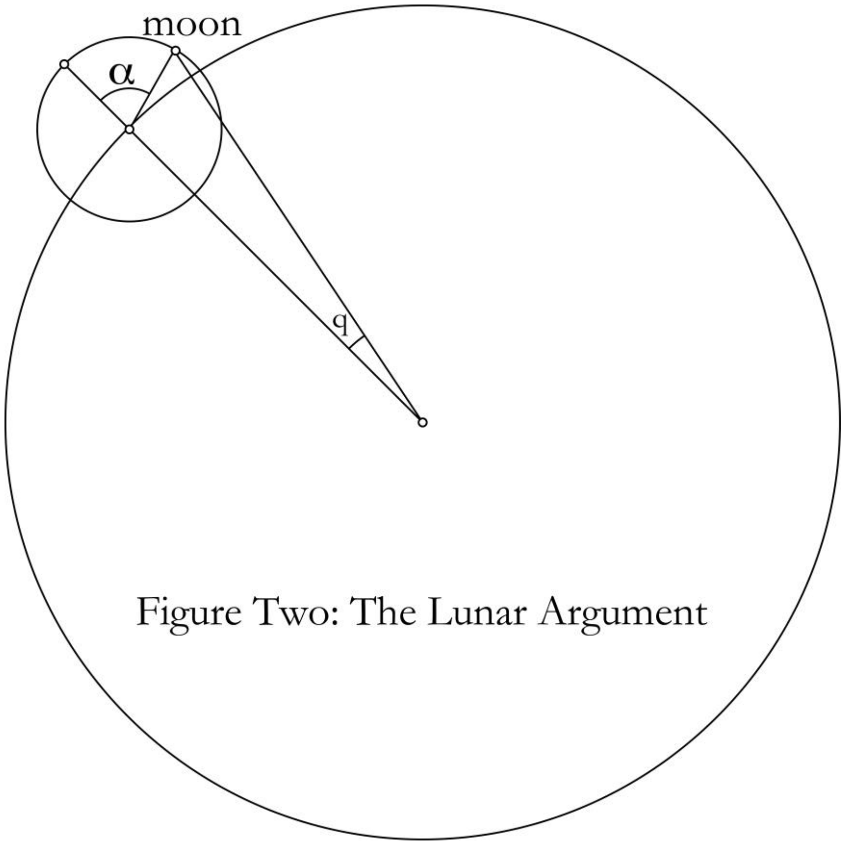

Figure Two: The Lunar Argument

B. Chapter 24 and Table B. This table provides the equation and hourly velocity of the Sun. The parameters in these tables are due to al-Battānī (N, II.78-83 and 88). Maimonides had also relied on Battānī's parameters, using the same maximum solar equation of 1;59°, though the table in *AE* is far more detailed than the one found in Maimonides' *Code* (Gandz 1956, 51). The instructions for using the table say that for signs zero through five, to read down, and for signs eleven through six to read up. When the Sun is going from its apogee to its perigee (0° to 180°), one subtracts the equation from its mean position, which is known from Table A, whereas when the Sun is going from its perigee to its apogee (180° to 360°), one adds the equation to the mean position. The maximum equation, which is 1;59,10°, occurs when the Sun's mean motion is two degrees into sign four. This table yields the true position of the Sun at the moment of the mean conjunction.

C. Chapter 24 and Table C. This table provides the equation and hourly velocity of the Moon. With the Moon, enter the *argumentum* of the Moon at the moment of the mean conjunction in the table of the lunar equation. If the *argumentum* is from zero to six signs, subtract the equation, and if it is from six signs to zero, add the equation. These parameters come from Battānī's *zīj* (an astronomical handbook with tables), although there are places where MS C and both printed editions differ uniformly from Battānī's *zīj* (N, II.78-83 and 88). Battānī's table for the hourly solar and lunar motion has a vertical argument with intervals of six degrees rather than one, which is what is found in *AE*. The common numbers in *Six Wings* have a vertical argument of thirty seconds. These tables yield the true position of the Moon at the moment of the mean conjunction.

D. Chapter 25 and Table D. This table provides the elongation of the luminaries and its twelfth. This table approximates the position of the luminaries at the moment of the true conjunction. The vertical rows give the elongation, the distance between the luminaries, in intervals of six minutes. The horizontal columns give 13/12 of the elongation for the Moon's position and 1/12 of the elongation for the Sun's position. First, one subtracts the lesser of the luminaries' true positions, based on which luminary is behind in the motion in the order of the zodiacal signs, which yields the elongation. 13/12 of the elongation is the approximate direct motion (*yosheir*) of the Moon and 1/12 of the elongation is the approximate direct motion of the Sun. Second, one looks back to the luminaries' known positions to see if the Sun was ahead of the Moon in their paths through the ecliptic. If so, then the true conjunction had not yet occurred. Add 1/12 of the difference to the Sun's position and the Moon's direct motion (13/12 of the difference) to the Moon's position. At that time the luminaries are in one position, at the moment of the true conjunction. If, though, the Moon had passed the Sun, then the true conjunction had already occurred. In that case, one subtracts 1/12 of the elongation from the position of the Sun's and the Moon's direct motion (13/12 of the elongation) from the position of the Moon. Then, the luminaries will be in one place, at the moment of the true conjunction. So far, one knows their locations at the moment of the true conjunction. As for the nodes, take the difference between the positions of the mean and true conjunctions. Add that difference to the position of the nodes' *argu-*

mentum if the luminaries' true position is greater than their mean position, or subtract the difference if not. What emerges is the approximate true position of the nodes at the moment of the true conjunction. *AE* has ignored the three minute daily retrograde motions of the nodes because there is less than a day between the mean and true conjunctions. One would need the following (§E) table of the hours of the distance to determine how long it would take for the luminaries to traverse the distance to yield the moment of the true conjunction. In fact, one could eliminate this table's approximations by using the next table to determine the moment of the true conjunction and, then, using tables for the solar motion to determine the Sun's location, and hence the Moon's at that moment.

Neither Maimonides' *Sanctification of the New Moon, Six Wings*, nor Battānī's *zīj* contained a table like this. Khwārizmī's *zīj* did have a procedure in which the distance between the Sun and the Moon plus its twelfth is added or subtracted. Khwārizmī's *zīj* reads: "Then, according to which (body) is ahead (of the other), the interval is called either 'of the preceding Sun' or 'of the (preceding) Moon.' If it is of the Sun, the above total is to be added to the place of the Moon, the half of the sixth, however, to the place of the Sun; if it is of the Moon, these (quantities) are to be added or subtracted from these (places)" (Neugebauer 1962, 60).

E1. Chapter 26 and Table E1. Table E1 provides the hours of the distance. The previous table, of the distance and its twelfth, approximated the location of the two luminaries at the moment of the true conjunction. The present double argument table helps one determine the time between the mean and true conjunctions. It computes, given the elongation as well as the luminaries' hourly velocities at the moment of mean conjunction, how long the Moon would take to close the gap. To use the table, determine the elongation by subtracting the lesser position from the greater. Then, subtract the solar velocity from the lunar velocity and the difference is the hourly direct motion, the hourly elongation. The table divides that distance by the hourly elongation. If the Sun is before the Moon, meaning that the conjunction has not yet occurred, add the hours (and their parts) obtained to the time of the mean conjunction, or subtract them from the mean conjunction if the Moon is ahead of the Sun (I, 17a). This table, expressed as an equation, is $t = \Delta\lambda/\Delta v$ (i.e. the difference of the lunar and solar positions divided by the difference in their velocities), and reflects the correct way to determine the time between the mean and true syzygies according to the *zīj*es available in the Byzantine Empire during K̲umaṭiano's lifetime (Tihon 1996, 246).[14] Less accurate methods for determining the true syzygy had been debated and rejected by K̲umaṭiano and Byzantine Christian scholars.

The horizontal argument is the hourly elongation, the difference of the Sun and Moon's velocities, and it ranges from 0; 28° to 0; 33, 30°. The vertical argument is the elongation itself. According to *AE*'s tables, the hourly solar velocity ranges from 0;2, 23° to 0; 2, 33° and the hourly lunar velocity ranges from 0; 30, 18° to 0; 36, 4°. Based on those parameters, the minimum elongation would be 0; 27, 45° and the maximum would be 0; 33, 41°.

[14] Bashyatchi eschewed the inexact method found in Chrysococcès' tables, a method that re-appeared in Sharbiṭ ha-Zahab̲'s Hebrew translation of Chrysococcès' tables.

Columns with elongations of 0; 27, 30° and 0; 34° would be at least as inaccurate as using the existing maximum and minimum columns of the hourly elongation. Taking into account the divergences between *AE* and Battānī's tables, the table of the equation of the hours of the distance provides no additional evidence for a source other than Battānī's tables for the solar and lunar velocities. In addition, if one computes the difference between the solar and lunar velocities with attention to the argument, one finds that 0; 2, 23° and 0; 30, 18° share the same argument, meaning that the minimum would be 0; 27, 55°. The maximum would be 0; 33, 31° (N, II.88). This method of dividing the distance by the hourly instantaneous elongation to yield the time necessary for the Moon to close the distance existed in Ibn al-Muthannā's commentary on al-Khwārizmī's *zīj* (Goldstein 1967, 95 and 343-4), but the production of double argument tables to display the results had a different history (Chabás and Goldstein 2019).[15] Ibn al-Kammād's table, in the *Zīj al-Muqtabis*, resembled the one in *AE*, but the vertical argument went to 12°, whereas the argument in *AE* went only to 7;0° (Chábas and Goldstein 1994, 14). Neither Maimonides' *Sanctification of the New Moon* nor *Six Wings* included such a table.

E2. Chapter 27 and Table E2. Table E2 provides the hours of mid-day. This table is latitude dependent and *AE*'s table was for the latitude of Constantinople. Once one knows the time of the true conjunction, one needs to know the amount of time between the moment of the true conjunction and nightfall, as that is the beginning of the day in Judaism. To make this determination, one uses the table for the hours of mid-day which is found in the 1966, 1870, and 1531 printed editions. There is a blank space for this table in MS C. MS A, a later MS, has this table. As the computed time of a conjunction is a certain time after mid-day (local noon), the table of hours of mid-day determines the amount of time between sunrise and mid-day; thus, at the first of Aries, there are six hours between sunrise and mid-day. Double the hours from sunrise to mid-day to yield the total number of hours in the day. Subtract that sum from 24 to obtain the number of hours in the night. Because the time of the conjunction is the time after mid-day, if the hours of mid-day are greater than the time of the conjunction, then the conjunction is during the day. If the time of conjunction is greater, subtract the hours of mid-day from the hours of the conjunction, and the remainder is the time of the conjunction from the beginning of the night. If that remainder is greater than the hours of the night, subtract the hours of the night from that remainder. The result is the time of the conjunction after the next morning.

F. Chapter 28 and Table F. This table provides values for twilight. Presuming that the conjunction is at night, the previous table informed the user of how many hours the conjunction was after sundown. *AE* wrote: "When one wishes to know how many hours the conjunction is distant from the beginning of the third evening, which is the beginning of evening according to our, the Karaites' opinion, add twilight, which is the time from the beginning of setting to the beginning of the third evening, to the hours of the distance from setting, and what emerges is the distance of the conjunction from the beginning of the third

[15] See also (Chábas and Goldstein 2012, 7) and (Chábas and Goldstein 2015, 42-46).

evening" (I, 19a).[16] *AE* has adjusted the beginning of evening from sunset to nightfall, meaning that the measure of time from local noon to nightfall is greater than the measure of time from sunrise to mid-day (local noon). This was an extremely important adjustment because the purpose of the tables was to investigate whether the Moon would be visible at the beginning of a certain evening. The user of the tables would not be determining the precise moment of visibility, but, rather, determining whether the Moon would be visible at a certain moment, the beginning of the evening that followed the moment of mean conjunction. If the Moon was not visible at the beginning of a given evening, it would certainly be visible by the beginning of the following evening.

Because the duration of twilight varies on each day of the year and also according to latitude, *AE* recommended interpolating for latitudes between those given and for times other than the beginning of the sign. Neither Battānī's *zīj* nor *Six Wings* covered twilight at all (King 1973, 366). Maimonides' *Sanctification* adjusted the mean position of the Moon depending on where the Sun was in the ecliptic, an adjustment which compensated for the varying angles between the horizon and ecliptic throughout the year (Gandz 1956, 55-56). Kennedy had listed *al-Zīj al-Sanjarī* as the only *zīj* that covered twilight, though since then other twilight tables in *zīj*es have come to light (Kennedy 1956, 159).[17] The values in *AE*'s table do not correspond either to the rising times method found in the tables that were produced by al-Qāyinī (fl. 11th century) and studied and provided by Davidian and Kennedy in 1961, nor to the method of hour angles outlined in Smart's textbook of spherical trigonometry (Davidian and Kennedy 1961, 149). The recomputation follows the method in Smart's textbook which is based on hour angles and is the method found in most of the twilight tables that King surveyed. The recomputation comes closer to the values of the table in *AE* if one defines the end of twilight as the Sun being 19° below the horizon, though pre-modern writers in Islamic societies considered twilight to be determined by a solar depression angle between 17° and 19° (Goldstein 1977, 98-99). The recomputed angles, following both methods, have symmetries that are not always found in *AE*'s tables, though Bashyatchi acknowledged the existence of such symmetries: "Every two points equidistant from the two equinoxes, their twilights are the same" (I, 19a). I am not sure how *AE* obtained its values.

In Chapter 29, *AE* explained that, in the latitude of Constantinople, if there were between 13h 50m to 27h 30m from the true conjunction to the beginning of third evening, then the Moon might be seen (I, 19a). In that chapter, he also referred to the view of a Greek astronomer, possibly the astronomer of Constantinople (*to<u>k</u>ein ha-'ir*), who held that there were about fourteen hours between the conjunction and first visibility. *AE* approached the question more systematically. To begin, take the time between the mean conjunction and time of visibility[18] to be investigated, i.e. the moment of the next beginning of

[16] (C, 29a) has exactly the same language about adding the time of twilight to the distance from the time of setting. The addition of the hours of twilight corrects for two different definitions of the beginning of night, i.e. sunset vs. nightfall.
[17] See (King 2004-2005, II.146-147) for a list of these tables for twilight and for the *zīj*es in which they are located.
[18] In what follows, I will not reiterate that one is investigating a moment of possible visibility.

evening, in hours and their parts with 40 minutes in an hour and 40 seconds in a minute (I, 19a).[19] Add the time to the position of the *argumentum* of the Moon to yield the corrected *argumentum* of the Moon at the moment of visibility. Enter the corrected *argumentum* of the Moon in the table of the lunar equation and take the Moon's hourly path. Multiply the hourly path by the time, in hours and minutes, between the true conjunction and the visibility. Add the result to the true position of the luminaries at the time of the true conjunction to yield the true position of the Moon at the time of visibility. Add that same result also to the *argumentum* of the nodes along with another three minutes, and the result is the corrected *argumentum* of the nodes at the time of visibility. It was also important to know the position of the Sun at the time of visibility: "Enter again the *argumentum* of the Sun in degrees and seconds and take what you find there of the hourly path of the Sun and multiply it by the hours and the minutes that there are between the true conjunction and the time of visibility. Add what emerges to the position of the luminaries and what emerges is the position of the Sun at the time of visibility" (I, 19b).

G. Chapter 29 and Table G. This table provides the latitude of the Moon. Now that the time of the true conjunction is known, as well as whether the true conjunction will be before or after nightfall, one can begin to compute the arc of visibility which is the distance between luminaries, after the true conjunction, necessary to observe the crescent. The precise computation of the arc of visibility is complex because the elongation is measured in the ecliptic which makes different angles with the local horizon throughout the year (Gandz 1956, 137). Knowledge of the lunar latitude, the angular distance of the Moon from the ecliptic according to which the elongation is measured, is necessary for calculating the arc of visibility more precisely. Although *AE*'s table and Battānī's *zīj* share the same parameter for the maximum latitude (N, II.80), the other values are close, but not always the same. Neither BnF MS Héb. 1075 or MS Héb. 1042 of *Six Wings*, have a table for lunar latitude. Khwārizmī's maximum lunar latitude was 4;30° (Neugebauer 1962, 98).

To use this table, one needs the corrected *argumentum* of the nodes at the nightfall for which one wants to investigate the Moon's visibility. To determine the first elongation, the elongation at the time of first visibility, subtract the position of the Sun at the time of first visibility from the position of the Moon at the time of first visibility. This is the arc of visibility in direct degrees (*ma'alot y^{e}sharot*). To find the lunar latitude, enter the corrected *argumentum* of the nodes at the time of first visibility. The result is the latitude of the Moon at the time of first visibility.

H. Chapter 31 and Table H. This table provides the parallax of the Moon when it enters various signs. At this point, the user has the Moon's elongation and latitude at the time of the first visibility. *AE* explains that the first elongation and first latitude have to be corrected before recomputing the arc of visibility. *AE* presented some views of how parallax modifies the calculation of the elongation. In Chapter 31, Bashyatchi wrote that if the true conjunction is before midheaven, then the parallax in longitude is added to the location of

[19] This procedure comes from Chapter 30 of *AE*.

the true conjunction. If the true conjunction is after culmination, the deviation in longitude would be subtracted. If the Moon is to the south, then the deviation in latitude is added, and vice-versa were the Moon to the north. The table in *AE* has columns for the latitude of Constantinople, 41.5°, as well as latitudes 30° and 45°. This is a much more simplified table than the one found in Battānī's *zīj* which has entries for different times of day. As for the table in *AE*, that for the parallax of Constantinople is very close to the table of lunar parallax for 41;15° in *Battānī*'s *zīj* (N, II.99).[20] The figures for latitudes 30° and 45° are particularly close to Battānī's (N, II.97-100). The tables in *AE* take the parallax at sunset for the fourth through ninth signs and the parallax at sunrise for the tenth through third signs. The parallaxes for the latitude of Constantinople in *AE* lack the symmetry of the corrected versions of the two other parallax columns in *AE*, but some of the lack of symmetry was due to confusion between a gimel and a ḥet (i.e. 23 for 28), as those letters look similar in Arabic. The gimel/ḥet confusion suggests that *AE* did not get its parameters, at least for parallax, directly from Bar Ḥiyya's tables, or from other Hebrew tables based on Battānī's *zīj* (P, 32b-34a). Maimonides mentioned Battānī in his discussion of calendar calculations, so I hypothesize that Bashyatchi or an intermediary procured Battānī's tables, or tables based on Battānī's, in Arabic to fill out *AE* (Gandz 1956, lvii).

In Chapter 32 of *AE*, Bashyatchi gave different instructions for how to use parallax tables to find the arc of visibility; the first elongation and latitude come from the initial arc of visibility computation:

> Know the position of the Moon, in which zodiacal sign it is, and enter it in the table of the anomaly in longitude [i.e. the parallax] in the horizon of Constantinople. Take the zodiacal sign that corresponds to it at its side and they are sixty seconds for one degree and subtract them forever from the first elongation and that which remains is called the second elongation.
>
> After that, know for yourself the latitude of the Moon, how much it is, from the table of the latitude of the Moon. Know if it is northern or southern and write it separately, and it is called the first latitude. After that, enter the sign of the Moon in the table of the variation (*ḥillup̱*) of the latitude and take what is there, next to it, from the variation of the latitude. Afterwards look, and if the latitude of the Moon is northern, subtract the difference in latitude from the first latitude of the Moon. If it is southern, add the difference in latitude to the first. Whatever the latitude is after you add to it or subtract from it is called the second latitude and write it separately with its name (I, 21a).

I. Chapter 31 and Table I. This table provides the deviation of the circle (*n^elizat ha-ma'gal*) of the Moon. Neugebauer described this equation as "a variation in the 'orbit' of the Moon," but I will use "deviation of the circle" so as to avoid the anachronistic connotations of "orbit" (Gandz 1956, 67 and 141). This correction is represented by c_3, which is multiplied by the second latitude (β_2), in Figure Three. *AE* used this table to modify further, after the correction for parallax, the arc of visibility (λ). The table accounts for whether the

[20] Battānī's value was identical to that found in the *Handy Tables* (Stahlman 1993, 276).

Moon's ecliptic longitude is ahead of or behind the point where a longitude arc passing through the Moon's right ascension would intersect the ecliptic. In each cell of the table, there are two values. The top value is for when one reads down the table, i.e. for signs zero through five according to the values on the right side of the table. The value at the bottom of the box is for when one is reading up the table, according to the values on the left side of the table. In two cases, 1/6 is provided in parentheses following the 1531 printed version, though neither in MS C nor the more recent printed editions. The value in the table is a fraction of the second latitude, and the product with the second longitude should be deducted from the second elongation if the Moon is to the north of the ecliptic and should be added to the second elongation if the Moon is to the south (I, 21a). The result is the third elongation of the Moon. With the signs of Cancer and Capricorn, there is no deviation of the circle. In those cases, the second elongation is the third elongation (I, 21a). The third elongation still measures the elongation according to the ecliptic, as was the case with the second elongation, but with great circle arcs from the pole of the celestial equator, not the pole of the ecliptic.[21] The terms "third elongation" and "fourth elongation" along with the associated steps in the computations, were Maimonides' contribution.

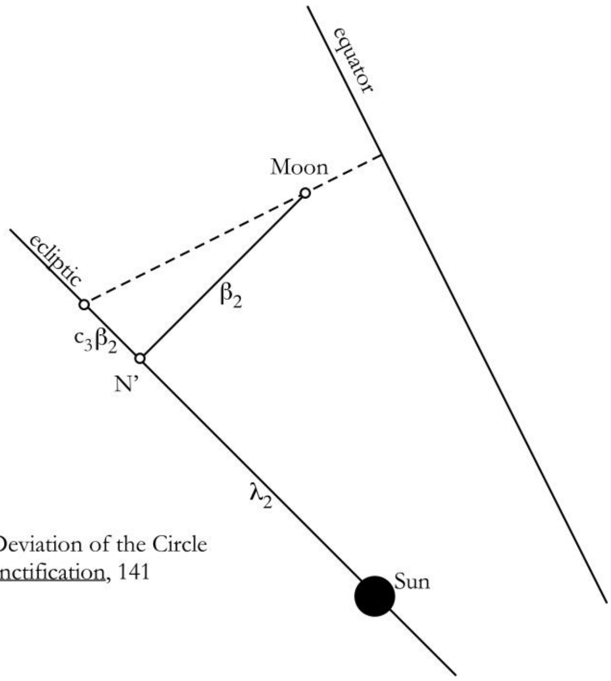

Figure Three: The Deviation of the Circle
See Maimonides, Sanctification, 141

A table for the deviation of the circle of the Moon does not exist in *Six Wings* (F) or in the Hebrew translation of the *Persian Tables* found in the same codex with F.[22] As well,

[21] For more on the use of ecliptic coordinates with arcs drawn from the pole of the celestial equator, see (Fischer, Kunitzsch, Langermann 1988, 274-275, n.46 - n.47).

[22] Bashyatchi wrote (I, 20a): "The scholar Rabbi Abraham Ibn Ezra of blessed memory, in his being a follower of the ancients, did not mention this deviation, though the scholar Maimonides mentioned it in his books due to his being a modern and he saw the books of the moderns. Behold, if the Moon has a northern latitude, subtract

the values in the table for the deviation of the circle that Neugebauer produced following Maimonides' *Sanctification* are not the same as the table found in *AE*. The tables differ in that Neugebauer, following Maimonides, grouped the twelve signs into three groups of four, whereas the table in *AE* displayed six groups of two signs. There are two cases in which *AE*'s table provided entries for 0 to 5/ 5 to 10 and 20 to 25/25 to 30, and the entries are different, whereas in Neugebauer's table, based on Maimonides' *Sanctification*, four signs shared the same values.

J. Chapter 31 and Table J. This table provides setting times for the latitude of Constantinople. A setting time is the arc of the celestial equator that sets with a certain arc of the ecliptic. Thus, the setting time for 1° is the rising time of 181°. Maimonides' *Sanctification of the New Moon* had one setting time for an entire zodiacal sign, whereas the table in *AE* computed the values for each degree of the ecliptic (Gandz 1956, 147). In the column for the 11th zodiacal sign (the first was zero), I have used R as the base text because that column is cut off in I.

In chapter 32 of *AE*, Bashyatchi explained how to use the table for setting times to compute the arc of visibility:

> After this, enter the position of the Sun in the table of rising times [in setting] with the signs from above and in degrees from the right and take the rising times [in setting] and write them separately. Afterwards add the third elongation to the position of the Sun, and enter what results in the aforementioned table of rising times [in setting] of the zodiacal signs. Write what you find of the rising times [in setting] together with the rising times [in setting] of the Sun, each species with its species, and subtract the lesser from the greater. What remains is that which is called the fourth elongation. Write it separately with its name.
>
> After that, know for yourself the first latitude of the Moon and take two thirds of it continually and write it separately, and it is called the quota of the zenith of the city (*m^{e}nat goḇah ha-m^{e}dinah*). If the latitude of the Moon is northern, add the quota of the geographical latitude of the zenith of the city to the fourth elongation. If the latitude of the Moon is southern, subtract the quota of the geographical latitude from the fourth elongation. Whatever the fourth elongation is after you add to it or subtract from it, write it separately and it is called the arc of visibility (I, 21a).

The fourth elongation is in equatorial degrees and, as such, could be converted to hours and minutes of time. Bashyatchi outlined the history of using the portion of the local zenith to compute the arc of visibility:

> Among what you need to know is that the scholar Maimonides followed this school of thought. The scholar Rabbi Abraham Ibn Ezra of blessed memory did not mention the variation (*ḥillup̱*) in longitude or the difference in latitude, and he also did not mention

this deviation from the arc of the distance and when it is southern, add this deviation to the arc of the distance." Islamic astronomers, according to Bashyatchi (loc. cit.), did notice this deviation.

> the deviation (*naliza*) of the circle [of the Moon]. He did say that when the Moon has a latitude we need, forever, to take 2/3 of it, either to subtract or to add it, according to the aforementioned conditions.
>
> Indeed, what appears to me is that one needs to take 2/3 of the first latitude if the Moon has a deviation of the circle (*n^e^lizat ha-ma'agal*), for the deviation in latitude in visibility has already been calculated. If, however, the Moon has no deviation of the circle, then its deviation in latitude in visibility is not calculated. This is impossible because in every way it is forever appropriate that there be calculated a deviation in visibility in latitude for the arc of visibility. Thus it is appropriate, if the Moon has a deviation of the circle, to take 2/3 of the first latitude according to how through the deviation of the circle, the deviation of the the latitude is corrected.[23] If, though, the Moon shall not have a deviation of the circle, we need to take 2/3 of the second latitude through which the deviation of the latitude is corrected.
>
> We call, indeed, 2/3 of the latitude the quota of the zenith of the city on account of how the degree of the ecliptic orb in which the Moon is is at the horizon and the Moon has no latitude at that time at which it is at the horizon and it sets with the setting of that degree (I, 21a).

K. Finally on 32b of MS C and on 21a of the 1531 printed edition (I), there is a small table for the limits of visibility entitled "The Table of the Limits (*Q^e^ṣei*)[24] of Visibility in Constantinople." Depending on the measure of the first elongation, different minima for the arc of visibility are necessary. In other words, if both conditions obtain, then the lunar crescent will be visible. Maimonides' *Sanctification* had correlated the first elongation and the arc of visibility, but gave different parameters (Gandz 1956, 71). The table in *AE* is:

first elongation		arc of visibility	
13°	50′	8°	10′
12°		10°	
11°		11°	
10°		12°	
8°	10′	13°	50′

Bashyatchi prefaced the table by writing:

> After you know the corrected arc of visibility through the conditions that we have mentioned, know for yourself the limits of visibility. They are, if the arc of visibility is greater than 8°10′, and the first elongation greater than 13°50′, know that the Moon will be seen. If the arc of visibility is less than this and the elongation is less than this, then know that it will not be seen. Thus, if the arc of visibility is greater than 10° and the first

[23] See (Gandz 1956, 69). Maimonides said here that one should *always* take two thirds of the first latitude.
[24] (I, 21a) reads: *ḥeiṣi*.

> elongation is greater than 12°, know that the Moon will be seen, and if they are less than this, it is not seen.
>
> Thus, if the arc of visibility is more than 11° and the first elongation is more than 11°, know that the Moon will be seen.[25] Thus if the arc of visibility is more than 12° and the first elongation more than 10°, know that the Moon is seen. If the arc of visibility is greater than 13°50′, and the first elongation is more than 8°10′ and know that the Moon is seen at the horizon of Constantinople whose latitude from the equator is 41°30′ (I, 21a).

That chapter, the final one that addresses the use of the lunar visibility tables, concludes with a discussion of conditions that might prevent a new Moon that was calculated to appear from appearing (I, 21a-21b). This could be due to clouds or other meteorological phenomena, or even a geological feature such as a mountain obstructing observations. *AE* also acknowledged that there were times when the new Moon appeared even when it was calculated not to be visible. That could be due to different meteorological phenomena, such as exceptionally pure and humid air, or because the location was much higher than surrounding locations. For the same reasons, stars would appear larger at the eastern or western horizons than they would at the center of the heavens, as there are more vapors, rising up from the earth, at the horizons. He wrote:

> This is why the masters of the tradition (*ba'alei ha-qabbalah*) said "the Sun knows its coming; the Moon does not know its coming" [Psalm 104:19] on account of how the processings of the Moon are variable. Thus our scholars, upon them be peace, said to act in accord with (*l^{e}-hitnaheig*) the judgments of the approximations in waxing and waning and to distance oneself from these calculations.[26] God forbid that we say that we use, for the sanctification of the months, these judgments. Rather, what we have mentioned is only that we not be known by our deficient stupidity in this; as well due to their having said it is not appropriate to forbid a great good on account of a small evil; as well to inform our contemporaries that the way that they follow in the judgments of the months is neither by approximation nor by observations due to them saying that between the time of the true conjunction and the time of setting 18 [hours] or at the least, 17 hours until until the arc of visibility is correct. From 18 [hours] and up they say that it is always appropriate to sanctify with the Moon being observed.
>
> They did not know, sometimes, between 11 1/2 and a little less that it would be seen. Sometimes, at 25 1/2 it is not seen. In many times people of perfect knowledge commented that they saw the Moon with there being a little more than 11. 1/2 hours between the conjunction and setting until they got confused and said that it was connected to vapors and a likeness of the Moon was seen in the firmament. Due to its speed it was

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[25] (King 2004-2005, I.695) reported that many Muslim astronomers adopted the Indian parameter of a twelve degree difference in setting times for the limit of whether the crescent would be visible. King studied a number of Islamic tables for lunar crescent visibility in (King 1987). For more such tables, see (Hogendijk 1988, 29-44).

[26] The tradition to which *AE* referred is the system of signals used to communicate the sighting of the New Moon discussed in *Babylonian Talmud, Rosh ha-Shanah*, 23b.

> seen with the Sun and it served for 2 hours.[27] Calculation extracted the arc of visibility according to the knowledge of Rabbi Aaron b Elijah of Nicomedia (d. 1369; lit: Aaron the author of *Eiṣ ḥayyim*) at 9° and they were amazed by this and returned from their first doctrine. Behold these are the ways of approximating and the ways of seeing the life and the good, and you should choose life (I, 21b).

It is striking that *AE* referred to a rabbinic source as "the tradition."

L. The 19th century and 1966 printed editions include some tables not found in the 1531 printed edition (I) nor in early MSS, such as a table of the positions of the two luminaries and their *argumenta* with many sexagesimal places (OR, 38-39). This table also contains columns for the mean lunar and solar positions. For example, in a table with the mean nodal and lunar motions in anomaly for months and for the cycles of 19 years, the mean lunar motion in anomaly is given to twelfths, and the mean nodal motion in anomaly is given to eighths (OR, 43). To illustrate how this printed edition of *AE* used the same entries for the mean motions at intervals of seconds (second from top) and thirds (at the top), here is the part of the table for mean solar position:

ninths	eighths	sevenths	sixths	fifths	Fourths	thirds	for thirds
eighths	sevenths	sixths	fifths	fourths	Thirds	seconds	for seconds
30	27	51	50	27	2	0	1
0	55	42	41	55	4	0	2
30	22	34	32	23	6	0	3
0	50	25	23	51	9	0	4

The next two pages contained the same table with the argument of minutes (instead of seconds and thirds) (OR, 40-41). Aside from the increased precision, the entries do not diverge much from the 1531 printed edition (OR, 44). For instance, in each 19-year cycle, the time of the conjunction advanced 2d 16h 31min 45sec in the 1531 printed edition, a value calculated from the table of the conjunctions per year, and 2d 16h 33min 3sec in the modern printed edition. The differences means that scholars re-computed the tables. Finally, that table of 19-year conjunction cycles is transposed for the Crimean peninsula (OR, 48). The intervals between values are the same, but the value for the position of the luminaries varies by an hour for the 300th 19 year cycle. At the bottom of that page is another table for the monthly motions, and the intervals between entries are the same as in the earlier table, although the entries themselves are not (OR, 43-44). That table also has rows for those three columns over each year of a 19-year cycle. There is also a table of the hours of mid-day for latitude 45° and a table of rising times for that latitude (45°) to go along with the tables for hours of mid-day and setting times for the latitude of Constantinople found in the 1531 printed edition (OR, 45-46).

[27] (C, 33a): induced to be.

The major difference between the earlier and later printed editions of *AE*'s instructions for using the tables is an extensive insertion in Chapter 22, right after "it is the conjunction of Tishri in the year 5240 in the horizon of Constantinople (I, 13b)." The insertion, found in slightly different forms in the 1835 and 1966 printed versions, began (in the 1966 version)[28]: "Inasmuch as the table of the roots upon which are added the numbers of the aforementioned motions, its time has already passed, so my lord, teacher and rabbi, my respected teacher the rabbi David the astronomer Qoḵizoḇ saw fit to calculate and to print tables of the mean speeds of the luminaries and their *argumenta* from creation to four hundred cycles and he called them 'the days of old' (Deuteronomy 32:7),[29] and brief general rules from the essence of the new Moon and the rest of the information necessary to extract the new Moons and the arc of visibility and he called them the manners of the world (*haliḵot 'olam*)" (OR, 26). "The days of old" and "the manners of the world" were the names of the two sections of this chapter. The first section explained how and why the months were divided into months of 29 and 30 days and how and why the cycles of ordinary and embolismic years were computed. The second part begins with definitions of 58 concepts from astronomy that are relevant for using the tables. For instance, Bashyatchi described his lunar model, which was the simple Ptolemaic lunar model (OR, 31). The addition of this exhaustive list of terms, to go with tables computed to a greater number of significant digits suggested that the producers of the latest printed edition hoped that readers of *AE* would continue to compute the calendar.

[28] After this opening explanation, the rest of the insertion was the same in all of the printed editions that I examined. The differences in the opening portion could be due to the fact that the scholar David b. Mordeḵai Qoḵizoḇ (1777-1855) died between the production of the 1835 and 1966 printed editions.

[29] This is a reference to Deuteronomy 32:7.

II. Sample Calculation for Iyyar, 5311
(with recourse to table entries not included in the edited tables)

N.B. “s”, in expressions of time, means *seconds*; in positions, it means *[zodiacal] sign*.

A. Determine the date of the mean conjunction

Tishrei, 5311:	4d 16h 44m 14s
Add for Iyyar, 5311:	3d 17h 08m 21s
=	1d 09h 52m 35s

Determine the location of the mean conjunction for Iyyar, 5311:

Tishrei, 5311:	6s 00; 33, 56°
Add for Iyyar, 5311:	6s 23; 44, 55°
=	0s 24; 18, 51°

Determine the mean solar, lunar, and nodal *argumenta* for Iyyar, 5311

1. Solar *argumentum* on 1 Tishrei, 5311

	2s 27; 59, 39°
Add for Iyyar, 5311:	6s 23; 44, 20°
=	9s 21; 43, 59°

2. Lunar *argumentum* on 1 Tishrei, 5311

	6s 16; 05, 23°
Add for Iyyar, 5311:	6s 00; 43, 00°
=	0s 16; 48, 23°

3. Lunar nodal *argumentum* on 1 Tishrei, 5311

	0s 08; 12, 50°
Add for Iyyar, 5311:	7s 04; 41, 55°
=	8s 12; 53, 45°

B. Determine the solar equation and hourly velocity

Because the solar *argumentum* is 9s 21; 43, 59° (≈ 9s 22°), Table B yields an equation of 1; 49, 0° and an hourly velocity of 0; 2, 26°

Because the Sun is moving from its perigee to its apogee, one adds the equation from the mean position:

mean position:	0s 24; 18, 51°
equation:	1; 49, 00°
=	0s 26; 7, 51°

C. Determine the lunar equation and hourly velocity
Because the lunar *argumentum* is 0s 16; 48, 23° (≈ 0s 17°), Table C yields an equation of 1; 26, 5° and an hourly velocity of 0; 30, 24°

Because the Moon is moving from its apogee to its perigee, one adds the equation to the mean position:

mean position:	0s 24; 18, 51°
equation:	1; 26, 05°
=	0s 25; 44, 56°

D. Now that we have the true position of the luminaries at the moment of the mean conjunction, Table D approximates the positions of the luminaries at the time of the true conjunction.

As the Moon is ahead of the Sun, we subtract the true position of the Sun from the true position of the Moon to obtain the elongation:

position of sun	0s 26; 7, 51°
position of moon	−0s 25; 44, 56°
=	0; 22, 55° ≈ 0; 23°, requiring interpolation between the entries for 0; 18° and 0; 24°

The table says that the observed motion between the mean and true conjunctions for the Moon is 3; 32, 20° and for the Sun is 0; 16; 20°. Because the Sun was ahead of the Moon, we add the observed motions to yield the luminaries' positions at the time of the true conjunction:

1. Moon

(At mean conjunction)	0s 25; 44, 56°
+	0; 25°
=	0s 26; 9, 56°, ≈ 0s 26; 10°

2. Sun

(At mean conjunction)	0s 26; 7, 51°
+	0; 1, 55°
=	0s 26; 9, 46°, ≈ 0s 26; 10°

3. We also find the true position of the nodes by first finding the difference between the mean and true conjunctions:

Mean conjunction:	0s 24; 18, 51°
True conjunction:	0s 26; 10°
=	1; 51, 9°

Since the true conjunction was after the mean conjunction, we add this difference to the nodes' *argumentum*:

Argumentum: 8s 12; 53, 45°
\+ 1; 51, 9°
= 8s 14; 44, 53° (approximate position of the nodes at true conjunction)

E1. Table E1 determines the time between the mean and true conjunctions.
From §D, the elongation is ≈ 0; 23°, which means interpolating between 0; 20° and 0; 30°
From §C, the lunar hourly velocity is 0; 30, 24°
From §B, the solar hourly velocity is 0; 2, 26°
The difference in velocities is 0; 27; 58° (≈ 28°)

We enter both values into the table and find that the true conjunction occurred 52m after the mean conjunction.

From §A, we found that the mean conjunction of Iyyar, 5311 was
1d 9h 52m 35s
\+ 0h 52m
= 1d 10h 45m (time of true conjunction)

E2. Table E2 determines whether the true conjunction was during the day or night.
The true conjunction was at 0s 26; 10° (≈ 0s 26°), so there are 6h 36m 30s between sunrise and mid-day and 6h 36m 0s between mid-day and sunset. Thus, the conjunction is after sunset.

F. Karaites defined nightfall from the beginning of the third evening, which is the end of twilight. The duration of twilight, from Table F, should be added to the hours from mid-day to sunset:

6h 36m 30s
\+ 1h 49m
= 8h 25m 30s (the time of visibility to be investigated)

Determine the time interval between the time of mean conjunction and the time of first visibility:
9h 52m 35s (time of mean conjunction)
− 8h 25m 30s (time of visibility)
= 1h 27m 5s
converted to a system of 40 minutes in an hour and 40 seconds in a minute = 58m 3s[30]

[30] (T, 5b) clarified that one takes 2/3 of the time interval and adds it to the earlier *argumentum* of the Moon.

One subtracts that value, as degrees, minutes, and seconds, from the Moon's *argumentum* at mean conjunction to obtain the Moon's *argumentum* at first visibility:

	0s 16; 48, 23°
–	0; 58, 3°
=	0s 15; 50, 20°, corresponding to an hourly lunar velocity of 0; 30, 23°

Following Chapter 30 of *AE*, we determine the position of the Sun, Moon, and nodes at the time to be investigated.

The time between true conjunction (1d 10h 45m) and visibility (1d 8h 26m) is: -2h 21m, so we multiply approximately
21h 40m × 0; 30, 23° (lunar hourly velocity at first visibility) = 10; 58, 18° (amount Moon has moved between true conjunction and first visibility on the subsequent day)

Lunar position at visibility:
0s 26; 10° (position at true conjunction on the next day) + 10; 58, 18° = 1s 7; 8, 18°

Solar position at first visibility:
21h 40m × 0; 2, 26° (solar hourly velocity) = 0; 52, 43° (amount Sun has moved in that time interval)

0s 26; 10° (solar position at conjunction) + 0; 52, 43° = 0s 27; 2, 43°

To find the *argumentum* of the nodes at visibility, add 2; 44, 4° (amount Moon has moved in that time interval), plus 3 minutes, to the *argumentum* of the nodes at true conjunction:
8s 10; 51, 34° + 11; 1, 18° = 8s 21; 52, 52°

G. Enter the *argumentum* of the nodes at true conjunction into Table G to find the first latitude:
4; 57° (southern)

The first elongation is the separation between the Moon and Sun at first visibility:

	1s 7; 8, 18° (lunar position at first visibility)
–	0s 27; 2, 43° (solar position at first visibility)
=	10; 5, 35°

H. Table H corrects the first longitude and first latitude for lunar parallax
Following the instructions in Chapter 32, we subtract the parallax in longitude from the first elongation

	10; 5, 35°
–	0; 23, 00°
=	9; 42, 35° (second elongation)

Because the Moon’s latitude is to the south, we add the parallax in latitude to the first latitude:

	4; 57°
+	0; 20°
=	5; 17° (second latitude)

I. Table I takes account of the deviation of the circle to derive the third elongation:
As the Moon is at 1s 7; 8, 18°, with a southern latitude, one takes 1/3 of the second latitude and adds it to the second elongation:
5; 17° × 1/3 = 1; 45, 40°

	9; 42, 35° (second elongation)
+	1; 45, 40°
=	11; 28, 15° (the third elongation)

J. Finally, one uses Table J, for setting times, to compute the fourth elongation:
Position of the Sun: 0s 26; 10°
Setting time: 33; 38° (interpolated)

Position of the Sun:	0s 26; 10°
+	11; 28, 15° (the third elongation)
=	1s 7; 38, 15° (position of the Moon reflecting the third elongation)

Setting time: 49° (interpolated)

Fourth elongation is 49° − 33; 38° = 15; 22°

According to *AE*, the fourth elongation is adjusted by the quota of the zenith of the city (*m*^{e}*nat gob̲ah ha-m*^{e}*dinah*). Because, in this case, the Moon did have a deviation of the circle, one takes 2/3 of the first latitude:
2/3 × 4; 57° = 3; 18°

Because the Moon’s latitude was southern, we subtract that quota from the fourth elongation, yielding 12; 4° for a modified fourth elongation.

Common sense and “The Table of the Limits of Visibility in Constantinople” found in §K both dictate that the new Moon will be seen at nightfall on the day following the one we originally investigated..

III. Key Conclusions

A. Bashyatchi most likely accessed at least some of Battānī's *zīj* through an Arabic MS and, thus, follows in the trend of Romaniot Jews who did not necessarily know Arabic well nevertheless procuring and accessing Arabic MSS. Other examples include Moses b. Elijah Galeano and Elijah Mizraḥi.[31]

B. The twilight table did not come from Bashyatchi's cited sources.

C. Bashyatchi modified Maimonides' technique and tables for computing the arc of visibility.

D. There are numerous commentaries on *Adderet Eliyahu* and studying them would provide more information about how the tables were used. MS NY JTSA 2597, 2a-14a, is a 1580 commentary on the tables in *AE*, including a worked calculation. MS SP IOS A96, 34a-54b, is a 17th century commentary on *AE* that focuses on the calendar. More commentaries were written in later centuries and I hope to examine them in a future publication.

E. This article has focused on the tables themselves. More work on the techniques of lunar crescent calculation found in *AE* and the commentaries on *AE* is necessary.

[31] On Moses b. Elijah Galeano, see (Morrison 2013). On Elijah Mizraḥi, see (Langermann 2012, 446-447).

Table A

		Time of the New Moon				Place of the luminaries' conjunction				Mean solar anomaly		
cycle	year	days	hours	minutes	seconds	signs	degrees	minutes	seconds	signs	degrees	minutes
10s	5292	2	0	12	29	6	0	26	1	2	28	11
11i	5293	0	21	45	8	6	18	49	26	3	16	33
12s	5294	5	6	33	44	6	8	6	26	3	5	49
13s	5295	2	15	22	20	5	27	23	26	2	25	5
14i	5296	1	12	54	59	6	15	46	51	3	13	27
…...	…...	…...	…...	…...	…...	…...	…...	…...	…...	…...	…...	…...
8 i	5309	2	23	7	2	6	21	59	56	3	19	27
9 s	5310	0	7	55	38	6	11	16	56	3	8	43
10 s	5311	4	16	44	14	6	0	33	56	2	27	59
11 i	5312	3	14	16	53	6	18	57	21	3	16	21
12 s	5313	0	23	5	59	6	8	14	21	3	5	37
	simple year											
	Ḥeshvan	1	12	44	50	0	29	6	25	0	29	6
	Kislev	3	1	28	6	1	28	12	50	1	28	12
	Ṭeib̲et	4	14	12	9	2	27	19	15	2	27	19
	Sheb̲aṭ	6	2	56	12	3	26	25	40	3	26	25
	Adar	0	15	40	15	4	25	32	5	4	25	31
	Nisan	2	4	24	18	5	24	38	30	5	24	28
	Iyyar	3	17	8	21	6	23	44	55	6	23	44
	Sivan	5	5	52	24	7	22	51	20	7	22	50
	Tammuz	6	18	36	27	8	21	57	45	8	21	57
	Ab̲	1	7	2	30	9	21	4	10	9	21	3
	Elul	2	20	4	33	10	20	10	35	10	20	9
	Tishrei	4	8	48	36	11	19	17	0	11	19	16
		5	21	32	39	0	18	23	25	0	18	22

	Mean lunar anomaly				Mean nodal anomaly				
seconds	signs	degrees	minutes	seconds	signs	degrees	minutes	seconds	
19	8	9	10	23	11		54	55	
39	7	14	47	23	1	8	40	2	
39	5	24	35	23	1	16	45	20	
39	4	4	23	23	1	24	50	2	
59	3	10	0	23	3	3	35	45	
……	……	……	……	……	……	……	……	……	
39	9	26	29	23	11	23	2	50	
39	8	6	17	23	0	0	7	50	
39	6	16	5	23	0	8	12	50	
59	5	21	42	23	1	17	58	15	
59	4	1	30	23	1	25	3	15	
									leap year
20	0	25	49	0	1	0	40	25	Ḥeshvan
40	1	21	38	0	2	1	20	50	Kislev
0	2	17	27	0	3		1	15	Ṭeiḇet
20	3	13	16	0	4	2	41	40	Sheḇaṭ
40	4	9	5	0	5	3	22	5	Adar
0	5	4	54	0	6	4	1	30	Adar II
20	6	0	43	0	7	4	41	55	Nisan
40	6	26	32	0	8	5	21	20	Iyyar
0	7	22	21	0	9	6	2	45	Siwan
20	8	18	10	0	10	6	42	10	Tammuz
40	9	13	59	0	11	7	22	35	Aḇ
0	10	9	48	0	0	8	2	0	Elul
20	11	5	37	0	1	8	43	25	Tishrei

Table B

		sign 0				
descending	ascending	equation			motion	
		degrees	minutes	seconds	minutes	seconds
0	30	0	0	0	2	23
1	29	0	2	1	2	23
2	28	0	4	1	2	23
3	27	0	6	1	2	23
4	26	0	8/A=5	2	2	23
……	……	……	……	……	……	……
27	3	0	52	26	2	24
28	2	0	54	14	2	24
29	1	0	56	2	2	24
30	0	0	57	49	2	24
		sign 11				
		sign 3				
0	30	1	59	3	2	28
1	29	1	59	8	2	28
2	28	1	59	10	2	28
3	27	1	59	8	2	28
4	26	1	59	3	2	28
……	……	……	……	……	……	……
27	3	1	47	45	2	30
28	2	1	46	50	2	30
29	1	1	45	55	2	30
30	0	1	44	56	2	30
		sign 8				

			sign 1					sign 2	
	motion		equation			motion		equation	
seconds	minutes	seconds	degrees	minutes	seconds	minutes	seconds	degrees	minutes
0	2	23	0	57	49	2	24	1	41
1	2	23	0	59	35	2	24	1	42
1	2	23	1	1	19	2	24	1	43
1	2	23	1	3	3	2	24	1	44
2	2	23	1	4	46	2	24	1	45/A=48
……	……	……	……	……	……	……	……	……	……
26	2	24	1	37	59	2	25	1	58
14	2	24	1	39	5	2	25	1	58
2	2	24	1	40	14	2	25	1	58
49	2	24	1	41	14/C=16	2	25	1	59
			sign 10					sign 9	
			sign 4					sign 5	
3	2	28	1	44	57	2	30	1	1
8	2	28	1	43	53	2	30	0	59
10	2	28	1	42	50	2	30	0	57
8	2	28	1	41	47	2	30	0	55
3	2	28	1	40	43	2	30	0	53
……	……	……	……	……	……	……	……	……	……
45	2	30	1	6/C=7	46	2	32	0	6
50	2	30	1	5/C=4	0	2	32	0	4
55	2	30	1	3/C=4	13/C=0	2	32	0	2/C=4
56	2	30	1	1/C=5	24	2	32	0	0
			sign 7					sign 6	

Table C

		sign 0					sign 1		
		equation			velocity		equation		
ascending	descending	degrees	minutes	seconds	minutes	seconds	degrees	minutes	seconds
0	30	0	0	0	30	18	2	19	45
1	29	0	4	50	30	18	2	24/C=23	3
2	28	0	9	40	30	18	2	28	20
3	27	0	14	29	30	19	2	32	34
4	26	0	19	18	30	19	2	36	44/N=43
……	……	……	……	……	……	……	……	……	……
27	3	2	6	43	30	32	4	0	24
28	2	2	11	6/N=5	30	33	4	3	22
29	1	2	15	26	30	34	4	6	16
30	0	2	19	45	30	35	4	9	6
		sign 11					sign 10		
		sign 3					sign 4		
0	30	5	0	0/N=2	32	42	4	31	50
1	29	5	0	26	32	45	4	29	27/N=56
2	28	5	0	44	32	47	4	26	27/N=56
3	27	5	0	54	32	51	4	24	23
4	26	5	0	59	32	53	4	21	44
……	……	……	……	……	……	……	……	……	……
27	3	4	38	31	34	5	2	56	39
28	2	4	36	33/N=23	34	8	2	52	1
29	1	4	34	39/N=9	34	11	2	47	20
30	0	4	31	50	34	14	2	42	36
		sign 8					sign 7		

		sign 2				
velocity		equation			velocity	
minutes	seconds	degrees	minutes	seconds	minutes	seconds
30	35	4	9	6	31	25
30	36	4	11	53/N=33	31	28
30	38	4	14	37	31	30
30	39	4	17	18/N=8	31	32
30	40	4	19(N)/(A,B,C,I)=14	54(N)/A,B,C,I=59	31	34
……	……	……	……	……	……	……
31	19	4	58	16	32	33
31	21	4	59	0	32	36
31	23	4	59	36	32	39
31	25	5	0	0/N=2	32	42
		sign 9				
		sign 5				
34	14	2	42	36	35	31
34	17	2	37	49	35	33
34	20	2	32	56/A,B,N=57	35	35
34	23	2	28/A,B=25	1	35	37
34	26	2	23	2	35	39
……	……	……	……	……	……	……
35	25	0	17	19/N=14,B=9	36	3
35	27	0	11	30	36	3
35	29	0	5	45/B=40	36	4
35	31	0	0	0	36	4
		sign 6				

Table D

degrees	minutes	visible lunar position			visitble solar position		
of distance	of distance	degrees	minutes	seconds	degrees	minutes	seconds
0	6	0	6	30	0	0	30
0	12	0	13	0	0	1	0
0	18	0	19	30	0	1	30
0	24	0	26	0	0	2	0
……	……	……	……	……	……	……	……
2	6	2	16	30	0	10	30
2	12	2	23	0	0	11	0
2	18	2	29	30	0	11	30
2	24	2	36	0	0	12	0

degrees	minutes	visible lunar position			visible solar position		
of distance	of distance	degrees	minutes	seconds	degrees	minutes	seconds
2	30	2	42	30	0	12	30
2	36	2	49	0	0	13	0
2	42	2	55	30	0	13	30
2	48	3	2	0	0	14	0
……	……	……	……	……	……	……	……
4	30	4	52	30	0	22	30
4	36	4	59	0	0	23	0
4	42	5	5	30	0	23	30
4	48	5	12	0	0	24	0

degrees	minutes	visible lunar position			visible solar position		
of distance	of distance	degrees	minutes	seconds	degrees	minutes	seconds
4	54	5	18	30	0	24	30
5	0	5	25	0	0	25	0
5	6	5	31	30	0	25	30
5	12	5	38	0	0	26	0
……	……	……	……	……	……	……	……
6	54	7	28	30	0	34	30
6	0	7	35	0	0	35	0
6	6	7	41	30	0	35	30
6	12	7	48	0	0	36	0

Table E1

30'	31°	0'	31°	30'	32°	0'	32°	30'	33°	0'	33°	30'
minutes	hours	minutes	hours	minutes	hours	minutes	hours	minutes	hours	minutes	hours	minutes
20	0	20	0	19	0	19	0	19	0	18	0	17
39	0	39	0	38	0	36	0	37	0	36	0	35
59	0	58	0	57	0	56	0	56	0	54	0	53
18	1	17	1	16	1	11	1	14	1	12	1	11
38	1	36	1	35	1	32	1	32	1	30	1	29
58	1	58	1	54	1	52	1	52	1	29	1	47
……	……	……	……	……	……	……	……	……	……	……	……	……
7	11	54	11	48	11	34	11	34	11	3	11	3
27	12	13	12	7	11	52	11	53	11	21	11	21
47	12	33	12	27	12	11	12	14	11	37	11	39
6	12	52	12	46	12	30	12	17	12	7	11	57
26	13	11	13	5	12	49	12	50	13	15	12	15
46	13	30	13	24	13	8	13	9	13	33	12	33

Table E2

	5			6			7	
	Virgo			Libra			Scorpio	
seconds	hours	minutes	seconds	hours	minutes	seconds	hours	minutes
0	6	45	0	6	0	0	5	16
0	6	44	0	5	58/A=57,B=56	30	5	15
30	6	42	30/A=0	5	56	0	5	14
30	6	42	0	5	55	0	5	13
30	6	39	30	5	54	0	5	12
……	……	……	……	……	……	……	……	……
0	6	7	30	5	22	0	4	45
0	6	6/B=7	0	5	20/B=22	30	4	44
0	6	4	0	5	19	0	4	43
0	6	2	0	5	17	30	4	42

	8			9	
	Sagittarius			Capricorn	
seconds	hours	minutes	seconds	hours	minutes
0	4	41	0	4	24
0	4	40	0	4	24
0	4	39	30/B=0	4	24
0	4	38	30	4	24
0	4	38	0	4	24
……	……	……	……	……	……
0	4	24	0	4	36
30	4	24	0	4	37
30	4	24	0	4	38
30	4	24	0	4	38

	10			11		
	Aquarius			Pisces		
seconds	hours	minutes	seconds	hours	minutes	seconds
0	4	39(A)/B=38;C,I=34	30	5	16	30
0	4	40	0	5	18	0
0	4	40	0	5	19	30
0	4	41	30	5	21	0
0	4	43	0	5	22	30
……	……	……	……	……	……	……
30	5	10	30	5	54	0
0	5	12	0	5	55	30
0	5	13	30	5	57	0
0	5	15	0	5	58	30

Table F

Zodiacal	Constantinople		Latitude 45°		Latitude 36°		Latitude 30°	
Sign	Hours	Degrees	Hours	Degrees	Hours	Degrees	Hours	Degrees
Aries	1	12	2	1	1	10	1	7
Taurus	1	13	2	2	1	11	1	8
Gemini	2	2	2	6	1	13	1	10
Cancer	2	7	2	10	1	14	1	14
Leo	2	2	2	6	1	6	1	10
Virgo	1	13	2	2	1	8	1	8
Libra	1	12	2	1	1	7	1	7
Scorpio	1	11	2	0	1	6	1	6
Saggitarius	1	10	1	14	1	5	1	5
Capricorn	1	9	1	13	1	4	1	4
Aquarius	1	10	1	14	1	5	1	5
Pisces	1	11	2	0	1	5	1	6/C=5

Recomputation								
Zodiacal	Constantinople		Latitude 45°		Latitude 36°		Latitude 30°	
Sign	Hours	Degrees	Hours	Degrees	Hours	Degrees	Hours	Degrees
Aries	1	10.53	1	12.37	1	8.69	1	7.08
Taurus	1	12.9	2	0.54	1	10.42	1	8.33
Gemini	2	2.59	2	7.56	1	13.59	1	10.55
Cancer	2	5.9	2	13.81	2	0.58	1	11.86
Leo	2	2.59	2	7.56	1	13.59	1	10.55
Virgo	1	12.9	2	0.54	1	10.42	1	8.33
Libra	1	10.53	1	12.37	1	8.69	1	7.08
Scorpio	1	10.44	1	12.15	1	8.71	1	7.19
Saggitarius	1	11.68	1	13.61	1	9.77	1	8.13
Capricorn	1	12.51	1	14.61	1	10.46	1	8.71
Aquarius	1	11.68	1	13.61	1	9.77	1	8.13
Pisces	1	10.44	1	12.15	1	8.71	1	7.19

Table G

ascending	descending	0 north/6 south		1 north/7 south		2 north/8 south	
		degrees	minutes	degrees	minutes	degrees	minutes
0	29	0	0	2	30(N,B)/C,I,R=30/A=34	4	20
1	28	0	5	2	34	4	23
2	27	0	11	2	39	4	25
3	26	0	16	2	44/C=34	4	27
4	25	0	21	2	48	4	30
5	24	0	26	2	52	4	32
6	23	0	31	2	57	4	34(N)/A,B,C,I=39
7	22	0	37	3	1	4	36
8	21	0	42	3	5	4	38
9	20	0	47	3	9	4	40
10	19	0	52	3	13	4	42
11	18	0	57	3	17	4	44
12	17	1	3	3	21	4	45
13	16	1	8	3	25	4	47
14	15	1	13	3	29	4	48
15	14	1	18	3	32	4	50
16	13	1	23	3	36	4	51
17	12	1	28	3	40	4	52
18	11	1	33	3	43	4	54
19	10	1	38	3	47	4	55
20	9	1	43	3	50	4	55
21	8	1	48	3	53/C=23	4	56
22	7	1	53(A,B)/I,C=??	3	57	4	57
23	6	1	58	4	0	4	58
24	5	2	2	4	3	4	58
25	4	2	7	4	6	4	59
26	3	2	12/A=15	4	9	4	59
27	2	2	16	4	12	5	0
28	1	2	21	4	15	5	0
29	0	2	26	4	17	5	0
		5 north/11 south		4 north/10 south		3 north/9 south	

Table H

Signs	Constantinople		Latitude 30°		Latitude 45°			Adjusted parallax for Constantinople		
	longitude	latitude	longitude	latitude	longitude	latitude		longitude	latitude	
0	22	17/B=47	30	41	20	47		22	17	
1	23(N)/A,B,C,I=28	20	31	40	20	47		23	20	
2	28(N)/A,B,C,I=29	29	36	38/A=35	25	45		28	29	
3	36(N)/A,B,C,I=37	38	40	28	32	33		36	38	
4	28	44	36	38	25	45		28	44	
5	23(N)/A,B,C,I=28	46	31	40	20	47		23	46	
6	22	47	30	41	20	47		22	47	
7	23	44	31	40	20	46		23	44	
8	28(N)/A,B,C,I=23	44	36	36	24	45		28	44	
9	36	38	28/A,B=25	26	32	38		36	38	
10	23	29	36	36	24	45		28	29	
11	23	20	31	40	20	46/B=40		23	20	

Table I

five	four	three	two	one	zero	degrees of
eleven	ten	nine	eight	seven	six	the sign
1/3	1/5	0	1/5	1/3	2/5	
						1 to 5
				1/3		
				1/3		
						6 to 10
1/3	1/5	0	1/5	1/3	2/5	
2/5	1/4	1/24	1/12	1/4		
						11 to 20
2/5	1/4	1/24	1/12	1/4	2/5	
	1/3	1/6	1/24	1/5	1/3	
						21 to 25
	1/3		1/24			
			0			
						26 to 30
2/5	1/3	1/6	0	1/5	1/3	

Table J

degrees of	0		1		2		3		4	
the sign	degrees	minutes	degrees	minutes	degrees	minutes	degrees	minutes	degrees	minutes
1	1	18	40	14	79	25	115	27	143	26/C=12
2	2	35	41	33	80	42	116	29	144	12
3	3	53	42	51	81	59	117	32	144	58
4	5	10	42	10	83	15	118	38/A=35	145	42/C=12
5	6	25	56	29	84	31/C=34	119	37	146	29
6	7	44/A,B=45	46	48	85	47	120	40	147	14/C=19
7	8	3	48	6	87	3	121	42	147	59
8	10	20	49	25	88	19	122	45	148	44
9	11	38	50	43	89	35	123	47	149	29
10	12	55	52	2	90	51	124	49	150/C=151	4
11	14	13	53	21	92	20	125	46	150	57
12	15	31	54	39	93	18	126	43	151	37
13	16	49	55	58/A,B=55	94	31	127	40	152	19
14	18	7	57	17	96	44	128	36	153	0
15	19	25	58	36	97	26	129	33	153	41
16	20	43	59	54	98	9	130	29	154	21
17	22	1	61	13	99	22	131	26	155	20
18	23	19	62	32	100	34	132	22	155	42
19	24	37	63	50	101	47	133	19	156	23
20	25	55	65	9	102	59	134	15	157	3
21	27	13	66	25	104	8	135	6	157	41
22	28	39	67	42	105	17	135	57	158	18
23	29	49	69	4	106	25	136	45	158	56
24	31	7	70	22	107	34	137	39	159	33
25	32	27/C=23	71/C=73	40/C=17	108	41	138	30	160	11
26	33	41	72	58	109	51	139	20	160	48
27	35	2	74	8/B=5	110	59	140	15	161	26(A,X)/C=46,I=106,B=56
28	36	20	75	34	112	7	141	1	162	3
29	37	38	76	51	113	15	141	51	162	40
30	38	56	78	9	114	23	142	42	163	17

5		6		7		8	
degrees	minutes	degrees	minutes	degrees	minutes	degrees	minutes
163/C=160	52/C=32	180/C=197	32/C=0	197/C=218	20/C=9	218/C=245	9/C=45
164	28	181	4	197	58	218	59
164	50/B=20	182	37	198	34	219	50
165	35	182	9	199	12	220	40
166	13	182	44/B=42	199	49	221	31
166	45/A,B=48	183	14	200	27	222	21
167	22	183	46	201	4	223	12
167	51	184(A,B)/C=182	19	201	42/A=22	224	3
168	30	184	51/C=250	202	19	224	54
169	30/C=6	185	24	202	57	225	45
169	32	185	56/C=52	203	37	226	41
170	13	186	29	204	18	227	38
170	46	187	1	204	58	228	34
171	20	187	34	205	39	229	31
171	53	188	7	206	19	230	27
172	26	188	46/A,B,C=40	207	0	231	27
172	59	189	14	207	41	232/C=234	20
173	31	189	47	208	43	233	17
174	4	190	21	209	4	234	14
174	36	190	44/B=55	209	46	235	11
175	9	191	29	210	31	236	13
175	41	192	3	211	16	237/C=235	15
176	14	192	38	212	1	238	18
176	46	192	12	212	46	239	20
177	18	193	43/C=47	213	31	240	23
177	57/B=50	194	22	214	17	241/C=245	25
178	24	194	57	215	2	242	28
178	56	195	32	215	48	243	31
179	8/B=28	196/C=216	28(A,X)/C=33,B=8,I=108	216/C=244	33	244/C=280	31/C=35
180	0	196	43	217	17/B=18	245/C=247	37/I=7

9		10		11	
degrees	minutes	degrees	minutes	degrees	minutes
246/C=283	45/C=9	283/C=322	9/C=22	322	22
247	53	284/C=289	26	323	40
249	1	285/C=287	24	324	58
250	9	287/C=285	2	326	19
251	15	288	20	327	17
252	26	289	38/X=28	328	56
253	35/C=34	290	56	330	11
254	43	292	13	331	29
255	32/B=52	293	32	332	42/X=44
256	1	294	51	334	5/C=27
258	13	296/C=??	17	335	23
259	26	297	25	336	41
260/C=208	38/A,B=35	298	48/B=45	337	29
261	51	300	6	339	19
263	3	301	17	340	35
264	16	302	43/B=23	341	53/X=59
265	29/A=49,B=20	304	2	343	11
266	42	305	21	344	29
267	56	306	39	345	47
269	9	307	58	347	5
270	25	309	17	348	22
271	41	310	35/X=3	349	40
272	57	311	54	350	57
274	13	313	12	352	15/X=19
275/A,C=??	29	314	31	353	32
276	45	315	50	354	50
278	1	316	9	356	7
279	18	318	27	357	35
280/C=319	38/A,B=35/C=47	319/C=??	46/C=??	358	42/R=45
281	51	321	4/C=??	360	0

Acknowledgments

I presented an early version of this research at New York University's Institute for the Study of the Ancient World in April, 2019 and thank Professor Alexander Jones for his kind invitation and helpful comments. Richard Kremer of Dartmouth College and Daniel Lasker of Ben Gurion University of the Negev both gave valuable advice at an early stage of the project. I would also like to recognize *SCIAMVS*' two anonymous referees for their erudite comments that certainly improved this article, and thank Nathan Sidoli and Taro Mimura for their guidance. All remaining deficiencies in this article are my own responsibility.

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(Received: May 17, 2019)
(Revised: October 31, 2019)